FUN*da*MENTALS of Design

Topic 10
Bearings

Bearings

Bearings are machine elements that allow components to move with respect to each other. Bearings are used to support large skyscrapers to allow them to move during earthquakes, and bearings enable the finest of watches to tick away happily. Without bearings, everything would grind to a halt, including people, whose joints are comprised of sliding contact bearings!

There are two types of bearings, *contact* and *non-contact*. Contact-type bearings have mechanical contact between elements, and they include sliding, rolling, and flexural bearings. Mechanical contact means that stiffness normal to the direction of motion can be very high, but wear or fatigue can limit their life.

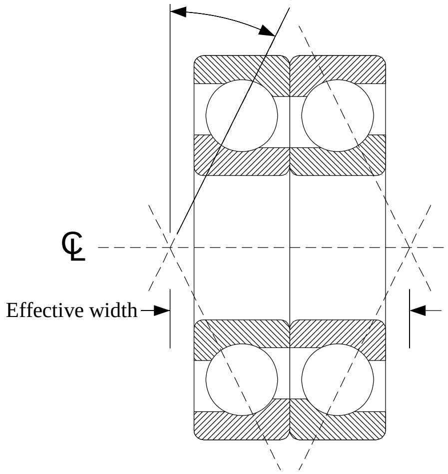
Non-contact bearings include externally pressurized and hydrodynamic fluid-film (liquid, air, mixed phase) and magnetic bearings. The lack of mechanical contact means that static friction can be eliminated, although viscous drag occurs when fluids are present; however, life can be virtually infinite if the external power units required to operate them do not fail.

Each type of bearing has its own niche application area, and thus design engineers must be familiar with different types of bearings, and their applications and limitations.

As with all other types of machine elements, it is important to understand the FUNdaMENTAL operating principles of different bearings in order to select the right bearing for the intended application. Indeed, the FUNdaMENTAL principles of design discussed in Topic 3 are of particular importance to the proper use of bearings in machines. These principles are used extensively in this chapter, although not always called out by name. Seek to identify and apply FUNdaMENTAL principles as you read about bearings!

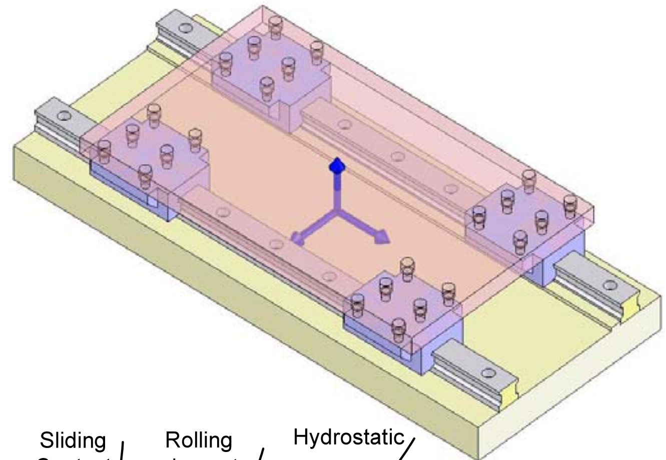
The principle of reciprocity is particularly useful in the design of bearing systems: Whenever you think you have a good design, invert it, think of using a completely different type of bearing or mounting, and compare it to what you originally considered.

Why did some civilizations discover bearings and others did not? Those with bearings moved farther faster, and history has yet to stop. Those with better bearings can make faster, more precise machines which increase productivity and competitive advantage. The future belongs to those who move with speed and accuracy!



Back-to-back bearing mounting

Topic 10
Bearings



Topics

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- Mounting
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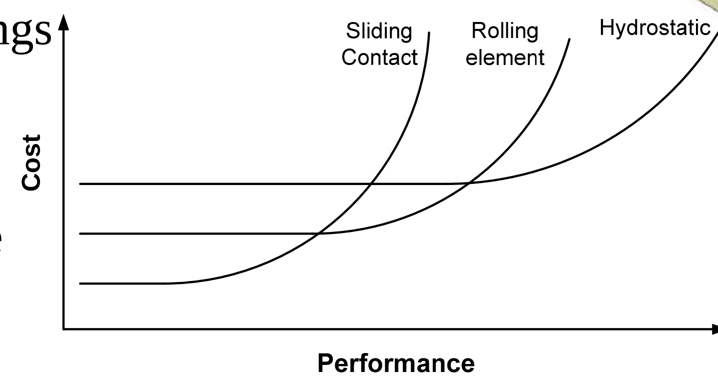

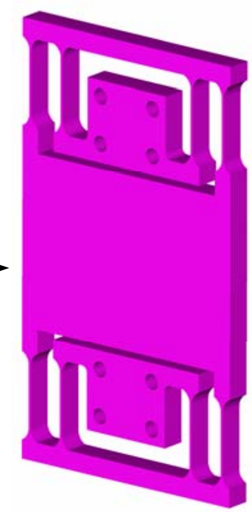

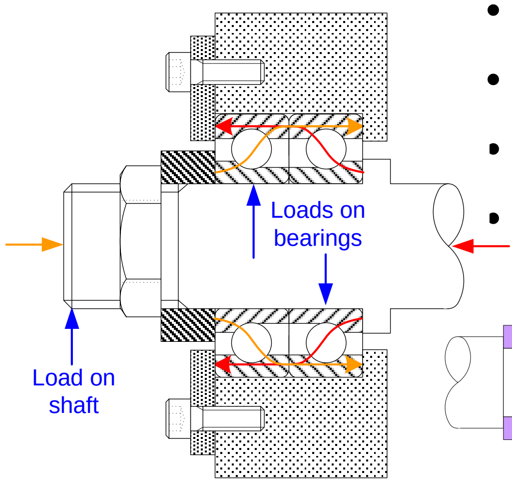

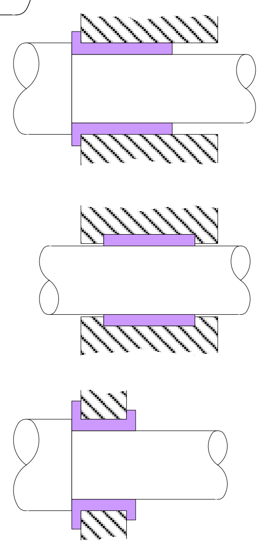

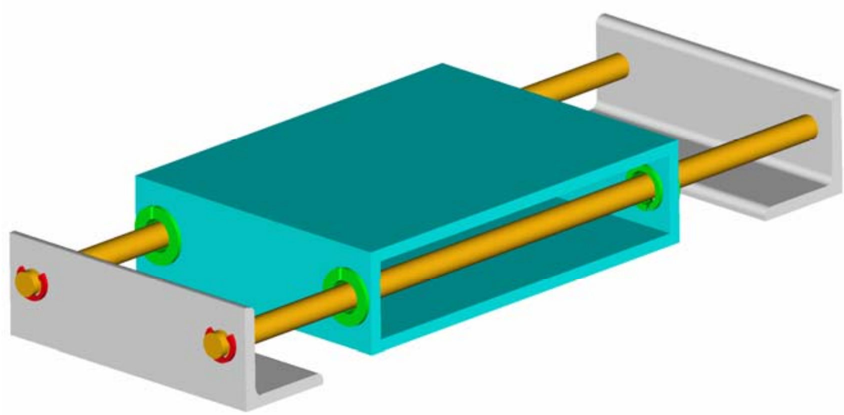

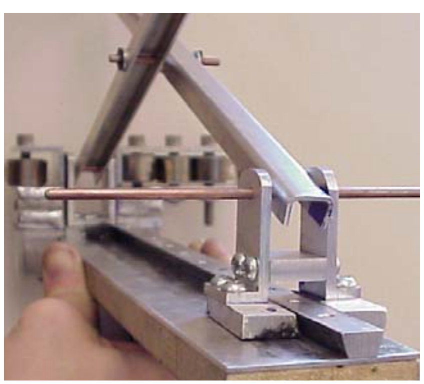

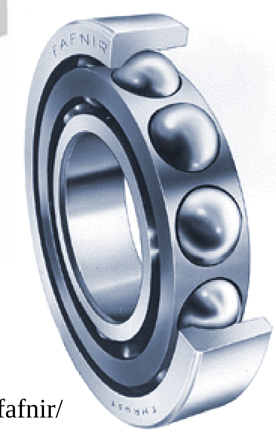

http://www.torrington.com/products/fafnir/

In the Beginning...

The evolution of technology is heavily dependant upon matching observations with needs. Did the idea for a rolling element bearing come from someone noticing a rock rolling down a hill, or from someone falling after stepping on a round log that rolled out from beneath them? Or was it found that a heavy object could be moved by rolling it, which led to the inspiration of putting a heavy object that couldn't roll on top of an object that could roll? And why did some cultures never develop the use of wheels at all, but merely dragged their belongings along on a stretcher-type device where the ends of two poles slide across the ground?

Archimedes discovered the secret of the lever, and perhaps realized that a large diameter wheel on a small diameter axle had little friction. For example, assume you have a cart of mass M supported by four wheels of diameter D on axles of diameter d, with coefficient of friction μ between the axle and wheel. Assume the wheels and ground are both hard, so rolling resistance itself is negligible. To push the cart, a force F on each axle is $Mg/4$ is required. The instant center of rotation is the contact point between the wheel and the ground which is distance $D/2$ from the center of the axle. The condition for rolling is that the moment about the instant center, $F*D/2$ must be greater than the moment due to friction between the axles and the wheels that resists rolling, which is $Mg*\mu*d/2$. Hence the person must push with a force of:

$$F > \frac{Mg\mu d}{D} \qquad \frac{F}{mg} = \frac{\mu d}{D}$$

Now consider the limits of the variables in this analysis: If D>>d, does it really matter that much in terms of a human or an oxen pushing the cart? If the coefficient of friction is at all reasonable, such as when μ = 0.05-0.1 for greased sliding contact bearings, and if D/d is on the order of 20, the force is 200-400 times less than the weight of the cart. Hence one can see why for a long period in human history that sliding contact bearings were perfectly adequate. As machinery advanced, better spindles were also developed to hold tools for drilling and boring, gun barrels and cannons in particular. It became apparent that as speeds increased, the spindles got very hot which used up energy and decreased accuracy. It is not clear who actually invented the first rolling element *anti-friction bearing*, but many different companies began to produce them in the late 1800s. It is interesting to note that the first biggest application for anti-friction bearings was one where power consumption was critical: the bicycle! Automotive and industrial applications, particular in railroads, rapidly developed, and the rest is history.[1]

The performance of a rolling element bearing is a function of the accuracy with which it is made; and the most accurate rolling element bearings are made on machines that use fluid films. In fact, the first machines used to grind rolling element bearing components used *hydrodynamic* bearings, where high speed relative motion between components caused a film of lubricant to build up between them which prevented mechanical contact. Any roughness or waviness of the components' surfaces was obfuscated by the billions and billions of oil molecules that separated them to create the ultimate in elastic averaging (see page 3-28). Hydrodynamic bearings prevent mechanical contact between components, and some large turbines supported by hydrodynamic bearings are still running decades and decades after they were first built.

The principal problem with hydrodynamic bearings, however, is that the thickness of the hydrodynamic layer, and hence the accuracy of the bearing, depends on speed. Perhaps this is what led to the development of *externally pressurized*, or *hydrostatic* bearings. Hydrostatic bearings use a pressurized film of liquid to precisely control the fluid film gap, and they can achieve mechanical accuracies of a few parts per billion. But they still suffer from heat generated by shearing of fluids on small gaps. *Aerostatic* bearings instead use air and have much lower heat generation.

Magnetic bearings go one step further to support a moving component and by using servo controlled electromagnetic fields. High speed centrifuges and natural gas pipeline compressors often use magnetic bearings to deliver the ultimate in speed and reliability. Next generation machines for manufacturing of nanometer-scale semiconductors rely on magnetic bearings.

Moving full-circle, flexural bearings are the ultimate in simplicity because they use material deformation in the place of pivot joints in linkages. From simple short-life consumer products to super precision instruments, flexural bearings are quite common and effective.

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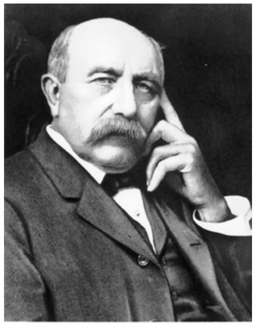
1. Special thanks to Michael N. Kotzalas of The Timken Corporation for his invaluable input, it kept things rolling smoothly along!



Henry Timken
1831-1909

Heinrich Hertz
1857-1894

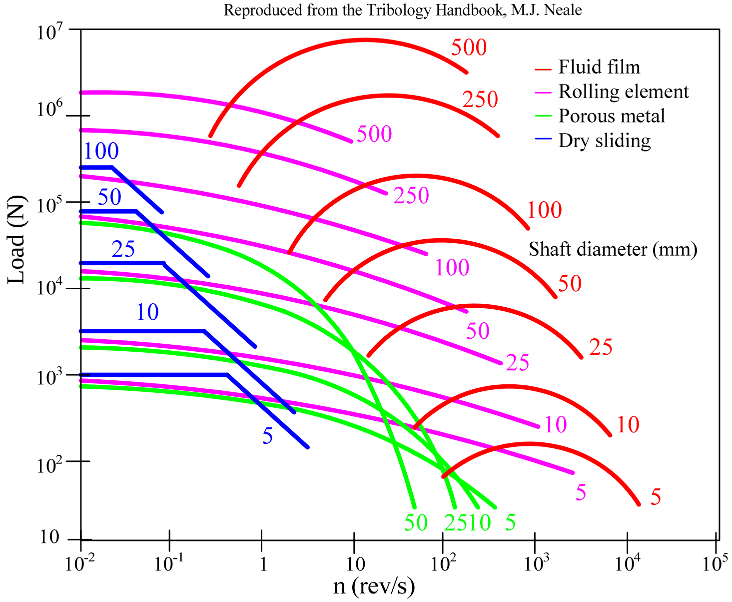
In the Beginning…

- *Bearing* is defined by Webster's to be "a support or supporting part"
  - A *bearing* is a component that allows for relative motion between parts
    - Your skeleton is the central structure that supports your body
    - Your body's joints are bearings that allow different parts to move
- Bearings can have many forms, but only two types of motions
  - Linear motion or rotary motion
    - Mechanical contact bearings: *Sliding, Rolling, Flexing*
    - Non-contact bearings: *Fluid Film, Magnetic*
  - One must understand the flow of power, the path of force transmission, and mechanical constraints
- Take care to select the right bearing for the right application!

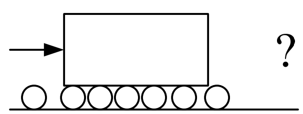
Leonardo da Vinci
1452–1519

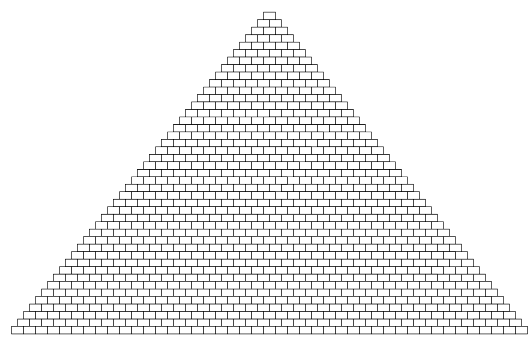
John Harrison
(1693–1776)

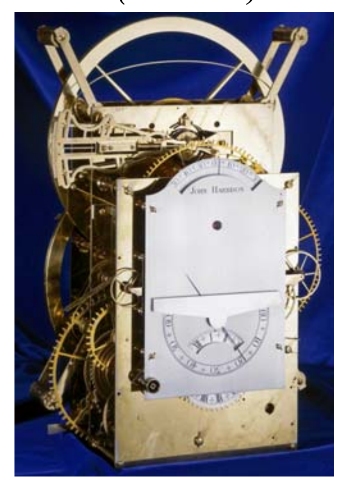
Reproduced from the Tribology Handbook, M.J. Neale



Cleanliness and lubrication are most important







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www.nmm.ac.uk

Contact Bearings: *Sliding Contact*

A bearing allows for relative motion between two surfaces, and perhaps the concept for *sliding contact* (*plain*) bearings were first discovered when Og slipped on a patch of ice? Since then, designers have sought ever more longer lasting slippery (lower friction) surfaces:

- *The product of the normal load and the coefficient of friction generates a friction force that resists motion. The lower the coefficient of friction, the lower the friction force and the greater the efficiency of the system.*
- *The pressure caused by the load acting on the contact area stresses the materials and causes them to slowly wear. The lower the contact pressure and the lower the sliding velocity, the longer the bearing will last.*

Lubrication can range from none on perfectly clean surfaces, which can result in a very high coefficient of friction, to forced lubrication. The former does indeed happen sometimes, but more typically from negligence, or when it is intended to create a brake. The latter is used on critical systems such as those operating at higher speed or with higher precision. For all types of lubrication, the coefficient of friction drops rapidly with increasing speed as a transition is made from mechanical contact to a mixture of sliding mechanical contact and fluid film shearing known as *boundary layer (or mixed) lubrication.* Eventually, as a function of speed and viscosity, hydrodynamic fluid film lubrication dominates, and there is no longer any sliding mechanical contact.

How can you measure the coefficient of friction between two bodies? Remember inclined planes: $\mu = \tan\theta$, where θ is the angle at which the object begins to slide. How is this derived? Start with a free body diagram...

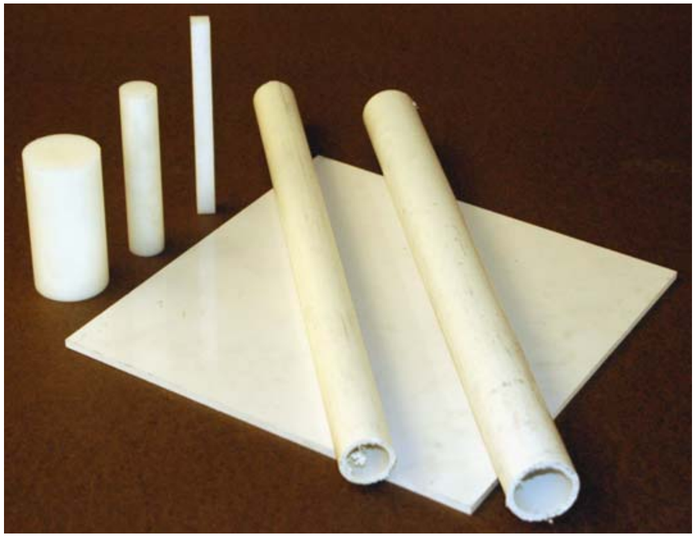
In general, sliding contact bearings do best when the materials in contact are dissimilar: A steel shaft should not run in a steel bore, but it can run in a bronze lined bore (e.g., a bronze bushing). If similar materials must be used, such as often the case in construction equipment where a steel pin runs inside a steel bore, one of the elements should be harder than the other. If one of the elements is harder than the other, wear can be concentrated in only one element, which can then be replaced instead of two. In addition, any abrasive particles tend to get forced into the softer material where they stay and instead of continually abrading the parts, they just wear a groove in one of the parts.

There are some materials, such as some polymers, that are inherently lubricious and do not require additional lubrication. In descending order these include Teflon, Delrin, and Nylon (note that in the presence of moisture, Nylon will rapidly abrade). Because Teflon cannot be injection molded, many molded bearings add Teflon (PTFE) particles to plastics. However, all perform better when some lubricant is applied. Plastic bearings can be easily machined from a solid block. They can also be purchased as specific bearing elements with features such as slit surfaces that allow them to be more easily pressed into a hole to snap in place. As discussed on page 10-29, the surface finish is critical. Ideally the surface has only valleys which act to trap pools of lubricant and no peaks which act as file teeth.

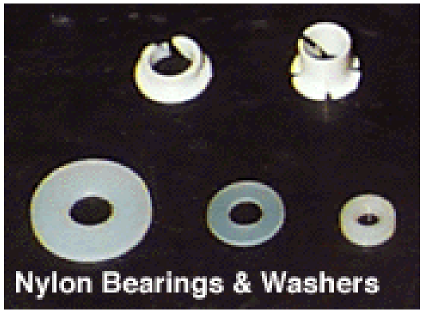
An issue with lubricants, either oil or grease, is that they attract dirt. For example, if you coat your hand with tanning oil and rub it on your leg, it slides smoothly (ahhhh). On the other hand, if you coat your hand with tanning oil, dip it in the sand, and then rub your leg, you will get a serious rash (ouch). Liquid lubricants attract dirt so they must either be used in a relatively clean environment, or seals must also be used to keep out dirt out, or in the case of a bicycle chain, parts must be periodically replaced. It only takes a very thin layer of lube to lubricate a bearing; unfortunately humans often think more is better, and excess lubricant just attracts more dirt.

So how can just the correct amount of lube be applied? Self-help comes to the rescue! Without lube the friction is higher which generates heat which decreases viscosity. This is why a porous bearing impregnated with lubricant can be self-lubricating. Bearings can also be impregnated with lubricants such as molybdenum disulphide or PTFE. These are known as *self-lubricating bearings*, or *self lubricating bushings*.

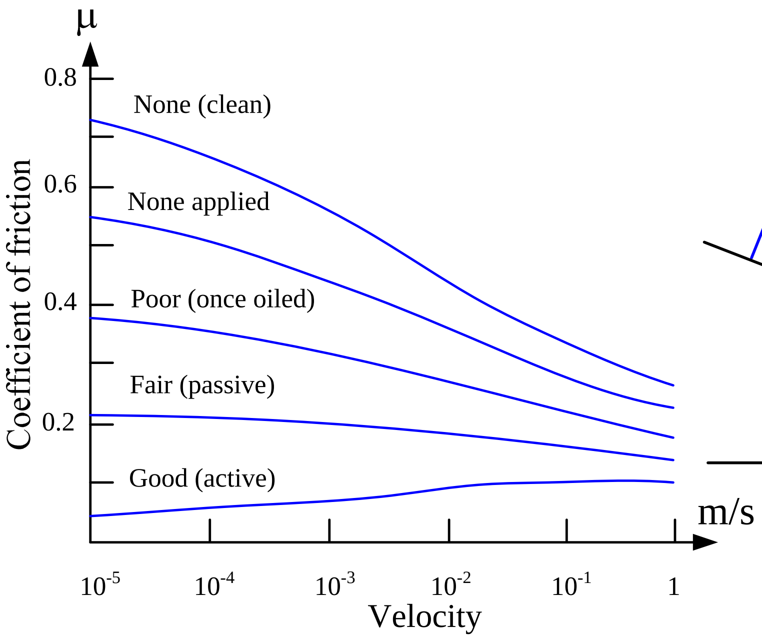
What types of sliding contact bearings do you think you might need in your machine? Create estimates for the forces applied and the efficiency that you can achieve with simple sliding contact bearings. Because sliding contact bearings can be so inexpensive, it is usually best to first try and create your design using sliding contact bearings.

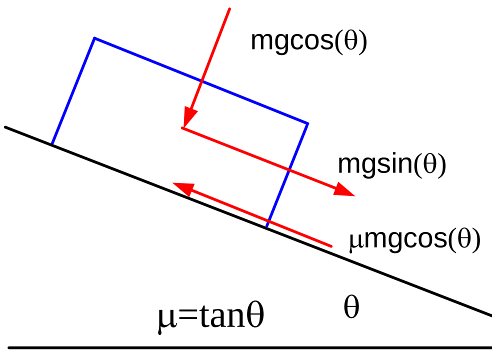


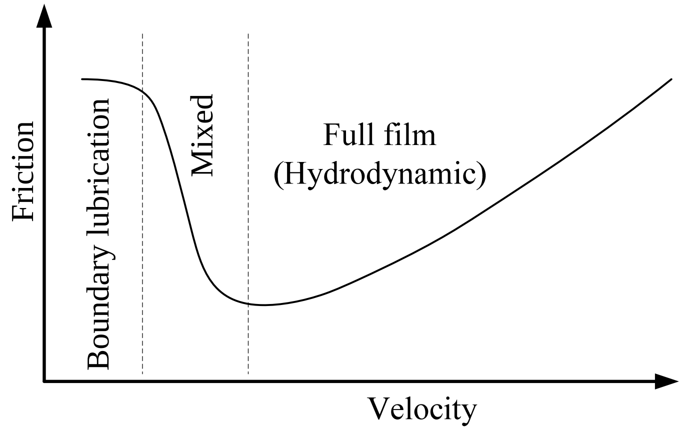
Contact Bearings: *Sliding Contact*



- Sliding contact bearings are commonly used for low-modest speed applications
  - Boundary lubrication reduces wear and friction
- Polymers, brass, and ceramics are commonly used
  - Hard materials (shaft) slide on soft materials (bearing)
  - Check maximum static pressure and also the PV value
  - Aluminum-on-aluminum is to be avoided!







Sliding Contact: *Rotary Motion*

A wheel on a simple axle with sliding contact between the two is perhaps the oldest type of bearing, and yet it is also one of the most common types of bearings in use today. A sliding contact radial bearing element used to support a shaft is called a *bushing*, and sometimes they have integral flanges to also support axial loads. As shown, there are many different types of mountings, which are discussed in detail starting on page 10-21[1]. When a bearing, such as a bushing, is placed in a modular structural housing, the assembly is called a *pillow block*. Hinges are another type of special mounting, and most also have sliding contact bearing elements between the two sides.

Sliding contact bearings have coefficients of friction on the order of 0.05 to 0.1, as compared to rolling element (anti friction) bearings with up to two orders of magnitude lower coefficient of friction. So why not always use rolling elements? The reason is cost. A rotary sliding contact bearing is typically a simple piece of molded plastic with an integral thrust washer face. It can be pressed into a bore, and since they are designed for clearance between the shaft and the bearing, the tolerances on the bore into which it is pressed and that shaft which it supports are not typically difficult to achieve. However, there is always the performance trade-off: Sliding contact bearings have more friction and are less accurate than rolling element bearings.

How good is good enough? Consider the need for a rotary bearing to support a wheel or pulley. When is a sliding contact bearing acceptable? It was shown on page 5-2 (see *Pulley_on_shaft_efficiency.xls*) that the efficiency η rapidly increases as the coefficient of friction μ decreases and the ratio γ of the wheel's outside diameter to inside diameter increases:

$$\gamma = \frac{D_{\text{outside diameter}}}{d_{\text{inside diameter}}}$$

$$\eta = \frac{\gamma - \mu}{\gamma + \mu}$$

1. These references to future page numbers, as opposed to putting all the information here in this section, are presented so that mounting, load-life...can be discussed as their own sections which then include subsections on sliding contact, rolling element, etc. Such is design itself: rarely is all information in one place, it is often distributed as in a treasure hunt!

In fact, when the ratio γ of the wheel's outside diameter to inside diameter is *5*, much of the potential gain in efficiency has been obtained. Saint-Venant applies here too!

What about at high speed? When does it make sense to move from a sliding contact bearing to a rolling element bearing? A 95% efficiency may be more than adequate for the bearings on a linkage for a bulldozer. On the other hand, a 5% loss of energy on a 5 kilowatt engine would not be desirable if only for the simple reason that 250 Watts of power would be dissipated as heat, which is not only a waste, it significantly contributes to the temperature rise of the system.

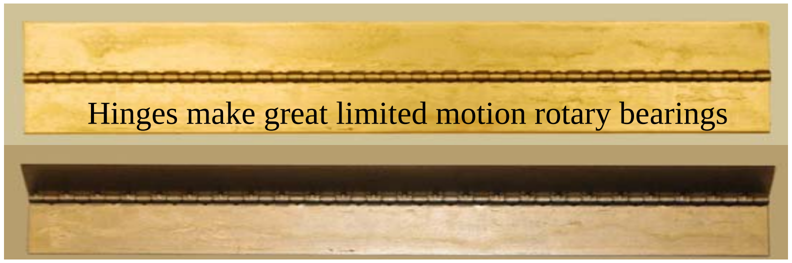
Heat generated in a bearing must be dissipated by conduction to other parts of the machine and by convection to the environment. A basic course in heat transfer is needed to estimate heat transfer from a system, but a very rough estimate (VRE) of the combined heat transfer coefficient for such a system is typically about 20 Watts/m^2/^{o}C. In other words, if a bearing has a surface area of 0.05 m^2, for every Watt of power that must be dissipated, the bearing temperature will rise 1^{o}C.

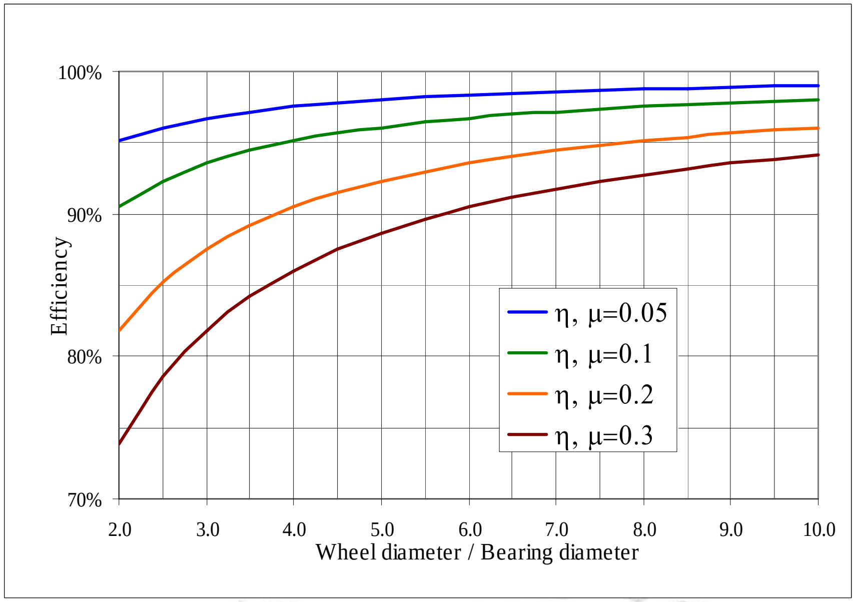
Other important considerations are the type of lubrication, materials, surfaces finish and the applied loads and speeds. The effect of contact pressure and relative velocity on the life of a sliding contact bearing is discussed in detail on page 10-31. The spreadsheet *Sliding_contact_rotary.xls* uses the analysis to help the designer deterministically design their system to use sliding contact rotary bearings.

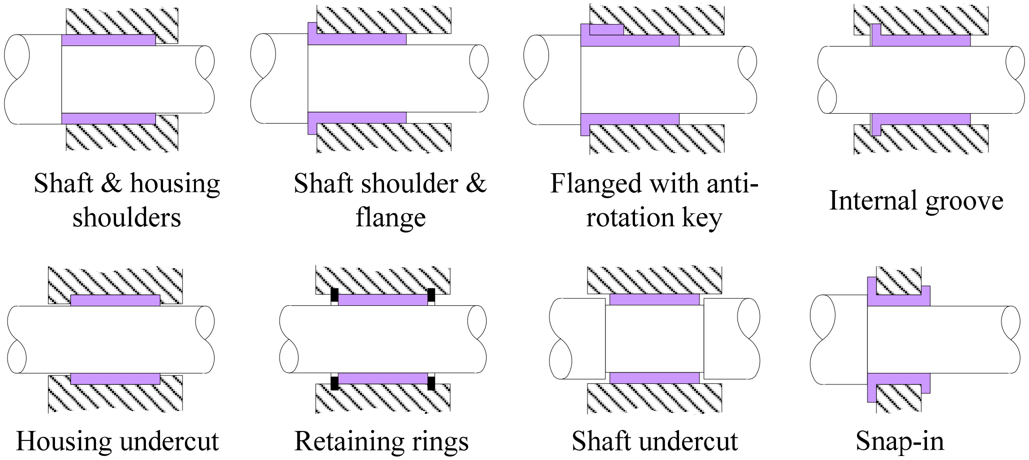
Evaluate the power likely to be dissipated by bearings in your machine. The product of the radial load, radius, and friction coefficient yields the drag torque generated by a sliding bering. The product of the drag torque and velocity in radians per second yields the drag power of the bearing. The drag power can be significant, particularly at high speeds. This is a critical part of the power budget for your machine (see page 7-29). Achieving a balance between the extra power required to operate a machine with sliding element bearings and their lower cost is the engineer's goal.

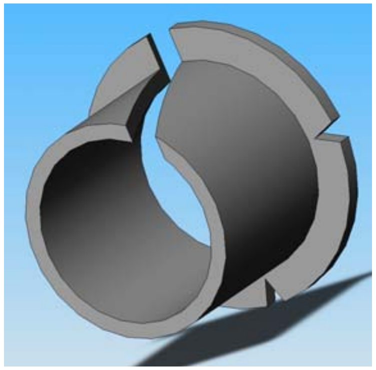
Sliding Contact: *Rotary Motion*

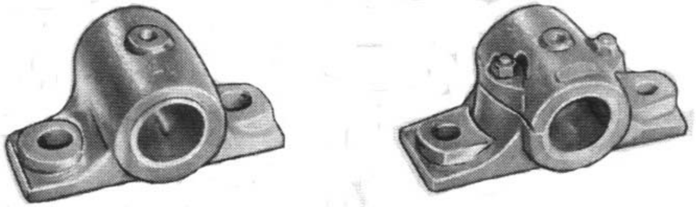
- Modular sliding contact bearings are found in many catalogs
  - Molded polymer and sintered bronze bearings can be impregnated with lubricants
    - Low-cost Nylon spacers ("standoffs") used in electronics work well for design contests
    - Also used for linear motion on round shafts
- Flat washers ((thrust washers) support thrust loads



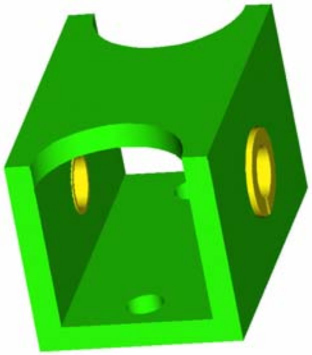




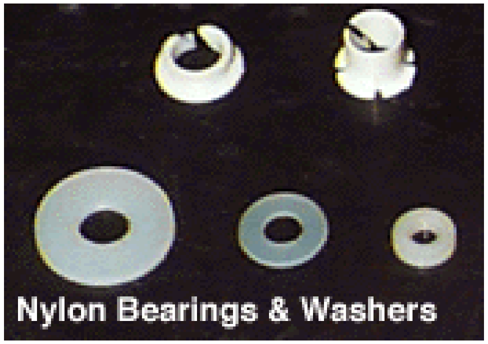




Pillow blocks from www.mcmaster.com



Can you imagine a split housing with a bolt to squeeze the housing and bearing to set bearing-shaft clearance?



Sliding Contact: *Linear Motion*

Sliding contact linear bearing systems are comprised of a moving structure, often referred to as the *carriage* or *slide,* and the bearing elements and the surfaces on which they slide are called the *rails, ways,* or *guides*. It is more complex to make the inner surface of the carriage match the outer surface of the rails than it is to bore a hole in a structure for a rotary bearing. If a snug fit is required for accurate motion, an adjustable plate, a *gib,* can be used to take out the clearance.

Sliding contact linear bearings are essentially just sliding contact rotary bearings with a very large radius of curvature, and thus all the issues from the previous discussion also apply. The most notable exception is that a portion of the linear bearing or the rail will be uncovered during some portion of the motion, and the exposed region is more likely to pick up dirt which then leads to wear of the bearing. For this reason, it is particularly important to not use too much lubricant, and to use wipers or bellows covers to help keep the system clean. Lubrication and susceptibility to dirt are reasons why revolute joint linkages are often preferred.

There are three basic configurations for linear sliding contact bearings: *boxway, dovetail,* and *twin-rail*. Boxway bearings support the largest loads in all directions. Dovetail bearings have less load capacity but fewer required precision surfaces and can be easier to adjust. Twin rails are the easiest to build, but the rails can be subject to bending deformations unless more complex supports are used. *Caveat emptor:* The right type for the right application!

Examine the doevtail bearings made by Bryan Ruddy to support the base of his lazy-tong actuators (see page 4-24). Although the angles may seem to add complexity, with the right tools, they are not difficult to make. Compare the dovetail system to a twin round rail system. To make sure that the holes for the shafts and bearings are all perfectly aligned, the holes should be made with a milling machine, so hole spacing accuracy can be on the order of 0.1 mm. This means that the holes for the bearings should be 0.1mm oversize to accommodate errors in machining and manufacturing...

Consider the design of a system that would use constant force springs to launch a projectile. In the simplest form, a bow-and-arrow type of system could be built, but how could multiple springs be attached to a single point that then applies the force to the projectile? As soon as some intermediate structure (carriage) is used as an anchor for the springs, the structure will have the potential to twist and jam because no two springs can ever be made to have exactly the same force. The FUNdaMENTAL principle of *Centers of Action*, as discussed on Page 3-26, suddenly becomes very important: The force from the springs should be applied through the centers of friction and mass in order to minimize the potential for angular motion of the carriage[1].

Carefully examine the figure. Some views are a bit hidden, so you will have to think about what may be hidden (or not). The blue element in the center is the carriage, and the projectile would fit in the hole in the end. The slots in the outer tube act as guides for the green bearing elements attached to the carriage. Each pair constrains one linear and two angular degree of freedom. Which ones? Can you draw a *stick figure* to visualize these degrees of freedom? Must there be two orthogonal pairs, or is just one pair and one bearing sufficient to restrain the two linear and one angular degree of freedom that need restraining? If two pairs are used for symmetry purposes, does it help or hurt? Remember *Occam's razor* (page 3-2) and *Maudslay's Maxims* (page 1-4)!

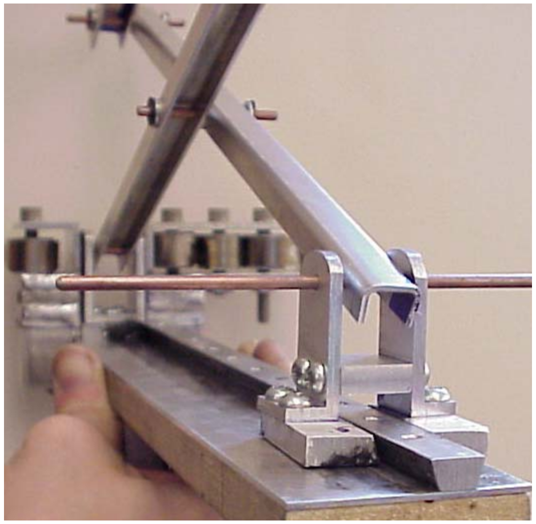
One end of each spring(s) would be attached to each end of the purple rod in order to transfer the forces from the springs to the carriage. The bearings (green) have aspect rations of about 3:1, so they resist any small moments caused by the springs not being of exact equal strength or any slight asymmetries in their mounting. To some this level of design detail may seem pedantic, but the difference in efficiency and repeatability gained by the management of friction and probability can lead to much greater reliability and hence performance and repeatability.

What linear motion sliding contact bearing systems do you anticipate needing? How will you select and engineer the best type of configuration? Remember, whenever you think of a design, also think about how the design will be manufactured. How will parts be aligned during assembly?

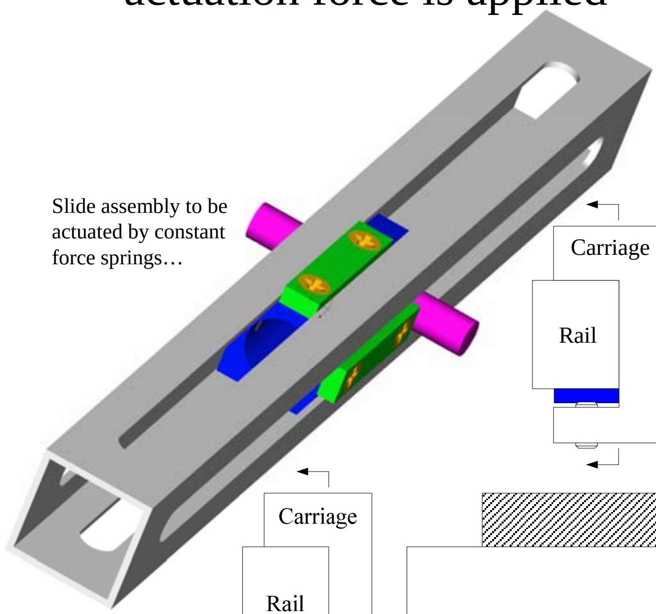
1. This type of design challenge faced the inventor of the cross-bow, which was described as one of the most destabilizing and devastating weapons ever invented, because a bolt fired from a crossbow could penetrate a Knight's armor. Suddenly, a peasant could bring down a knight!

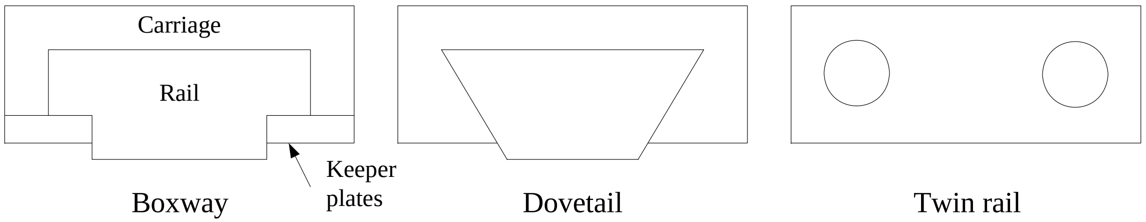
Sliding Contact: *Linear Motion*

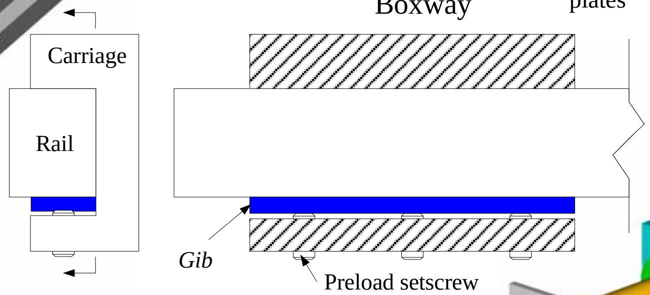
- Linear bearings are essentially rotary bearings with a really large radius of curvature
  - There are many configurations: boxway, dovetail, twin rails…
  - Clearance between bearing and rail or shaft can be removed by circumferential clamping or with *gibs*
- To prevent jamming, apply *Saint-Venant's* principle to the ratio of the length of the carriage to the spacing of the bearings
- Beware *centers of mass*, *stiffness*, *friction*, and where the actuation force is applied

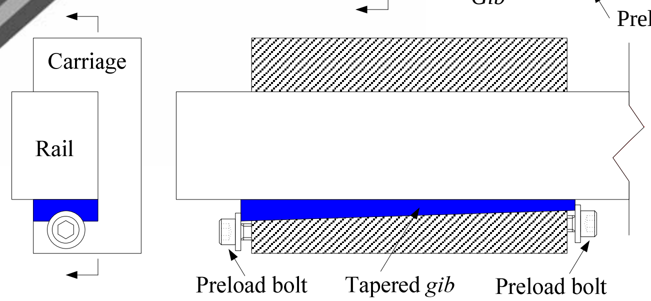


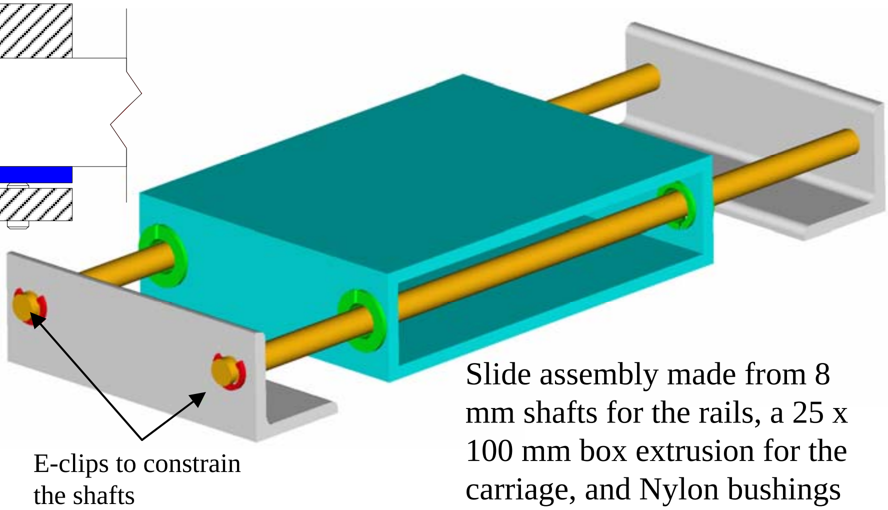
Bryan Ruddy used sliding contact dovetail bearings to guide his scissor linkage











Contact Bearings: *Rolling Elements*[1]

Rolling element bearings are essential for reducing friction which enables most machinery to operate efficiently and indeed even to exist. In fact, the first widespread application of rolling element bearings was to enable the first transportation revolution to take off: the bicycle. Rolling element bearings also enabled a revolution in manufacturing by making it possible to rapidly design and manufacture low-cost high precision machines, such as lathes, mills, and robots which in turn has helped to rapidly increase productivity. Although rolling element bearings are ubiquitous, designing with them requires significant care:

- To the first order, radial and axial loads can be added and this load must not exceed the rated static load for the bearing. If the maximum Hertz contact stress is exceeded, little craters will form which will rapidly lead to destruction of the bearing.
- A bearing's L_{10} life is how many millions of rotations can occur under load before 10% of the bearings fail. For ball bearings, the L_{10} life is proportional to the cube of the ratio of the combined load to the maximum dynamic load (power 10/3 power for rollers).
- Bearings are often overloaded by incorrect mounting. The first great mistake is to over constrain them such that when mounting bolts or nuts are tightened, the load capacity of the bearings is exceeded. In addition, even if a bearing appears to be properly constrained, heat generated by the bearings can sometimes also cause expansion and overloading of the bearings. The second great mistake is to misalign the bearings so the resulting geometric overconstraint overloads them each other when they move.
- Bearings should be thought of as stiff springs. Small mounting misalignments act as spring displacements. The force created by the imposed displacement δ acting on the bearing ($F = k\delta$) must be added to the total force considered in the load/life analysis! See *Bearing_stiffness_alignment.xls*.
- In general, bearings are meant to be used in pairs so they can support moment loads by acting as force couples.
- Accuracy standards are known as ABEC (Annular Bearing Engineers Committee) or RBEC (Roller Bearing Engineers Committee) of the American Bearing Manufacturers Association (ABMA). ABEC 3 & RBEC 3 rotary motion ball and roller bearings are common and low cost. ABEC 9 & RBEC 9 rotary motion ball and roller bearings are used in high precision machines. The International Standards organization (ISO) has a similar standard (ISO 492)

These are just highlights of issues which are discussed in detail in following sections. Again, it is stressed that a bearing is a very stiff spring, and small displacements caused by mounting misalignments and thermal growth create forces that must be added to the total load considered to be applied to the bearing:

$$F_{\text{load}} = k_{\text{bearing \& mount stiffness}} \times \left(\delta_{\text{misalignment}} + \delta_{\text{thermal \& load deformations}}\right)$$

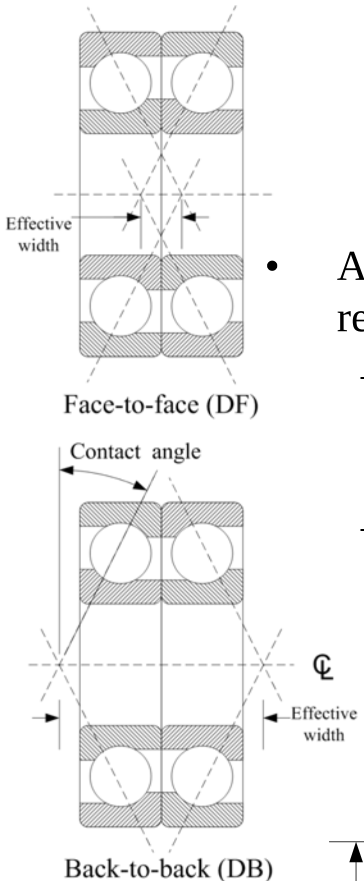
The key to reducing forces generated by misalignment is to think about how the elements will be manufactured, aligned, and fastened into place. How will the supporting structure itself deform? Good designers systematically examine a system supported by bearings and sketch arrows to illustrate the flow of forces through the system and the resulting displacements. They make sure that given thermal growth or misalignment, bearings either have the load capacity to accommodate the resulting generated forces, or they are mounted in a manner to allow displacements to occur without creating additional loads on the bearings. The structure is thus supposed to support the bearings which support a moving component. An interesting variation on rolling element bearings is where the rolling element is itself part of the structure.[2]

Once the mounting structure is detailed, the next most important issue is cleanliness and lubrication. Most bearings can be lubricated with grease kept in by rubber seals which also keep out dirt and moisture.

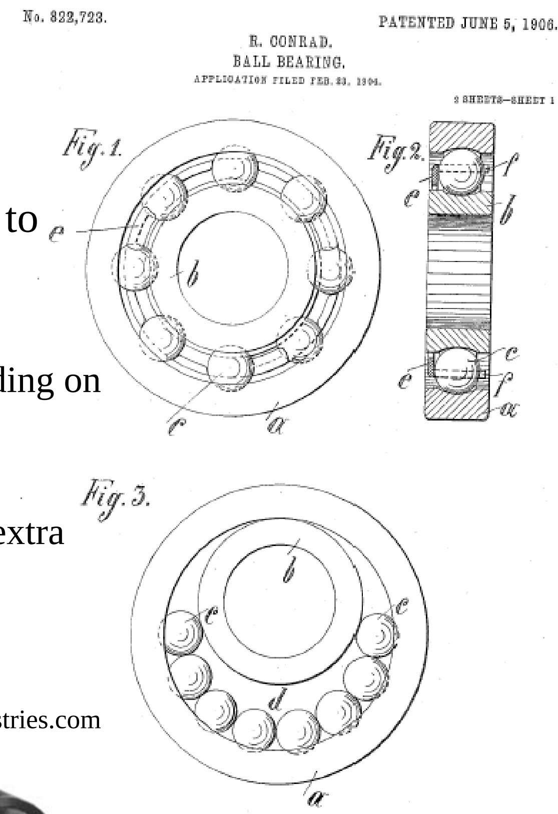
Perform a careful design review of your machine and draw arrows and/or lines to illustrate the flow of forces through your bearings, the components they support, and the support structure. Can your analysis justify the use of rolling element bearings? Make sure that the bearings are not over constrained. If the system deforms, can the bearings withstand the additional load? May the Force equals kx calculation be with you!

1. Special thanks to Michael N. Kotzalas of Timken for his detailed review of rolling element bearings and for Timken providing images of their bearings!

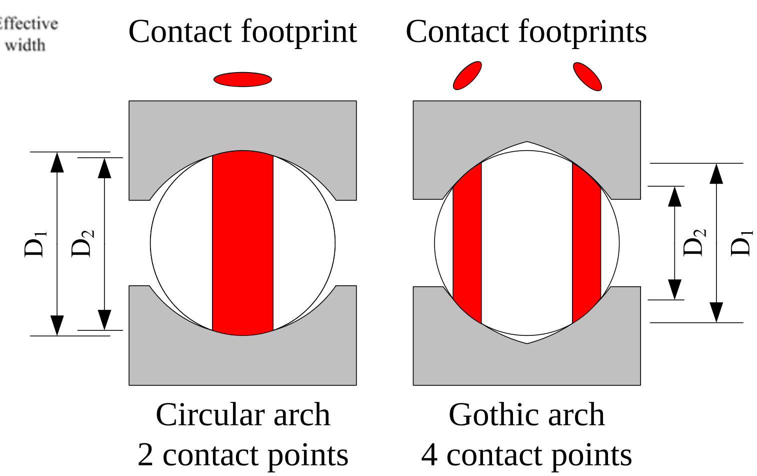
2. A. Jeanneau, J Herder, T. Laliberte, C. Gosselin, "A Compliant Rolling Contact Joint and its Application in a 3-DOF Planar Paralle Mechanism with Kinematic Analysis", Proc. DETC'04, ASME 2004 DETC, Sept. 28-Oct. 2, 2004, Salt Lake City, Utah, USA.



Contact Bearings: *Rolling Elements*

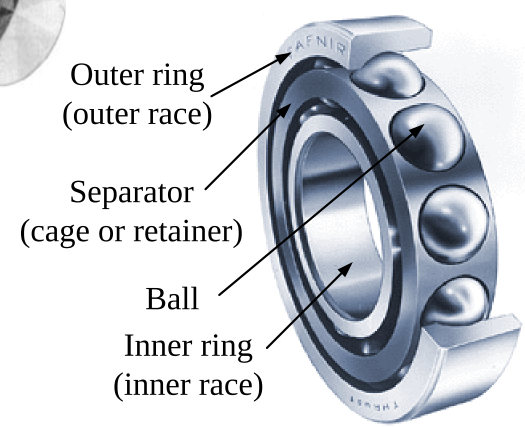


- Anti-friction bearings use rolling elements (balls or rollers) to reduce friction
  - Rotary bearings
    - Axial, thrust, and moment loads can be supported depending on the bearings and how they are mounted
  - Linear bearings
    - They can be non-recirculating (limited range of motion, extra low friction) or recirculating (unlimited range of motion)

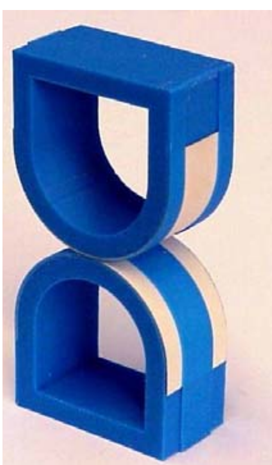


$$\% \text{ slip} = \frac{D_1 - D_2}{D_1} \times 100$$

http://www.thomsonindustries.com



http://www.torrington.com/products/fafnir/



Rolling joint model by Dr. Just Herder of the Delft University of Technology

Rolling Elements: *Rotary Motion*

Anti-friction bearings use rolling elements (balls or rollers) to reduce friction. The rolling elements are constrained between an inner race (ring) and an outer race (ring). Generally, a *separator* (cage) keeps the rolling elements spaced apart so they do not rub against each other which would result in skidding. *Full-complement* bearings are fully loaded with rolling elements and have no cage, which enables them to carry greater loads at low speeds. There are a vast array of different sizes and types of rolling element bearings to meet virtually any need. From subminiature bearings for instruments, to giant bearings for the bases of huge cranes, somebody somewhere makes the bearing you need. The challenge is to define your need and then select the best type.

In general, a sphere can be made more accurate and for less cost than a cylinder; and Hertz contact theory clearly indicates a roller can support far more load than a ball. Hence *ball bearings* generally are used for lower cost, lower load, or higher precision applications. *Roller bearings* are most often used when maximum load capacity is required in a minimum of space.[1]

A common ball bearing is the *Conrad* or *deep-groove* ball bearing which is designed to support radial or bi-directional axial loads. The nominal contact region between the ball and the raceway is oriented perpendicular to the axis of motion, but as axial loads are applied, the region tilts as the ball starts to push on the side of the raceway. Even though the angle of contact is slight, because there are so many more balls resisting axial loads, in general, unless stated otherwise, the axial load capacity of a bearing is generally equal to the radial load capacity. To support a moment load, two bearings spaced apart on a shaft should be used.

4-point contact bearings have ball-raceway contact at four points and one bearing can support bidirectional radial, axial and moment loads. However, the Hertzian contact ellipses are at 4 points and there is substantial slip as contact is made at different diameters with respect to the ball's axis of rotation.

1. Note the terminology here, where "ball bearing" is commonly implied to mean a bearing that uses a rolling ball element, but it can be a rotary or linear motion bearing, depending on the application. The same is true for roller bearings.

Angular contact ball bearings' contact regions are inclined from the start, and this enables them to be mounted in thermally stable configurations so they generally are used in high speed and high accuracy applications; however, a pair, at least, must be used in order to resist bi-directional thrust loads.

The *DN number* is the product of the average diameter of the bearing rings (millimeters) and the rotational speed (rpm). The higher the DN number, the greater the potential for heat generation. Heat causes expansion which can lead to improper constraint, that can further increase loads and cause an unstable situation that leads to failure. How bearings are mounted, and how they constrain shafts can have a big impact on heat generation. Bearing mounting for proper constraint is discussed in detail starting on pages 10-20.

The coefficient of friction for a sliding contact bearing may be 0.05-0.1, but the coefficient of friction for a rolling element bearing is typically 0.005. Why is it not much lower? It depends on the amount of deformation of the rolling element, the cage, the lubricant, and the shape of the rolling element-contact surface interface. In general, to minimize cost and maintenance, and to increase reliability, rolling element bearings typically are packed with grease and seals to keep the grease in and the dirt out. At high speeds, viscous drag becomes too great, and so instead of grease, oil must be used which incurs the expense of a lubricating system.

Lubricant shearing generates heat in a bearing. Shear power is the product of velocity $(r*\omega)$ and force $(r*\omega*\mu*A/h)$ which equals $(rw)^2\mu A/h$. Centrifugal forces cause Hertzian loading which at very high speeds can be significant enough to cause failure. Centrifugal forces are proportional to $r*w^2$. Thus the DN number has limits as an indicator of the true effect of speed on a bearing.

Rolling element bearings perform much better when they are *preloaded*. This is due to the fact that when the rolling elements are preloaded, they roll and do not skid, the stiffness increases rapidly due to the non-linear nature of Hertzian contact. In addition, with all the elements in contact, more "springs" act in unison so the stiffness increases greatly (see page 10-20).

The decision to use a rolling element bearing is often based on a need for efficiency or accuracy. Can you justify where and why you need them?

Rolling Elements: *Rotary Motion*

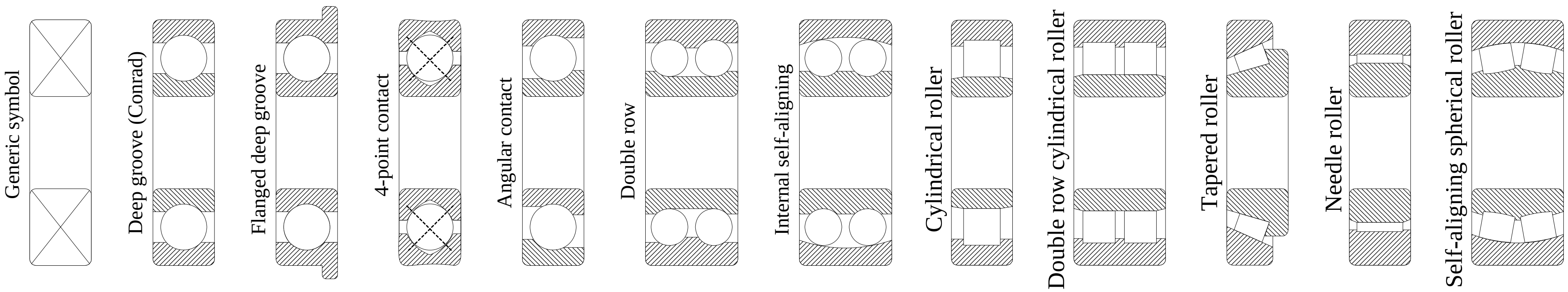
Bearing	Cage	ABEC-1		ABEC-3		ABEC-7		
		Grease	Oil	Grease	Oil	Grease	Oil	Oil Mist
Radial and angular contact, single row (rpm)	Molded nylon	300,000	350,000	300,000	400,000	400,000	600,000	750,000
	Composite	300,000	350,000	300,000	400,000	400,000	600,000	750,000
	Metallic	250,000	300,000	-	-	-	-	-

http://www.timken.com

- Bearing speed is limited by thermal and centrifugal effects:
  - Shear power is the product of velocity ($R\omega$) and force ($R\omega\mu A/h$) = $(R\omega)^2\mu A/h$
  - Centrifugal load is proportional to $r\omega^2$
    - DN value (D = (OD+ID)/2 or sometimes ID, (mm)) and N is the rpm
    - An older methodology, still useful to provide a quick indicator of performance
    - ISO 15312 speed rating standard is more accurate

$$qA = \frac{n\pi}{30\times 10^3}\left(10^{-7} f_o (\nu n)^{2/3} d_m^3 + f_1 F d_m\right)$$

    - n = speed rating
    - ν = ISO specified viscosity value
    - d_m = bearing pitch or (OD+ID)/2
    - F = applied load
    - q = ISO specified heat flux (function of surface area and bearing type)
    - A = ISO specified area (function of bearing type)
  - ***Scrutinize mountings and beware over constraint!***



Thrust

Rotary Motion: *"Ball Bearings"*

"Ball bearings" use balls that roll on conformal raceways on the inner and outer rings' (races) outer and inner surfaces respectively. By having the raceway closely conform to the ball, rather than rolling on a pure cylinder, orders of magnitude greater load capacity are obtained[1]. So common has this basic design become, that the term *ball bearing* has come to mean not just a spherical metal ball used in bearings, but a bearing itself that uses balls. As shown, there are many different types of ball bearings, but in general, when the term *ball bearing* is used, people are usually referring to a *deep-groove radial bearing*, which is sometimes called a *Conrad bearing* after Robert Conrad who invented its means of manufacture.

The problem with ball bearings is how to assemble them with a minimum number of pieces and effort? Conrad[2] was awarded British patent no. 12,206 in 1903, and U.S. Patent 822,723 in 1906 for placing the inner ring inside of the outer ring so they touch. The balls are then loaded in the large remaining space. As the balls are moved circumferentially towards the contact point between the rings, the inner ring moves to become concentric with the outer ring. A *retainer* (separator or cage) keeps the balls separate which ensures that they do not rub against each other, and that the inner ring remains concentric with the outer ring. A *full compliment* bearing uses a filling notch to fully load the annulus with balls to achieve 30% higher load capacity; however, there is no cage, so speeds are limited, and axial loads can cause the balls to run against the filling slot which leads to early failure.

Ball bearings were critical in the launching of one of the greatest personal freedom inventions of all time: The bicycle. The first bicycle appears to have been invented by Leonardo da Vinci[3], who also appears to have invented the first ball bearing. However, it took the 1800's industrial revolution and its capability to precisely work metal on a large commercial scale to make bikes ubiquitous. With industry came the ever greater need to commute to work...

1. See for example ISO 15 or ABMA 20 which defines the boundary plan (ID, OD, width) of standard ball, cylindrical, and roller bearings. See ISO 76 or ABMA 9 for ball bearing static load ratings, and ISO 281 or ABMA 9 for ball bearing dynamic load ratings.
2. The person who often deserves and gets the most credit for an idea is often the person who figures out how to best manufacture the idea.
3. James McGurn, An Illustrated History of Cycling, John Murray Publishers, London, 1987. TA Harris, Rolling Bearing Analysis, 1991 John Wiley & Sons, also has a very nice historical introduction.

As discussed on page 10-17, misalignments create moments which can overload bearings. In 1907, Sven Wingquist invented the *self-aligning ball bearing*, where the outer ring raceway is spherical so the inner race is free to tilt with the shaft. This results in reduced radial load capacity, because the raceway is less conformal; however, there is no additional effective radial load caused by misalignment or deformation of the structure. As a result, the net life of the bearing can be increased in some demanding applications. This major invention helped to launch SKF Corp. which today is one of the largest producers of ball bearings in the world.

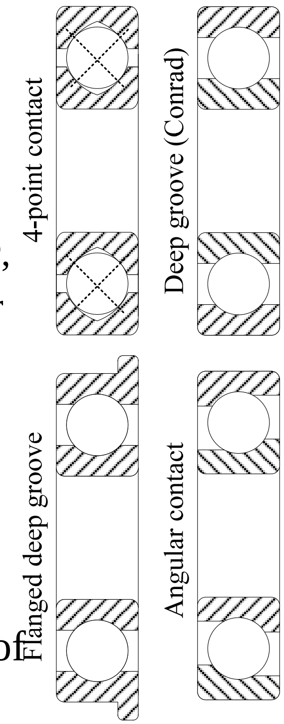
Flanged bearings have a flange integral with the outer ring to axially constrain the bearing which helps to reduce mounting complexity and cost. *Four-point contact bearings* have gothic arch grooves on the inner and outer raceways that enable the balls to make 2 points of contact on each of the inner and outer races, and hence they can support a moment load applied to the races; however, this also means that there is more differential slip and thus lower efficiency. *Angular contact bearings* have their raceways ground such that the balls only make contact with one side. This allows the other side of the raceway to be left relatively open so more balls can be loaded. Because the balls are always contacting the raceway at a nominally known angle and there are more of them, much greater accuracy and axial load capacity are obtained. *Double row bearings* have 2x load capacity without worrying about raceway size variations, and they can also support moment loads. *Spherical* or *self-aligning ball bearings* have larger diameter conformal raceways with a common center so the inner ring is free to pivot and thus they do not suffer from misalignment-induced forces. Mounting and alignment, lubrication, and predicting life are all very important and will be discussed on following pages.

Selecting a "ball bearing" requires a designer to first decide what type of bearing is required. Adhering to Occam's razor, one should always start with the simplest, and hence lowest cost bearing, which is a simple deep-groove bearing. If size is not the principle constraint, then one can often find a commonly used size which means that the price should be lower. In fact, for many commercial products, if you can design your product using the same size as roller blade ball bearings, you may have a large cost advantage!

Where in your machine is it critical to reduce friction or increase accuracy, and should you even be using ball bearings? Can you analytically justify the need?

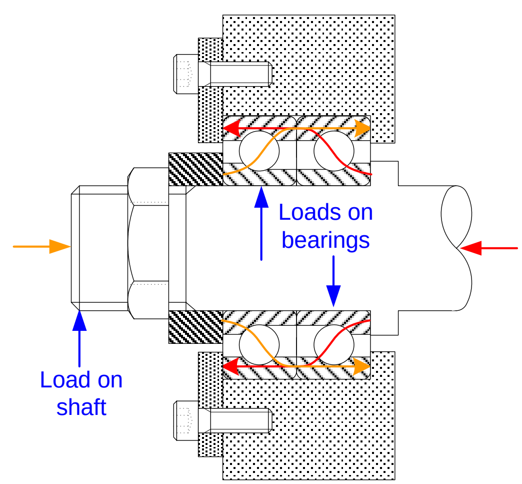
Rotary Motion: “*Ball Bearings*”

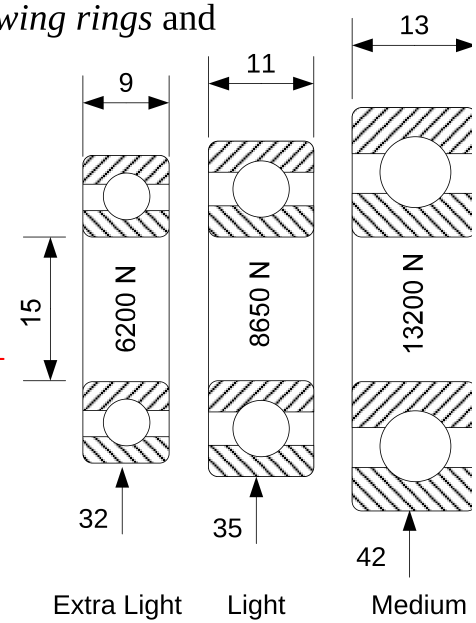
- Deep groove, “Conrad”, ball bearings are the most common form of ball bearings
  - For bidirectional axial loads, balls share the load
  - axial load capacity C_a / radial load capacity C_r depends on the ball-groove contact angle α^o:
  - $C_a/C_r = 1.58$ for $\alpha^o = 20^o$, $C_a/C_r = 1.18$ for $\alpha^o = 15^o$, $C_a/C_r = .77$ for $\alpha^o = 10^o$, $C_a/C_r = .38$ for $\alpha^o = 20^o$,
    - Balls roll from one side of the contact groove to the other: bidirectional stiffness is non-linear
    - Same bore-size, different duty bearings have different outer dimensions and load capacities
- Angular contact bearings’ contact grooves are preferentially ground on one side, and can accurately support both radial and uni-directional axial loads
  - The most common form of precision ball bearings, often used for high speed applications
- Deep Groove or Angular Contact ball bearings in pairs support shaft moment loads
- Four-point contact ball bearings have gothic arch grooves so each ball contacts two points of each raceway, and thus allow a single bearing to support radial, axial, and moment loads
  - Giant forms of these bearings, with gear teeth on the inner or outer race, are called *slewing rings* and they are used as turntable bearings for cranes, excavators…



Extra Light 9100	10	26	8	1960	5160
	15	32	9	2800	6360
	280	420	65	355000	360000
Light 200K	10	30	9	2650	6800
	15	35	11	3450	8650
	280	500	80	710000	560000
Light 200W (full compliment)	15	35	11	5060	11000
	280	500	80	1120000	765000
Medium 300K	10	35	11	3460	9200
	15	42	13	5240	13300
	280	580	108	780000	585000

Data corsy of The Timken Company for Fafir® ballbearings.





Rotary Motion: *"Roller Bearings"*

Were roller bearings invented before ball bearings? For how long had logs been used as rollers under heavy objects? Perhaps we will never know, but analytical motivation can be found in Hertz contact analysis. Compare the load capacity of a roller of length and diameter equal to the diameter of a ball. Assume the major diameter of the raceway is 10x the ball diameter, and the ball rolls in a raceway whose minor diameter is 1.5x that of the ball, while the roller rolls on a flat surface. At a stress level of 1500 N/mm^2, a single roller can support 8160 N, whereas a single ball can support 418 N.

Perhaps the modern roller bearing was created as a result of the *Longitude Prize* announced by the British government in 1714 for the precise determination of a ship's longitude: *£10,000 for any method capable of determining a ship's longitude within one degree; £15,000, within 40 minutes, and £20,000 within one half a degree.* Latitude could be easily determined by the stars, but longitude could not, and as a result, sailing vessels had the nasty habit of bumping into large rocks (continents) in the fog. The Longitude Board was established, and the royal astronomers set about looking for an astronomical solution. It took a true genius to realize that the solution lay elsewhere, in the design of an accurate clock. John Harrison (1693–1776) was an English clock designer (horologist), who developed and built the world's first successful maritime clock. Around 1750, Harrison invented the caged roller bearing as a means to achieve the low friction levels required for his clocks. It took him a lifetime and four tries but he did it, and forever changed the technology of navigation and the destiny of the world. However, jealous astronomers refused to concede defeat, and Harrison only claimed his prize in 1773 when King George III persuaded Parliament to bypass the Longitude Board and award the prize to Harrison.[1]

Caged roller and ball bearings became more and more commonplace as reducing friction allowed for the increase in speed and hence efficiency. More efficiency meant more bearings which meant more precision machine tools to make the bearings, and the wheels of industry started rolling faster and faster. Henry Timken (1831-1909) was a successful carriage maker when he retired, grew restless, and then got back into the business. He invented the *tapered roller bearing* so carriages could carry greater loads more efficiently. Conventional roller bearings were well known at the time, but they only carried radial loads, and thus two radial and two thrust bearings had to be used to support shafts and withstand radial, bi-directional thrust, and moment loads. Perhaps inspired by conventional ball bearings, which when preloaded in a back-to-back configuration could support all these loads with just two bearings, Timken invented the tapered roller bearing[2] which uses tapered rollers on conical races where all the surfaces have a common apex. This allows the rollers to roll on the conical race surfaces without translating.

The innovation wheel kept on rolling. The Torrington Company had been making needles for weaving and sewing industries, and their swaging machines also enabled them to manufacture bicycle spokes. They also began manufacturing ball bearings and they used roller bearings in their own machines. The ability of many small rollers (needles) to carry far more load than a few larger elements in a confined space was well-known; however, the needle rollers lacked a cage and they often went in all directions during assembly. In the 1930's, Edmund Brown, a new research engineer invented the *drawn cup needle bearing* where the ends of the sheet metal outer raceway were folded over needles' rounded ends to keep them in place.[3] This made needle roller bearings easily usable, such as in universal joints for automotive driveshafts (1962).

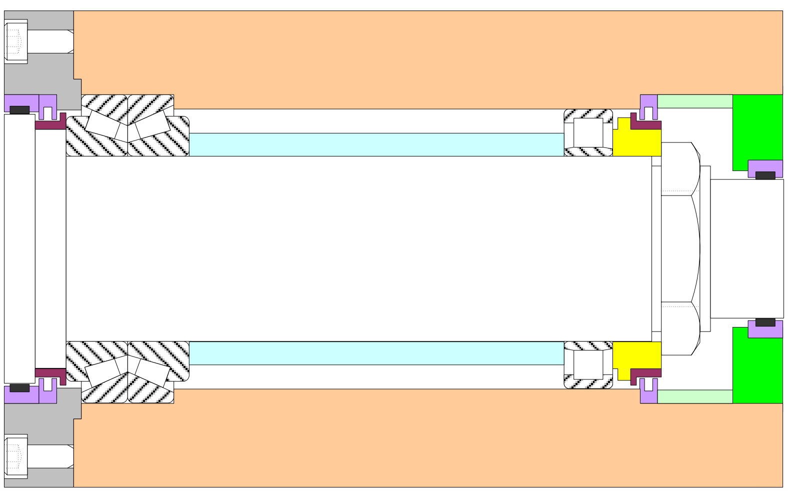
The expression "roll on" might be attributed to truckers, or perhaps it should be traced to Henry Timken who is said to have told his children:[4] "To be independent, you must be successful. If you want to lead in any line you must bring to it independence of thought, unfailing industry, aggression and indomitable purpose. If you have an idea which you think is right, push it to a finish. Don't let anyone else influence you against it. If we all thought the same way, there would be no progress. But above all, don't set your name to anything you will ever have the cause to be ashamed of."

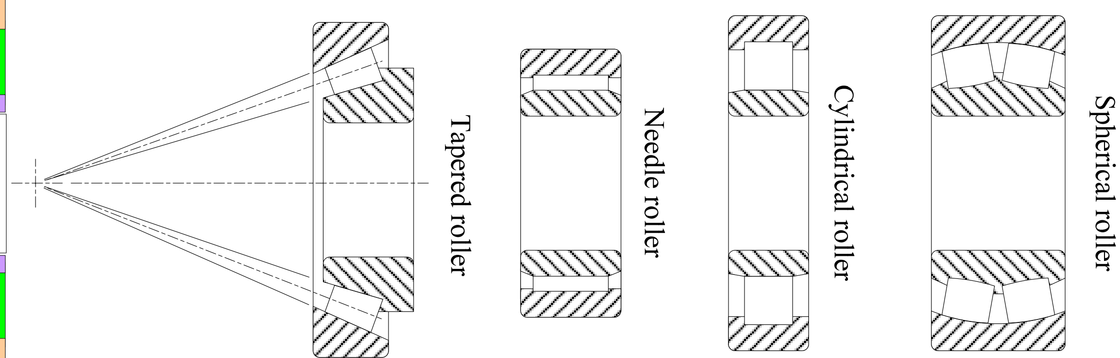
1. See Dava Sobel, Longitude: The True Story of a Lone Genius who Solved the Greatest Scientific Problem of his Time, Penguin Books, 1996, for a history of Harrison's quest for the design of a clock accurate enough for nautical navigation. Also see the National Maritime Museum at the Royal Observatory Greenwich: http://www.nmm.ac.uk/site/request/setTemplate:singlecontent/contentTypeA/conWebDoc/contentId/355/viewPage/1

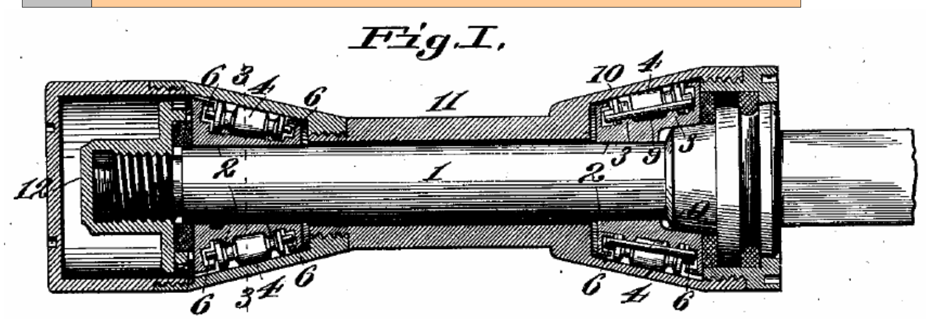
2. US Patents 606,635 and 606,636.
3. See Edwin Lieberthal, Progress Through Precision: The first 125 Years at The Torrington Company, 1992, The Torrington Company, and US patents 2,038,474, 2,038,465, and 2,063,787.
4. From http://www.timken.com/aboutus/history/pdf/history.pdf

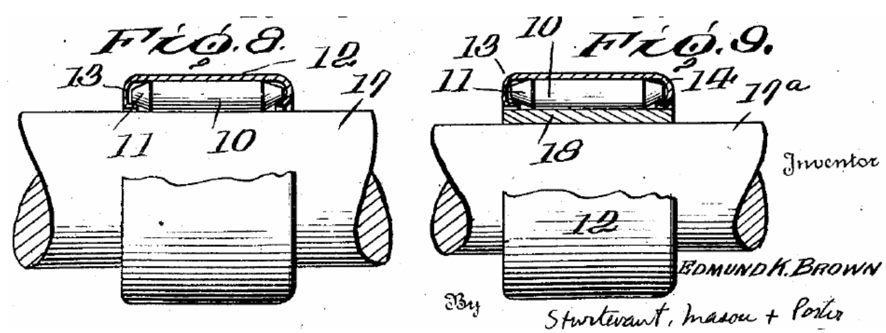
Rotary Motion: *“Roller Bearings”*

- Line contact gives roller bearings many times the load capacity of a similarly sized ball bearing
  - Beware deformations that can cause edge loading of rollers
    - Many rollers actually have a slight barrel shape
    - For large shaft deformations, e.g., dynamically heavily loaded shafts, use spherical roller bearings
- Tapered roller bearings are the roller equivalent of angular contact ball bearings
  - Invented by Henry Timken in 1898, it revolutionized modern industry by simplifying the design of heavy machinery such as railroad axle bearing supports
- Spherical roller bearings revolutionized paper making by allowing huge rollers to be supported on rolling element bearings without structural deformations overloading bearings









Rolling Elements: *Linear Motion*

Rolling element linear motion bearings do not move at nearly the speeds that most rotary motion rolling element bearings do; however, their ball-raceway interface is similar in that it is governed by Hertz contact. The stresses in a system with Hertz contact are proportional to Force$^{1/3}$ for point contact and Force$^{2/3}$ for line contact (see pages 9-16, 9-17). Hence most rolling element bearing systems use hardened steel or ceramic rolling elements and raceways. Unlike rotary motion systems, linear motion systems do not have the mechanical advantage of a wheel diameter being much larger than an axle diameter; hence frictional forces are often of greater significance.

There are two principal types of rolling element linear motion bearings: *non-recirculating* and *recirculating*. The former are used for short range of motion systems. The rollers move at half the forward speed of the supported object: If an object of length L is to move a distance X, it must be supported by an assembly of rollers that is $L + X/2$ long. The primary advantage of such a system is its simplicity, and its smoothness of motion. For short stroke systems, this bearing can provide high load capacity and accuracy in a small space. A cage contains the rolling elements and limits their range of travel. This rolling element bushing is often used in stamping dies to guide the motion of one die with respect to another, and is called a *die set bushing*.

The most basic type of recirculating linear motion rolling element bearing is a *rolling wheel*, supported by ball bearings, that rides on a rail. A train on tracks would be a very large example of such a bearing system. Typically the wheel-to-rail interface is not lubricated, and the systems are used for very long range of motion systems.

Recirculating rolling element linear motion bearings have a *carriage* or *bearing block* or *truck* which contains raceways with rolling elements that are opposite the *raceways* in the *rail* along which the carriage moves. *Ball bushing bearings* are recirculating linear motion bearings that ride on round shafts. In fact, the *Ball Bushing*^{TM}, invented by John Thomson in the 1950s was the first commercially successful recirculating linear motion ball bearing. The load capacity of a ball bushing bearing is somewhat limited because the curvature of the shaft is opposite that of the balls, rather than conformal to them. Over the years others evolved the concept to have greater load capacity by grinding longitudinal grooves into the round shafts. Eventually, grooves were ground directly into rectangular shafts which could be more easily bolted to surfaces, and the *linear motion guide*, or *linear guide*, was born. Arrangements of rolling elements are typically made so that the rolling elements are inherently preloaded, and high accuracy motion is obtained.

Recirculating element linear motion bearings use pick-up paths (often tubes) into which flow the rolling elements that come out from underneath the carriage. The rolling elements are going at only half the translational speed as the carriage which they support, and the pick-up path redirects the rolling elements back to the other end of the carriage where they begin the cycle anew. As they roll, they need lubrication, just as rotary motion bearings do. Unfortunately, a portion of the raceway is left exposed, which means that dirt can collect on it, and even be attracted to it by the lubricant. To minimize harm from dirt, linear motion bearings can use *wipers* and *seals*. Wipers act as little plows that scrape dirt off of the raceways. Seals keep the lubricant inside the bearing and minimize the layer of lubricant film left on the raceway as the carriage moves along.

Linear motion rolling element bearings can have 10-20 times less friction than sliding contact linear motion bearings. In addition, a typical linear guide has equal load capacity in both directions normal to the axis of motion. Their primary drawback is the fact that it takes a finite amount of room to house the rolling elements, and the rolling element mechanisms cost more to produce than sliding contact bearings.

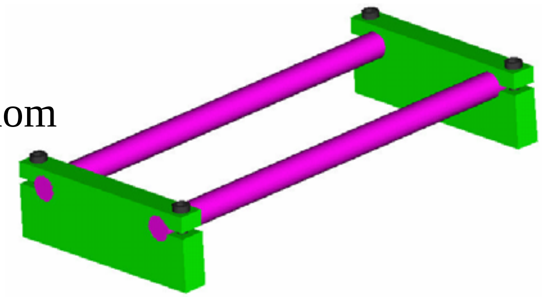
How does one select a bearing for the application? First, write down the Functional Requirements for the design and analyze the loads to be supported to determine if a rolling element bearing is even warranted. Next, sketch designs that uses different types of bearing, and do a cost-benefit analysis for each. In robot design contests, rolling wheel linear bearings are often used because they are the simplest solution.

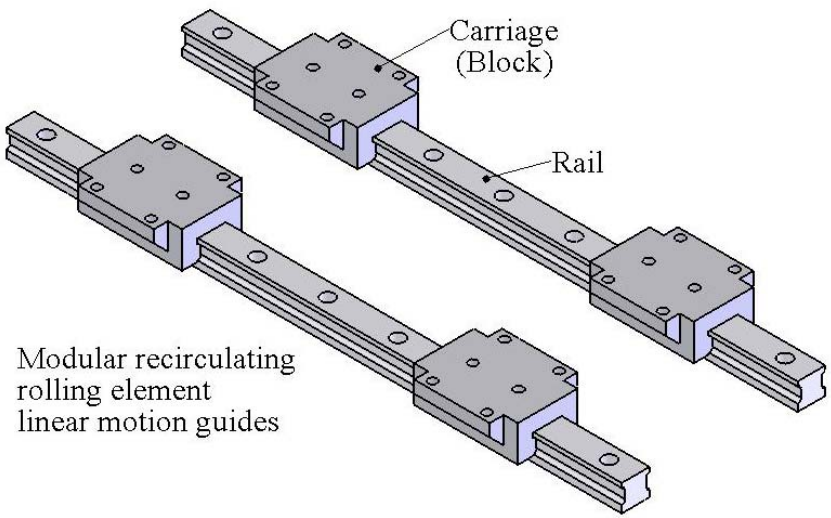
Make an assessment of your machine's required accuracies of linear motion and the loads that must be supported. What are the trade-offs of using a sliding contact versus rolling element bearing system? How would you make/mount each type of system? To what forces will the bearings be subjected? Analysis of forces on a carriage supported by linear motion bearings is discussed in detail on page 10-35.

Rolling Elements: *Linear Motion*

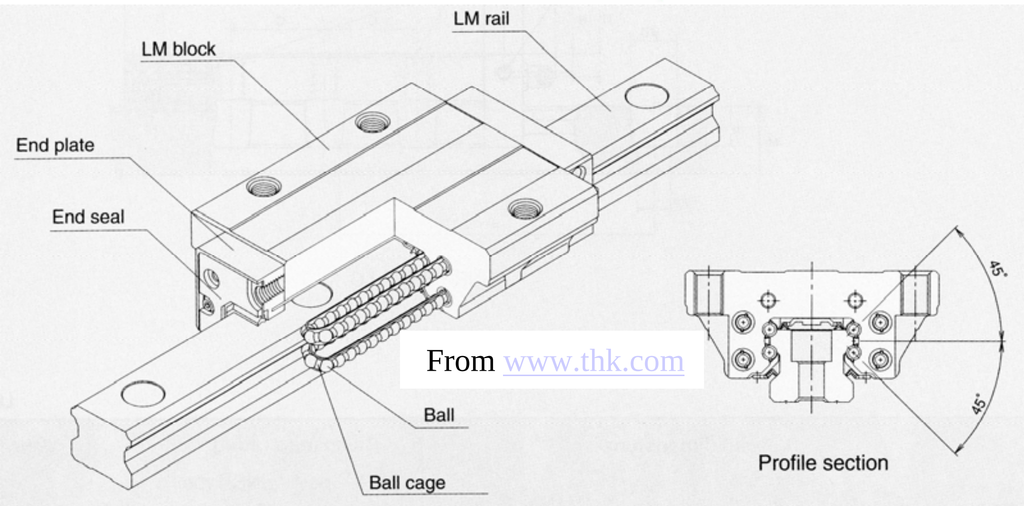
- Linear rolling element bearings can be non-recirculating or recirculating
- Non-recirculating types: wheels (cam followers) & rolling elements
  - More complex arrangements are needed to constrain 5 degrees of freedom
  - Wheels on rails can roll very far
  - Rollers between two surfaces travel half-as-far as the moving surface
- Recirculating types: *linear guides* & *ball bearing bushings*
  - Modular form makes then easier to use to constrain 5 degrees of freedom
  - Travel distance is only limited by ability to splice together rails
- *Scrutinize mountings and beware overconstraint!*

BallBushing™ from www.thomsonindustries.com

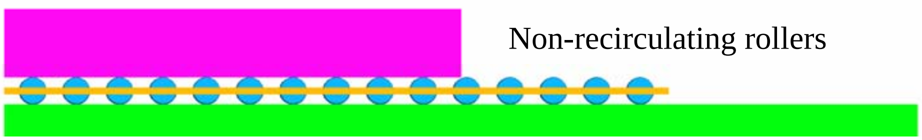




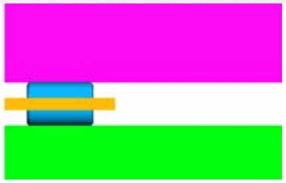
Modular recirculating rolling element linear motion guides



From www.thk.com



Non-recirculating rollers



Linear Motion: *Wheels on Rails*

Since the accuracy of rotary motion rolling element bearings is so high and they are so inexpensive, one of the simplest ways to get accurate linear travel is to use a rotary motion bearing as a wheel riding on a hard surface (rail). When the surface is curved, it is referred to as a *cam*, and wheels are then called *cam followers.* Flanged wheels help keep the wheels on rails and the surfaces of the wheels can also be curved. Railroad wheels have a special shape that works in unison with the rails to minimize slippage between the wheels, which are attached to solid axles, and the rails.

Rotary motion bearings are usually designed so their races receive support from the housing and shaft; thus one cannot simply use the load ratings for rotary motion bearings in this type of application. Cam followers are intended for this type of application and have thicker outer rings. Cam followers often have integral threaded shafts but are also available with a through hole or the shaft mounted eccentrically in the inner race. As an adjustment nut is turned, the center of the inner race is displaced radially with respect to the shaft, thereby preloading the system.

A design goal is often to minimize the number of wheels required. Ideally only five wheels are required to define the motion of a carriage, where four wheels roll on a Vee-shaped rail and one wheel acts as an outrigger rolling on a flat surface. This is referred to as a *kinematic system* because there is only one wheel for each degree of freedom that is constrained. Practically, this configuration often has stability problems like a three-legged chair and so more rollers are used where the structure to which they are mounted has enough compliance to ensure that all the wheels maintain contact with the rails. Hertz contact theory is used to make sure the rollers are not overloaded, but which form, point or line? The former implies a large radius of curvature or crown on the wheel. The latter implies a cylindrical wheel. The former is more tolerant of alignment errors including misalignment caused by deflection of the shaft under load. Non-kinematic systems must rely on elastic deformation of the components or careful design to balance thermal expansion.

Kinematic configurations have no alignment requirements other than requiring the outer race of the cam followers must ride flat on the rails. If the wheels do not ride flat, the system will still be kinematic, but enhanced wear on the rolling edge will result. To ensure proper contact, as mentioned above, the outer race can have a crowned profile. Non-kinematic systems must be aligned so that all rollers' preloads stay within desirable limits, and desired accuracy of motion is achieved. Kinematic systems are preloaded by the weight of the carriage being supported or by preloading with magnetic attraction. Non-kinematic systems are often preloaded by an opposed bearing configuration. If a friction drive is used, the capstan roller can be used to preload the kinematic arrangement of rollers. In most cases, static and dynamic coefficients of friction on the order of 0.01-0.005 can be expected.

Make sure that when you design a rolling wheel system (or any bearing system), that you visualize angular deformations and have a plan to accommodate them. Either use compliance inherent in the structure and the ability of the selected bearings to withstand the loads, or provide for movable (self-aligning) mounts. Rolling wheels are particularly prone to edge loading of the wheels which can result in early failure.

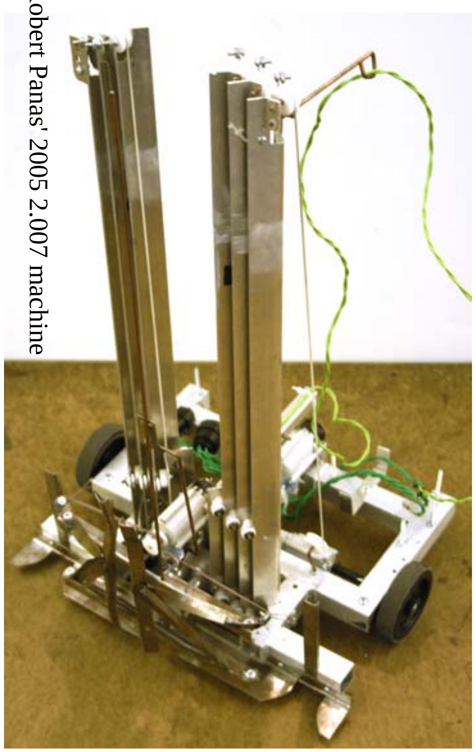
Rolling elements can also reduce friction between an extending member and the axis from which it extends. Four rollers at the opening of a tube and four rollers attached to the end of a mating tube support the extending tube at both end by rollers. If the roller outside diameter is 3-5 times the inside diameter, the roller can be placed over a simple shaft with a sliding bearing interface between the two. The solid model shows one such concept which is also straightforward to manufacture. Also shown are students' machines that used rollers to support extending axes. The rollers greatly reduced friction and enabled the deployment of long booms that would otherwise not have been possible given the small motors available.

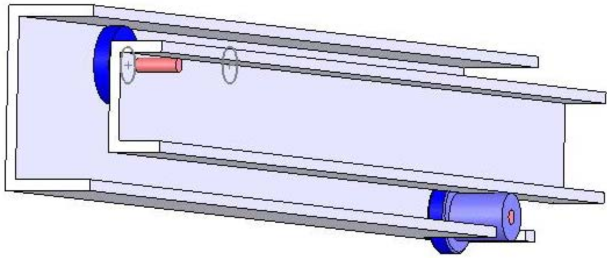
How would you design the shafts and rollers for the extending axis supported by rollers described above? Are eight rollers really needed or perhaps just two rollers and six sliding contact bearings? A force and motion analysis of the system can help determine when its worth the cost of using rolling elements.

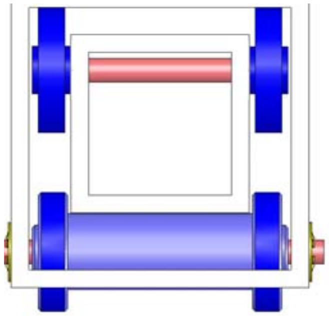
Linear Motion: *Wheels on Rails*

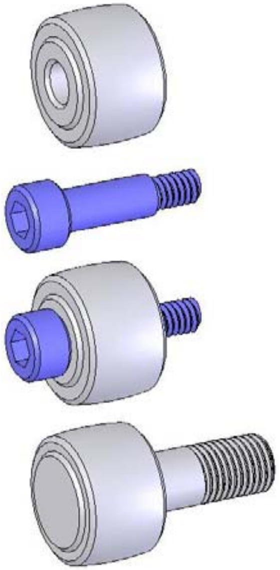
- Wheels can roll on tracks to enable linear motion
  - Soft materials (plastic) provide compliance and misalignment forgiveness
    - Plastic may take a set and develop a flat spot under heavy static loads
    - The bore of a plastic wheel makes a nice bearing for a shaft
  - Hard materials (steel) provide load capacity and accuracy
    - Rolling element bearings often support steel wheels
  - Very simple and effective
- Examples: Extending axis, Trains, factory automation systems

Robert Panas' 2005 2.007 machine









Teri Hall's 2005 2.007 machine

Linear Motion: *Ball Bushing Bearings*

A sliding contact bearing that fits over a round shaft is often called a *bushing*. One of the first mass-produced modular rolling element linear motion bearings for use in machines and instruments was the *Ball Bushing*^{TM} (ball bushing bearing), invented by John Thomson. Ball bushing bearings use recirculating balls that roll on round shafts and are constrained by grooved raceways in the bushing that surrounds the shaft. They allow for rotational alignment but not continuous rotational motion about the shaft. Ball bushing bearings are typically available in the form of single units that are pressed into a bore in the machine structure or they are available already mounted in aluminum or steel pillow blocks or flange blocks. Self-aligning bearings' raceways' outer surfaces are curved so they allow the bearing to tilt within a housing. They must be used in pairs on a shaft in order to support moment loads. Twin pillow blocks can support radial and moment loads.

Shafts can be simply supported and used with closed ball bushing bearings or shafts can be supported along their lengths and used with open-type bearings. As shown, the latter have asymmetric load capacity. The former are used primarily where short strokes are needed or where the carriage serves as a guide for reciprocating motion and straightness of travel is not of primary concern. The latter allow heavier loads to be supported along the entire range of travel, but stiffness in a direction outward from the shaft is less than toward the shaft. Off-the-shelf shafts are available in many different diameters (metric and inch) up to 5 m long. Rail supports can be purchased from stock or machined into the structure to which the shafts are attached.

Single ball bushing bearings are meant to support radial loads. A linear motion carriage will thus typically be supported by three or four self-aligning ball bushing bearings at two corners of a carriage and one as an outrigger, or at four corners of a carriage respectively. The use of three bearing units would make the assembly quasi-kinematic, so vertical parallelism of the shafts is not as critical, although lack of horizontal parallelism can still cause the carriage motion to bind. The self-aligning feature refers only to angular misalignment; if the shafts are divergent, then the open-type bushings will be spread apart as the carriage moves along the diverging rails. Self aligning bushings enable the carriage to move smoothly even if the rails are slightly curved. Analysis of forces on a carriage supported by linear motion bearings is discussed in detail on page 10-32.

When heavy loads and moments are applied to a machine tool table from many directions and locations, the carriage must be supported at each corner. If very heavy loads are encountered, then more than four bearing units may be required. Once one crosses over from quasi-kinematic to overconstrained, it is generally acceptable to add more bearing units to increase load-carrying capability. Remember that the bearings themselves are never perfect and they must be spaced 3-5 bearing carriages apart so they do not over constrain each other. Beware that static and dynamic friction will generally increase with an increase in overconstraint, as more bearing mounts must elastically deform to accommodate relative error motions.

To assemble a system where the rails are supported along their length, one rail is located with respect to a machine reference and then bolted down securely according to a procedure provided by the manufacturer. The second rail is made to be parallel to the first through the use of spacer blocks (e.g., gage blocks) and then the second rail is bolted in place. Measurements are made along the lengths of the two rails to make sure that alignment is better than the desired running parallelism. The pillow blocks are then placed on the rails and the carriage bolted to them. The assembly is tested by measuring the force required to sustain slow steady motion and noting any deviations in this force along the length of travel.

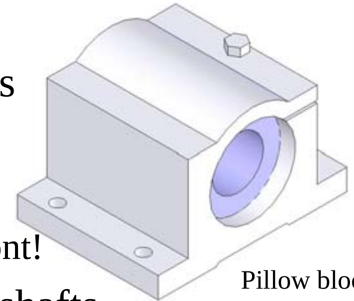
Closed-type ball bushing bearings can be preloaded by using oversize balls. Closed or open type ball bushing bearings can be preloaded with a circumferentially squeezing clamp. As with most rolling element bearings, static and dynamic coefficients of friction can be anywhere in the range of 0.01-0.001 depending on load, preload, alignment accuracy, and lubrication.

Ball Splines use nominally round shafts with axial raceways ground in them with radii of curvature typically equal to $1.5D_{ball}$. Conformal raceways greatly increase the load capacity as indicated in Chapter 9's discussion of Hertz contact stresses. In fact, the evolutionary process of the ball bearing bushing into the ball spline to carry more load (and transmit torque), also lead to the *profile rail bearing*, or *linear guide*, which is more easily mounted on a flat surface along its entire length.

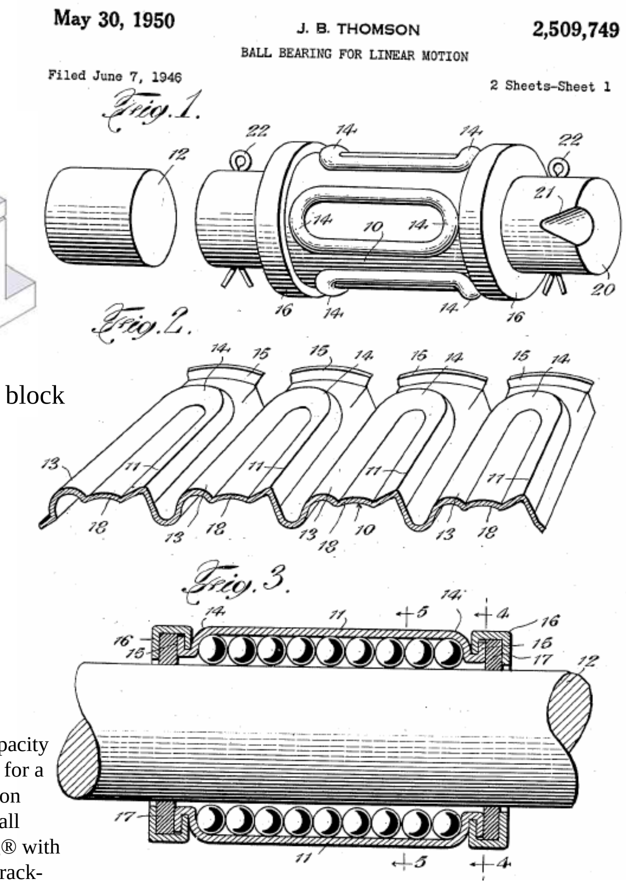
How would you use ball bushing bearings in your machine if you had the option? Always play "what if" games when learning about new things!

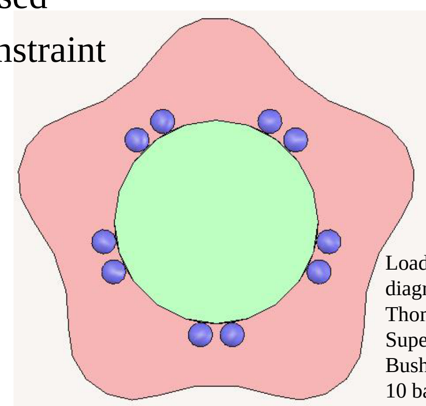
Linear Motion: *Ball Bushing Bearings*

- Ball bearings revolutionized rotary motion machines
  - Why not use ball bearings for linear motion?
    - What to do with the balls at the end of travel?
      - Pick them up and put them back at the front!
- *Ball bushing bearings:* recirculating balls on round shafts
  - First "linear ball bearing", and still widely used
  - No rotational stiffness helps prevent overconstraint
  - Open configuration
    - Rails supported along their length
    - Load capacity greatest in compression
  - Closed configuration
    - Rails supported at ends
    - Load limited by deflection of rails
  - Self-aligning bushings use pivoting grooved ball raceways
  - Pillow block housings allow for preloading by squeezing the bushing

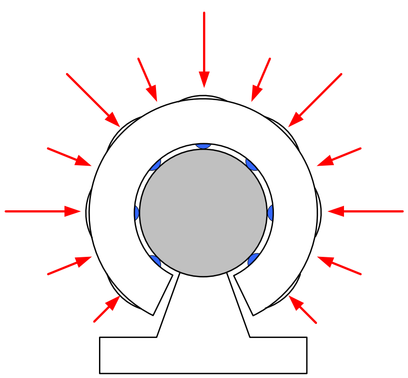


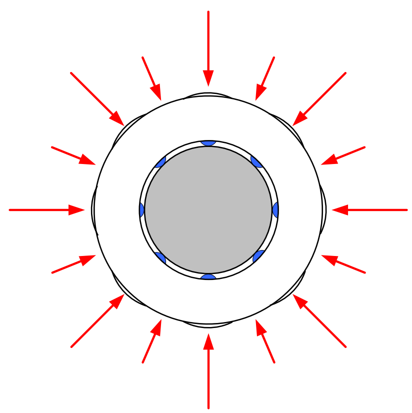
Pillow block



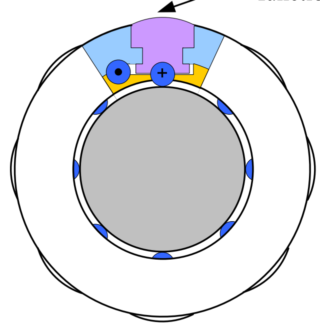


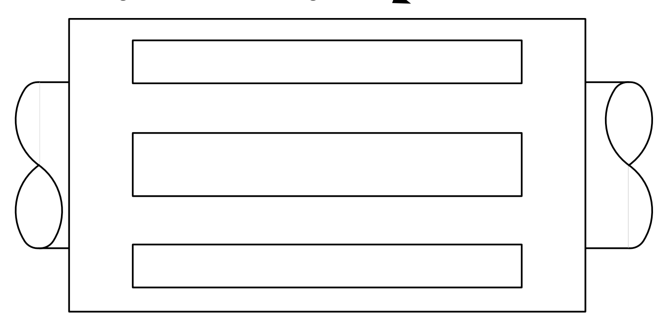
Load capacity diagram for a Thompson Super Ball Bushing® with 10 ball track-shaft contacts





Curved insert raceways enable self-aligning function when bushing is held in a housing





Linear Motion: *Linear Motion Guides*

Linear motion guides have an essentially rectangular cross section rail and rectangular box-shaped carriages that contain recirculating balls that roll in curved raceways on the rails and in the carriage. This type of design is said to have been invented in France in the 1930's, but commercial versions did not become widely available until better linear grinding machines evolved in the 1960's. Since then there have been many improvements, and many manufacturers of linear motion guides currently exist.

Typically, two rails and four carriages (blocks) are used to support a moving axis, although one can sometimes use a single rail and one or two carriages if the roll moments are modest. For purposes of accurately analyzing the load-carrying capacity and stiffness, a finite element model is needed that includes characteristics of the bed and carriage; however, one should use back-of-the-envelope calculations to preliminarily size members and choose bearings. The structure of the carriage should be designed so that its deformations are less than those of the bearing. For the purpose of estimating bearing reaction forces, assume that the carriage and the structure to which the rails are mounted behave like a rigid body. Page 10-33 details the analysis method.

There are a number of different manufacturers of linear motion guides, but there are basically two different types: those with circular arc grooves and those with Gothic arch grooves. Circular arc grooves contact balls at two points; hence in order to have bidirectional load capacity, four grooves per rail are required. The ball-groove contact vectors can be arranged in face-to-face or back-to-back configurations. Back-to-back designs have a higher moment capacity than do face-to-face designs. High moment capacity is useful for single-rail applications. For machine tools, moment capacity is obtained by spacing rails far apart and relying on the bearings' normal load capacity; thus high individual bearing carriage moment capacity can actually make a multiple bearing system more difficult to align and assemble without providing a much greater moment capacity for the system. Gothic arch grooves that contact balls at four points can have significant differential slip caused by the contact footprints being inclined to the ball's axes of rotation. Thus often the arches are offset so that the balls nominally make contact at two points and achieve rolling motion with lower friction. In the event of an overload, the deflection of the balls in one of the grooves causes the balls to make four-point contact with that groove.

It is not possible to generalize and say that one design type can carry more load or is stiffer than the other. One can always select a unit from a design type that has the desired load capacity and stiffness. Design engineers should also consider other important factors such as accuracy as a function of life, lubrication and sealing, and availability and service. Regardless of the type of linear bearing selected, it is also important to be careful if it will be subject to very slow motion, extended periods of vibration or impact loads, which can greatly decrease life. Manufacturer's catalogs usually derate the load capacity under these circumstances.

Running parallelism is a measure of how the carriage moves with respect to the rail as it moves, so accuracy of the bearing is highly dependant on how the rail is mounted to the machine. Linear motion guide rails are often ground when bolted to a grinding jig. To ensure that the rails deform to the same shape at which they were when the grooves were ground, it is important to specify that the tapped mounting holes be cleaned out and the manufacturers' suggested torque levels and tightening procedure be carefully followed. After tightening the bolts, the rail straightness should be checked. If necessary, the bolt torquing procedure may have to be modified.

To mount linear guides, one rail, which is referred to as the primary rail, is fully constrained. The secondary, or subsidiary, guide rail is made parallel to the primary rail using precision spacer blocks or a dial indicator. Alternatively, the carriages can be bolted to precision surfaces on the moving axis and moved along the rails. The secondary rail is forced by the bearing carriages to be parallel to the primary rail, and it is bolted in place as the axis is moved along. This is known as *zippering* the rail in place (see page 10-27). For heavily loaded systems, precision surfaces can be machined against which the bearing rails are wedged and then bolted.

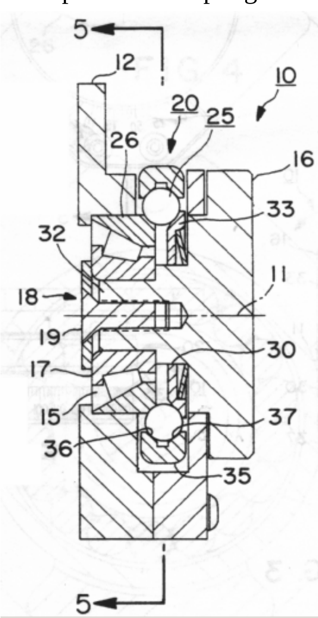
Linear guides are preloaded by the manufacturer with the use of oversize balls which will deform the carriage structure. Proper carriage shape is achieved when the carriage is bolted to a rigid flat surface. Hence many linear guide carriages have six mounting holes in them, and the purpose of the center set of holes is to pull the carriage surface flat against a reference surface to ensure that the proper preload is set and maximum stiffness is achieved.

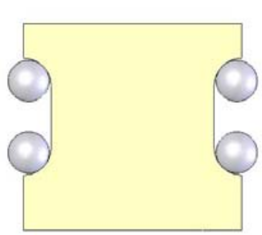
Don't you wish you had linear guides in your kit of parts?!

Linear Motion: *Linear Motion Guides*

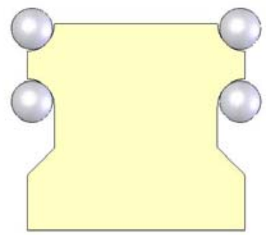
- Linear guides evolved from ball bushing bearings when it was noticed that overloaded bushings' balls cold-formed conformal grooves in round shafts
  - The next step was to grind conformal grooves in round shafts (rails)
  - Next the rails evolved into rectangular profiles which were easier to support along their length
- Linear guides are one of the most widely used modular linear motion components
- Many modern machine tools use linear guides to support their axes

US Patent 5,435,651 Reciprocity: it turns the "problem" of differential slip into an opportunity! Look it up on www.uspto.gov

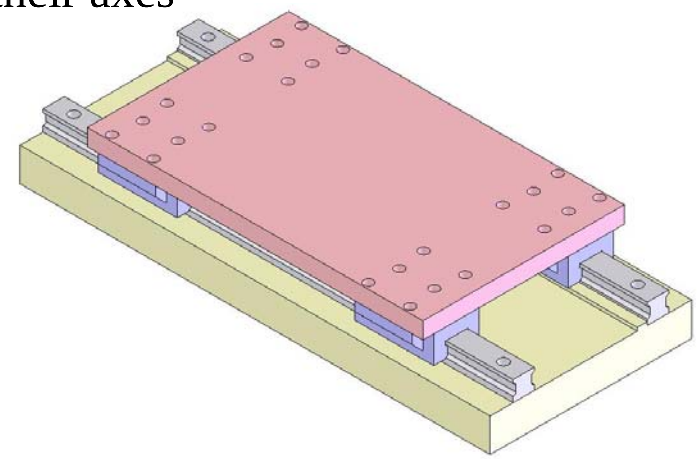




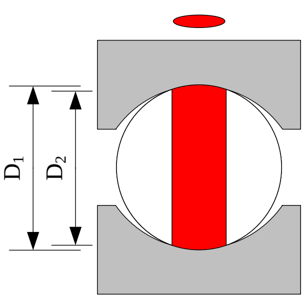
Back-to-back ball-guide track linear guides support moment loads with a single rail



Face-to-face ball-guide track linear guides are more tolerant of misalignment

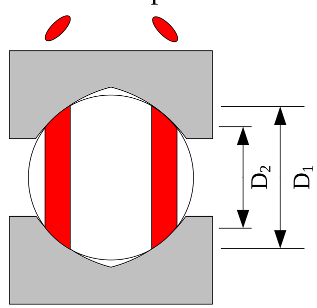


Contact footprint



Circular arch
2 contact points

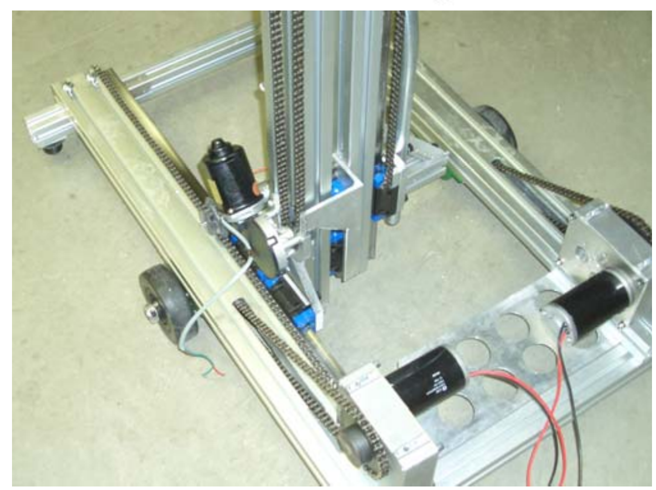
Contact footprints



Gothic arch
4 contact points

$$\%\ \text{slip} = \frac{D_1 - D_2}{D_1} \times 100$$

Linear guides used to support long travel axes on Bishop Brady High Schools 2005 robot (Manchester Regional Rookie of the Year). Vertical Z axis uses single rails. Horizontal X axis also has a sliding contact outrigger bearing



Contact Bearings: *Flexural*

Flexural bearings rely on elastic deformation to attain smooth motion. Since there are millions of planes of atoms in a typical flexural bearing, an averaging effect is produced that allows flexural bearings to achieve atomically smooth motion. For example, flexural bearings allow the tip of a scanning tunneling microscope to scan an object with subatomic resolution. There are two types of flexures, *monolithic* and *clamped*.

Flexural bearings' major drawback is their limited range of motion. For monolithic bearings, the ratio of range of motion to bearing size is on the order of 1/20. For flexural bearings made from high strength spring steel clamped in place, the ratio of range of motion to bearing size is on the order of 1/10. Unlike other bearings, flexural bearings are inherently preloaded and they generate a restoring force because they are essentially self-guided springs. Their applications range from silicon Micro Electro Mechanical Systems (MEMS), to fine instruments, to large trucks and trains where leaf springs also guide the motion of axles. They can also be used as "living hinges" in consumer products where plastics can endure very large deformations for the intended limited life of the product. In robot design contests, they can be made from sheet metal and used to create small range of motion adjustment mechanisms, or as flexural Vees for kinematic couplings (see page 9-20).

Because flexural bearings are elastic elements, they are easily designed using the basic principles of strength of materials.[1] Flexures are easily designed and made from either bolted-together components or cut from a plate. Most flexures are used as either pivots, or as four-bar-linkages. For a simple pivot as shown:

$$\alpha = \frac{3MR}{2Ew\left[\gamma^2 - R^2\right]}\left\{\frac{1}{\gamma} + \left[\frac{1}{\gamma^2 - R^2}\right]\left[\frac{2R^2+\gamma^2}{\gamma} + \frac{3R\gamma\left(\frac{\pi}{2} - \tan^{-1}\left(\frac{-R}{\sqrt{\gamma^2-R^2}}\right)\right)}{\sqrt{\gamma^2-R^2}}\right]\right\}$$

For a 4-bar flexure with blades of length *L*, thickness *t*, and width *w*, subject to a force *F*, each blade is subject to *F/2*, and the maximum bending moment in the blades will be FL/4. The first-order deflection δ of the platform in the desired motion direction, and the maximum stress will be:

$$\gamma = t/2 + R \qquad \delta = \frac{FL^3}{2Ewt^3} \qquad \sigma = \frac{3FL}{2wt^2}$$

There are two *parasitic error motions* that occur with simple 4-bar flexures: motion of the platform normal to the direction of motion and pitching motion that occurs when the force is applied to the platform to overcome the spring force of the blades. For small displacements δ, distance between flexure blades *b*, and force applied at a distance *a* above the fixed end of the springs, the parasitic error motions are:

$$\delta_{parasitic} \approx \frac{\delta^2}{2L} \qquad \theta_{pitch} \approx \left(\frac{6(L-2a)t^2}{3Lb^2 - 2Lt^2 + 6at^2}\right)\frac{\delta}{L}$$

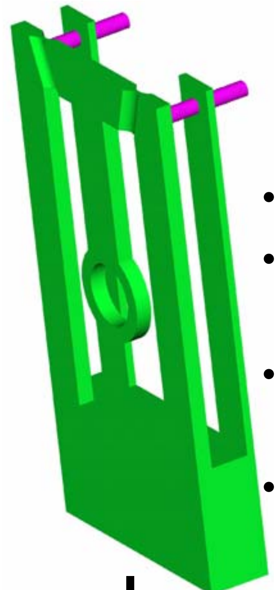
These errors can be overcome by twice applying reciprocity to flip the flexure twice, which results in a *folded flexure* design as shown in purple. There are four sets of 4-bar flexures, and the motion is $\delta_{folded\ flexure} = 2\delta_{4\text{-}bar}$ Also note the green flexure with the purple screws and the round holder located 2/3rds of the way back from the ends of cantilever beams. Recall that $\delta = FL^3/3EI$ and the slope $\alpha = FL^2/2EI$. Here a sine error $L\alpha$ is used to cancel deflection so the round holder (of a lens!) undergoes pure rotation. Why are the purple screws threaded into thin cantilevers that push on thicker cantilevers? Can flexures act as transmission elements? What is the Transmission ratio? This clever device was created by Dr. William Plummer of Polaroid.

Flexures can be made using an abrasive waterjet, or milled, but thin blades must be supported. For precision flexures, wire EDM can be used to cut them from hardened steel.

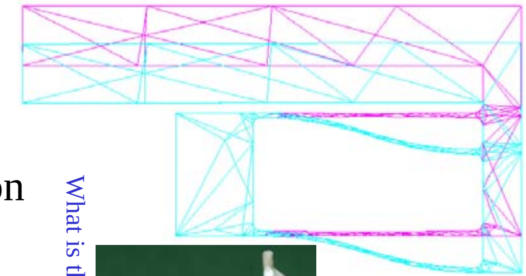
Identify small range of motion elements in your machine, such as triggers or alignment devices, where flexures could provide the desired motion without any parasitic friction forces. Play with the flexure design spreadsheets *Flexures_4_bar.xls* and *Flexures_hourglass_pivot.xls.*

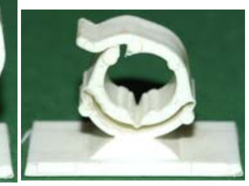
1. For detailed analysis, see for example Awtar, S., Slocum, A.H., and Sevincer E., "Characteristics of Beam-based Flexure Modules", ASME Journal of Mechanical Design, Vol.129, Issue 8, Aug 2007

Contact Bearings: *Flexural*

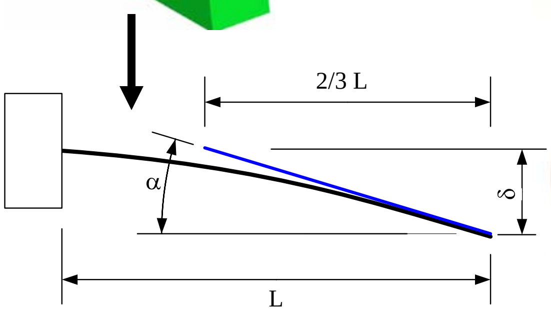


- Flexures use elastic deformation to provide the desired motion
- Linear and rotary motion flexures can be created
  - The challenge is to manage stress and constraints
- "Infinite-life: flexure will be 20x larger than the range of motion that it can provide (e.g., precision machines and instruments)
- "Short-life": flexure can be much smaller than other bearings (e.g., "living hinges" for flip-top bottles and lids)



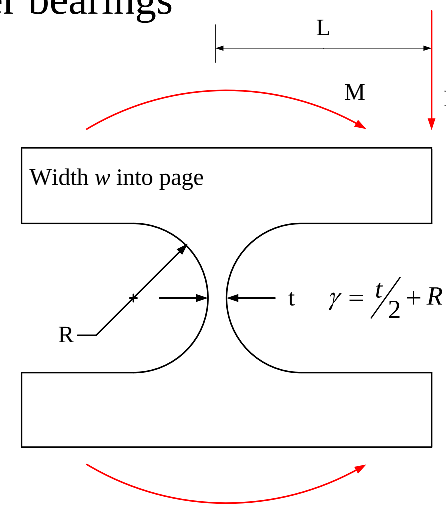


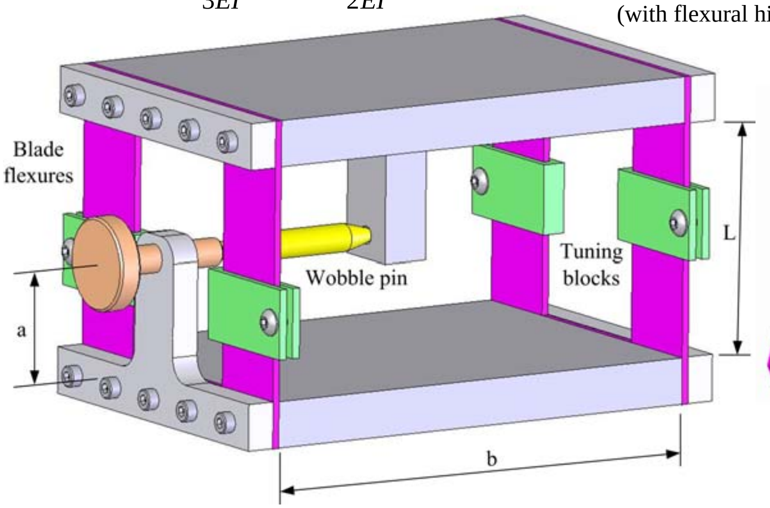
What is this thing used for?

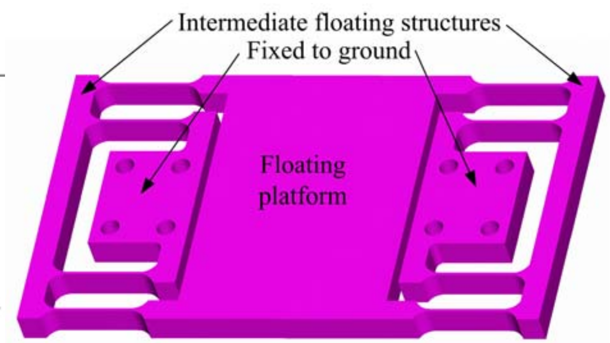


$$\delta = \frac{FL^3}{3EI} \qquad \alpha = \frac{FL^2}{2EI}$$

Its either the wonderfully designed top and flip-top cover (with flexural hinge), OR a happy space alien couple!







Folded Flexure

Cross-section rendering through a C'Flex flexural pivot bearing (see www.c-flex.com Special thanks to William R. Herbstritt of *Pageboys Web Design* for the rendering!)

Contact Bearings: *Flexural Rolling*[1]

As is often the case, a hybrid system can combine the best of both worlds. So it is with rolling contact flexural bearings. These bearings allow for compact rolling element joints to be designed. These joints can sustain large normal compressive forces due to the rolling contact, and respectable shear and tensile forces due to the flexible bands. This also make them inherently preloaded. Most importantly, they allow for rolling motion with a minimal number of elements and no lubrication which makes them particularly well-suited for precision instruments medical device applications.[2]

Since there is only one rolling contact, this type of joint is exactly constrained, and if the materials are corrosion resistant, no lubrication is required. Because there is rolling without slip, there will be very little wear. Beware, however, that under heavy loads, elastic deformation in the bands and Hertz contact deformations will occur. The flexible bands undergo bending just like the band drives described on page 5-6, and the analysis is well-understood and accurate, so the bands can be designed with a high degree of confidence.

The rolling contact flexure design can be applied to a revolute joint or to a linear joint. The former can be seen used in a pair of medical laparoscopic forceps. In addition, two rolling joints at right angles to each other can form an approximation of a spherical joint. Other medical applications such as artificial joints are likely to be developed (maybe by you!).

Flexural rolling bearings' kinematics of motion is straightforward given the constraint of rolling without slip. The radii are known and the angle that one member rolls is the input, and the other two angles can be readily calculated:

$$r_1\theta_1 = r_2\theta_2 \Rightarrow \theta_1 = \frac{r_2\theta_2}{r_1}$$

$$\theta = \theta_1 + \theta_2$$

Linear motion flexural rolling joints can also be created. The Bell-Everman SuperZ/Macro-Flexure™ stage uses rolling shear members whose width and orthogonal arrangement provides high shear and moment stiffness to the moving platform even though they are as thin as 0.1 mm. As the stage moves up by δ, the rollers translate upward by $\delta/2$. The flexural elements themselves are inherently preloaded. Parallelism of the stage from top to bottom of 1 micron is achievable throughout the stroke. A typical stroke is ½ of the over-all height, with no possibility of binding. Other important features of this design are low particle generation, vacuum compatibility and zero maintenance.

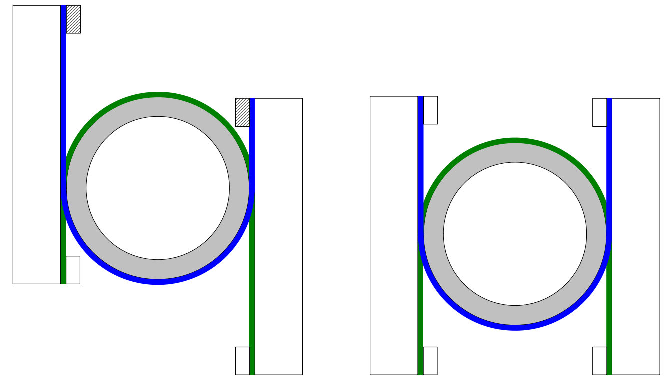
The actuation mechanism to control the linear motion can be a direct-drive electromagnetic system, such as a Lorentz-force actuator (see page 7-12), or a mechanical actuator such as a ballscrew (see page 6-3). Coupling a mechanical actuator to the system must be done with care so small parasitic error motions and reaction forces of the actuator do not cause unwanted error motions in the stage.

How might a flexural rolling element bearing be useful in your machine? Can you make the flexural elements out of rubber bands? Do you have thin-enough metal strips? Do not forget to calculate the bending stresses in the bands! Never use technology for its own sake, make sure that the functional requirements of the system warrant its use.

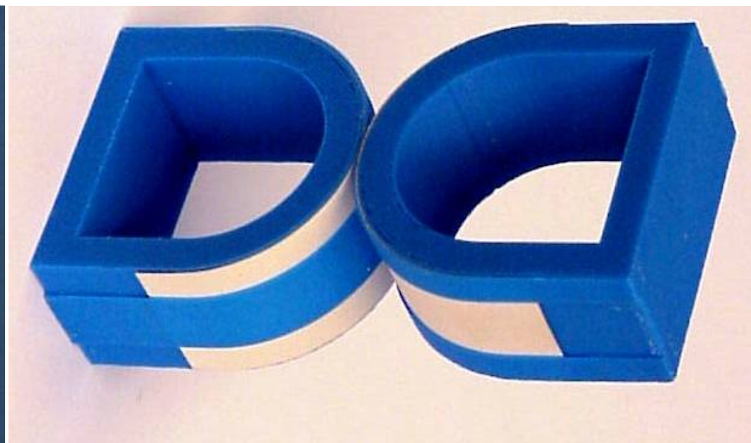
1. See for example Jeanneau, A., Herder, J., Laliberte, T., Gosselin, C., "A compliant Rolling Contact Joint and its Application in a 3-DOF Planar Parallel Mechanism with Kinematic Analysis", Proc. ASME Design Eng. Tech. Conf., Sept 28 - Oct 2, 2004, Salt Lake City, Utah, DETC2004-57264. Special thanks to Dr. Just Herder (J.L.Herder@WBMT.TUDELFT.NL) of Delft University of Technology for his input to this section.
2. See for example: Jobin J-P, Buddenberg HS, Herder JL, "An underactuated prosthesis finger mechanism with rolling joints", Proc. ASME Design Eng. Tech. Conf., Sept 28 - Oct 2, 2004, Salt Lake City, Utah, DETC2004-57192; Riele FLS te, Herder JL, "Perfect static balance with normal springs", Proc. ASME Design Eng. Technical Conf., Sept 9-12, 2001, Pittsburg, Pennsylvania, DETC2001/DAC21096; Herder JL "Force directed design of laparoscopic forceps", Proc. ASME DETC 25th Biennial Mechanisms Conf., Sept 13-16, 1998, Atlanta, Georgia, DETC98/MECH-5978; Visser H de, Herder JL, "Force directed design of a voluntary closing hand prosthesis," Jouo. Rehabilitation R&D, 37 2000, (3)261-271.

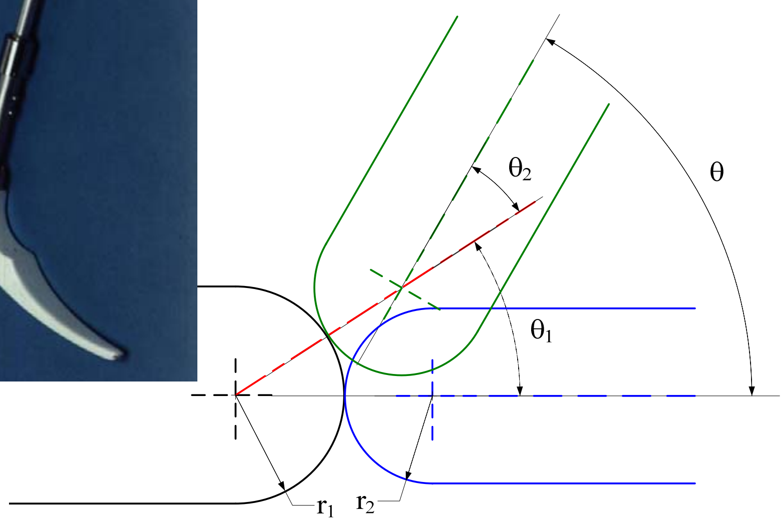
Contact Bearings: *Flexural Rolling*

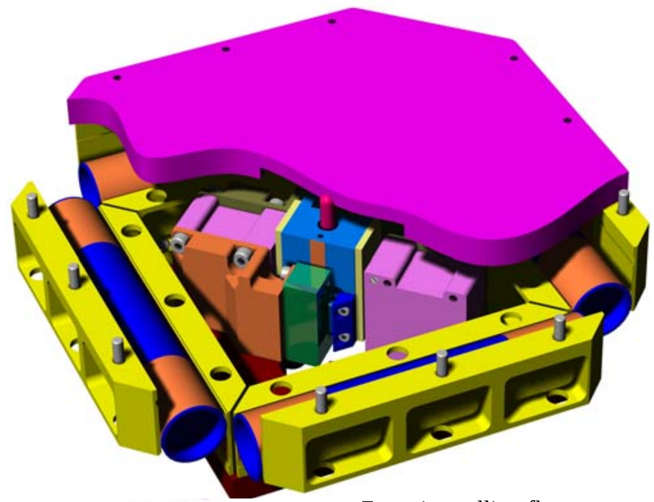
- The hybrid combination of flexing & rolling can provide the best of both worlds
  - Large rolling surfaces can provide accurate motion
  - Thin flexible bands constrain the rolling surfaces

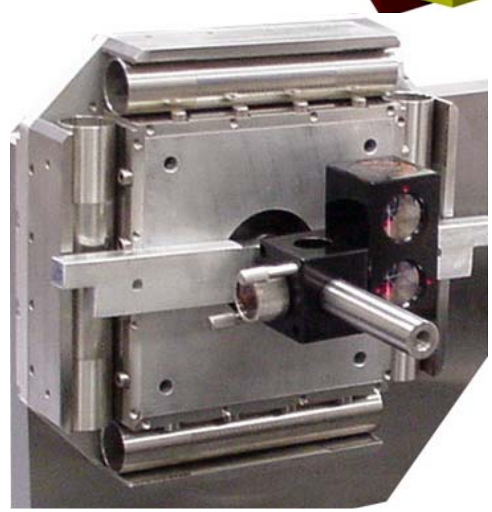


Rolling joint model and forceps images provided by Dr. Just Herder of the Delft University of Technology









Z-motion rolling flexure stage designed by Mike Everman
www.bell-everman.com
Patent pending

Non-Contact Bearings: *Hydrodynamic*

Hydrodynamic bearings were probably used for many years before Lord Rayleigh, who was one of the pioneers of fluid film analysis (as well as many other things!), introduced the idea of a step to help build up the hydrodynamic wedge. In 1912, Albert Kingsbury discovered that for large bearings, such as those used to support turbines, it was impractical to create a flat-enough continuous thrust surface. He thus separated the surface into independently tiltable regions. The axis of tilt was positioned so the tilting pads would form a hydrodynamic wedge to withstand thrust loads. His company, as well as some of his original bearings, is still in business today! Indeed, hydrodynamic bearings are commonly used, e.g., in internal combustion engines, because of their simplicity and very high load capacity.

Hydrodynamic bearings can be configured to support radial and thrust loads with cylindrical and axial surfaces. The speed of the object being supported acts on the viscous lubricating fluid to drag it into the gap between the bearing surface and the moving object. They can be customized, or standard units can be purchased.[1] Cone-type bearings can also be created, but the coupled radial and axial motion make them more difficult to design and then the radial and axial motions can become coupled which can cause accuracy issues.

When is a bearing simply sliding contact or boundary layer lubricated and when does it become a full hydrodynamic bearing? The *Stribeck* curve shows the different operating regimes:

- *Boundary:* At low speeds, low lubricant viscosities, and high loads/unit area (load pressure), material-to-material contact occurs. Some lubricant will be dragged into the contact zones which prevent adhesions, so the effect is not as bad as if no lubricant was present. Most sliding contact bearings operate in this regime, and have a maximum pressure*velocity limit (PV limit).
- *Mixed:* As the speed and/or lubricant viscosity is increased and/or the load pressure is decreased, more and more lubricant is dragged into the bearing region, and fewer and fewer material-to-material contact points occur. As a result, the friction rapidly drops.

1. See for example, http://www.kingsbury.com and http://www.kingsbury.com/pdfs/universe_brochure.pdf

- *Hydrodynamic:* Once hydrodynamic lift is achieved, there is no mechanical contact between the bearing and the supported object, and friction only rises as a function of speed and viscosity due to viscous shear.

The characteristic factor for hydrodynamic bearings is the dimensionless expression $\mu n/P$, where μ is the lubricant's dynamic viscosity (Pascal * seconds Pa-s = N-s/m^2), n is the rotational speed in revolutions/second, and P is the load per area (F/A). The dynamic viscosity relates the relative shear stress to the velocity and the gap between objects:

$$\tau_{N/m^2} = \frac{\mu_{N-s/m^2} V_{m/s}}{h_m}$$

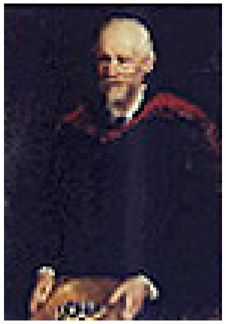
The force F to move an object with surface area $A = length*diameter$ at a speed V (m/s), or torque Γ required to turn a shaft of radius R with a radial clearance c between the shaft and bore are given by:

$$F_N = \frac{\mu_{N-s/m^2} A_{m^2} V_{m/s}}{h_m} \qquad \Gamma = R_m \frac{\mu_{N-s/m^2}\left(2R_m L_m\right)\left(2\pi_{rad/rev} n_{rev/s} R_m\right)}{c_m} = \frac{4\pi^2 \mu n L R^3}{c}$$

The design of hydrodynamic bearings requires extensive analysis, Fortunately, many resources exist. Just search the web[2]. In general, a designer who wishes to design a hydrodynamic bearing can expect to achieve an average constant unit load P = F/LD of about 10 atm (10^6 N/m^2) given appropriate lubricant viscosity, bearing gap, and a Length/Diameter ratio of at least 1. A thin oil film can support rapidly changing dynamic loads, such as experienced by an automotive engine crankshaft, up to 100 atm.

These guidelines enable a designer to estimate the size required for the a hydrodynamic bearing so an overall conceptual layout can be completed for a machine. If the concept is acceptable, the designer can then invest the time to detail the bearing design, or work with a bearing design company.

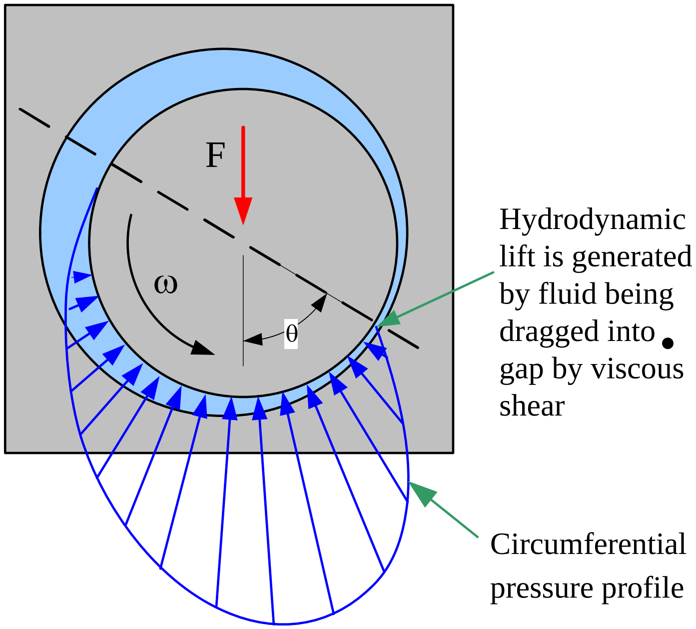
2. See for example J. Shigley & C. Mischke, Standard Handbook of Machine Design, Chapter 28 Journal Bearings (by T. Keith), Mcgraw-Hill Book Company, NYNY, 1986. Also, http://www.waukbearing.com/download_library

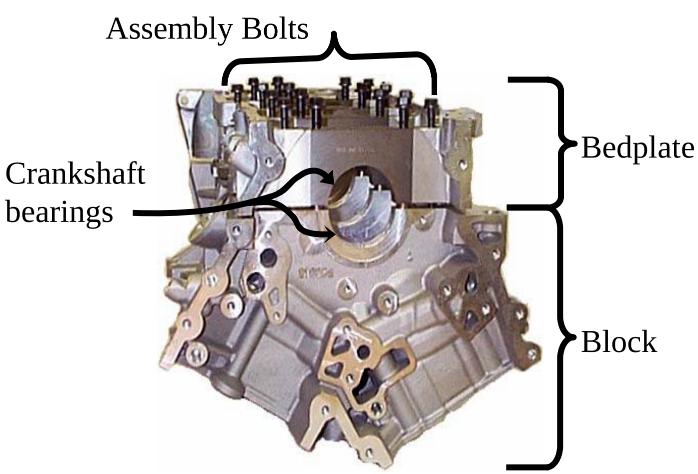


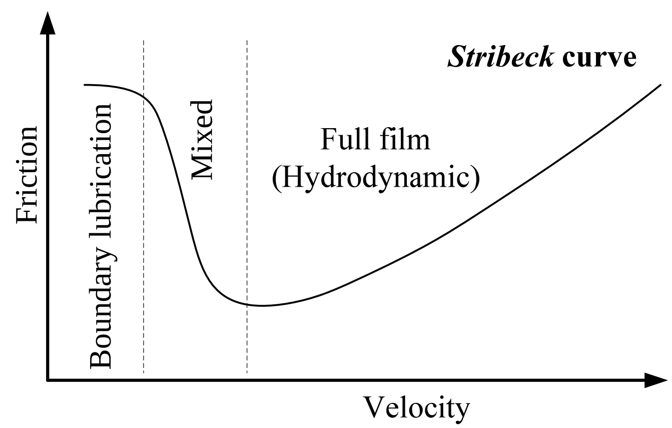
Osborne Reynolds (1842 - 1912)

Non-Contact Bearings: *Hydrodynamic*

- The industrial revolution was made possible by rotating shafts supported by thin films of lubricant induced by *hydrodynamic* shear
  - Load capacity dependant on area, velocity, and lubricant viscosity
    - L/D Aspect ratio of the area is important: L/D = 1:1 to 2:1 is best
  - They can last "forever" if they are never stopped
  - Their simplicity and low cost makes them widely applicable
- Compared to anti-friction (rolling element) bearings
  - They consume more energy via lubricant shear
  - They are less accurate
    - The axis of rotation position varies with load & speed
      - If load and speed are constant, they are extremely accurate







Albert Kingsbury 1863-1943

Non-Contact Bearings: *Aerostatic & Hydrostatic*

Aerostatic and *Hydrostatic bearings* utilize a thin film of pressurized air or oil, respectively, that is supplied from an external pressure or flow source to support a load. The fluid is pumped into a distribution manifold and through inlet restrictors that regulate the flow of fluid into each *bearing pad* that faces a surface such as a *rail* or a *way* in the case of a linear bearing, or a *bore* or *shaft* in the case of a rotary bearing. Each pad acts like a support point for the system because an individual pad cannot typically support a moment, only a force. When pressurized, the bearing pad never touches the surface, and bearing gaps are on the order of 0.005-0.1 mm. Aerostatic and hydrostatic bearings have been built with parts-per-billion accuracy, which is about two-orders-of magnitude better than rolling element bearings can achieve.

Aerostatic (often just called *air bearings*) and hydrostatic bearings can be configured in just about any form. The most common form utilizes sets of two pads that oppose each other about a rail or shaft to provide a system that has load capacity and stiffness in both directions. Both linear and rotary bearings are thus possible. Air bearings are typically limited to operate at less than 10 atm. of pressure for safety reasons, and hydrostatic bearings typically operate at 20-40 atm., although they can operate at up to 200 atm. When space is at a premium and large loads must be supported. The primary advantage of air bearings is that the air can flow out of the bearing without having to be collected, whereas a hydrostatic bearing requires a fluid collection system. The primary advantage of hydrostatic bearings is that they can support much larger loads in the same space, and can accommodate greater surface irregularities and have higher damping than air bearings. A prime example is the support of large telescopes.

For a linear motion air or hydrostatic bearing system, a carriage would typically have 6 sets of opposed pads to constrain five degrees of freedom. This means that the system is over constrained, but because of the fluid layer, a high degree of *elastic averaging* occurs as the fluid film accommodates surface irregularities. Another type of linear system uses an arrangement of pads to support loads and moments on a carriage. Gravity (weight), vacuum, or magnetic attraction[1] can be used to hold the pads down to the rail surfaces.

For a rotary motion air or hydrostatic bearing system, a housing would typically have 4, 5, or 6 sets of opposed pads at one end, and 4, 5, or 6 sets of pads at the other end to radially support a shaft at two places. The shaft would have a flange that is between air or hydrostatic thrust bearing surfaces. In all, five degrees of freedom would be supported. Alternatively, conical or spherical surfaces can be used. Each bearing pad must have an inlet restriction between it and the pressure source. This resistor acts in series with the fluid resistance leading out of the bearing pad. As a load is applied and the gap increases on one pad and decreases on the opposed pad, the pad resistances decrease and increase respectively:[2]

$$\Delta P = P_{\text{upper pad}} - P_{\text{lower pad}} = P_{\text{supply}} \left(\frac{R_{\text{upper gap}}}{R_{\text{upper restrictor}} + R_{\text{upper gap}}} - \frac{R_{\text{lower gap}}}{R_{\text{lower restrictor}} + R_{\text{lower gap}}} \right)$$

Since the fluid resistance is proportional to the gap cubed, the response to a load is very substantial. The electrical analogy is that of a resistance network where pressure is analogous to voltage: Without the inlet resistances, the pressure in the pads would not change even under load. The efficiency η is typically 25%-40%. The load capacity and stiffness of an aerostatic or hydrostatic bearing pad can be estimated from:

$$F_{\text{load capacity}} = \eta_{\text{efficiency}} P_{\text{supply pressure}} A_{\text{pad area}} \qquad K_{\text{stiffness}} = \frac{F_{\text{load capacity}}}{h_{\text{gap}}}$$

It is unlikely that an air or hydrostatic bearing would ever be needed in a robot design contest, but these bearings are of such immense importance, that it is vital to be aware of them and their characteristics. So next time you are playing air hockey, look how the puck is supported by a bunch of little air jets, each acting like a little aerostatic bearing pad, and tell yourself that only a geek would be worried about air bearings.

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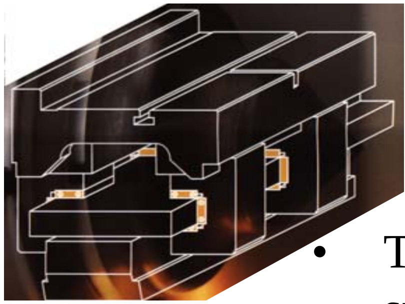
1. See A. Slocum, et-al., "Linear Motion Carriage with Bearings Preloaded by Inclined Iron Core Linear Electric Motor", Journal of the International Societies for Precision Engineering and Nanotechnology, 27 (2003) pp. 382-394. Also see US Patents 5,488,771 & 6,150,740.
2. This is for fixed inlet resistances. Self compensated hydrostatic bearings channel fluid from collecting regions to corresponding pockets on the opposite side of the shaft or rail. See Kotilainen, M., Slocum, A. "Manufacturing of Cast Monolithic Hydrostatic Journal Bearings", Journal of the International Societies for Precision Engineering and Nanotechnology, Vol. 25 (2001), pp. 235-244. Also see Slocum, A.H., Scagnetti, P.E., Kane, N.R., Brünnner, C., "Design of Self Compensated Water-Hydrostatic Bearings", Jou. Int. Soc. of Precision Engineering and Nanotechnology, Vol. 17, No. 3, 1995, pp 173-185. Diaphragm restrictors use a flexible diaphragm as part of a mechanical controller to minimize flow and greatly increase stiffness. See www.hyprostakik.de

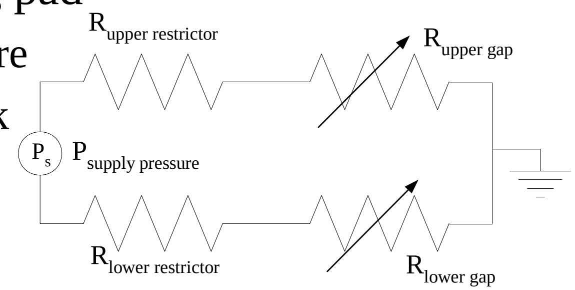
Non-Contact Bearings: *Aerostatic & Hydrostatic*

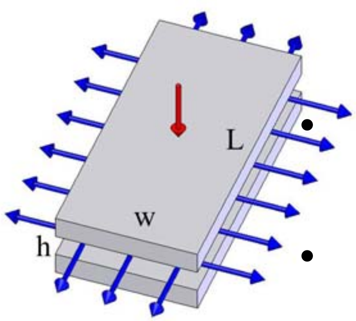


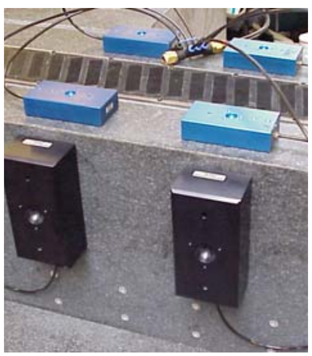
Weldon's 1632 Gold grinding machine carriage

What does it take to design a machine for this environment that can hold sub-micron tolerances? Thanks Reg Maas at CoorsTek!

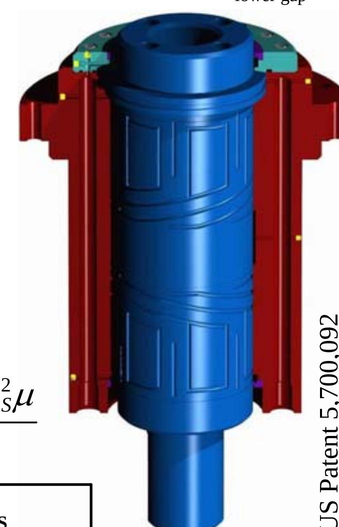
- These bearings rely on an external pressure source to supply flow through an inlet restrictor to a bearing pad
  - The bearing pad has resistance to flow to atmosphere
  - This forms a pressure (i.e., voltage) divider network
- Gas bearings do not require collection systems
  - Pressures are typically limited to 8 atm
- Liquid bearings typically use collection systems unless cutting fluid (e.g., water for ceramics) can be used
  - Pressures are typically 40 atm, but can be 200 atm
- Principle relations Force F, Stiffness k, Damping ζ, Flow Q, Pump power P_{pump} (bearing area L x w, viscosity μ, supply pressure P_S, gap h)



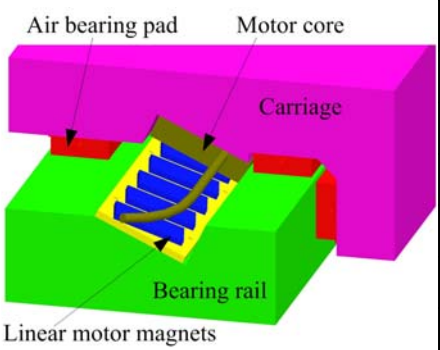




$$F \propto LwP_S \qquad k \propto \frac{LwP_S}{h} \qquad \zeta \propto \frac{Lw^3\mu}{h^3} \qquad Q \propto \frac{(L+w)P_S\mu}{h^3} \qquad P_{pump} \propto \frac{(L+w)P_S^2\mu}{h^3}$$



US Patent 5,700,092 Assigned to Hardinge, Inc



Constant Pump Power (P_{pump}) with Δμ		
Viscosity (μ)	10	1
Gap (h)	1	0.46
Flow (Q)	1	1
Pump power (P_{pump})	1	1
Stiffness (k)	1	2.15
Damping (ζ)	1	1

Constant Load Capacity (F) with ΔP_S			
Supply pressure (P_s)	2	1	1
Pad length (L)	1	2	1
Pad width (w)	1	1	2
Load capacity (F)	1	1	1
Pump power (P_{pump})	1	0.5	0.5
Damping (ζ)	1	2	8

Non-Contact Bearings: *Magnetic*[1]

It was shown analytically in 1842 by Samuel Earnshaw[2] that attempting to use only a magnet to support an object represented an unstable equilibrium; however, it was found in the 1930s that by using an electromagnet while measuring the air gap as a feedback parameter, the system could be stabilized. Although it is beyond the scope of this book to discuss how magnetic bearings are designed, an attempt will be made to introduce some of the characteristics of magnetic bearing-supported systems. Magnetic bearings will most likely become more commonplace in the machine design engineer's world as the quest continues for machines with nanometer accuracy to manufacture next generation microelectronic and optical components.

The biggest issue with magnetic bearings is that they require a closed-loop control system for stability. Typically, an analog control loop is used for coarse position control and a digital loop is superimposed on it for fine motion control and to compensate for analog component drift. An analog gap measurement sensor would be used with the analog control system. A laser interferometer, which has digital output, can be used as a high-resolution, high-bandwidth sensor for a fine-motion control digital control system.

Magnetic bearings do not limit the speed or acceleration of components they support. Rotary systems of 100,000 rpm and higher have been built for applications ranging from pumps to spindles for ultrahigh speed machining. Linear motion systems have been designed for ultra precision semiconductor manufacturing equipment with nanometer precision, and of course for large systems such as magnetically levitated trains

Motion control resolution of the bearing gap is also limited by the sensor and control system. There is virtually no mechanical damping in a magnetic bearing-supported system unless the suspended object is also in contact with a viscous fluid. Hence disturbance forces acting on the system play an important role in determining motion control resolution of the bearing gap. At low frequencies, performance is determined almost completely by the ability of the controller to cancel disturbances. The primary control system parameter affecting disturbance cancellation is the controller gain, which determines suspension stiffness. The higher the suspension stiffness, the greater the ability to reject disturbance forces. Depending on the nature of the source, at high frequencies, the disturbance forces are generally absorbed by the supported object's inertia and internal damping characteristics. In addition, a magnetic bearing's closed-loop control system can be used in conjunction with feedback from accelerometers to produce forces opposite to those created by vibration. The net effect is to cancel the vibration, which can be reduced by -20 dB using this type of system.[3]

All the electric current running through the coils generates significant amounts of heat and may require external cooling. For systems where the load does not vary greatly, a large percentage of the load can be supported by permanent magnets which minimize coil size and electric current required to levitate the load. As long as the coils are protected, magnetic bearings can operate in virtually any environment. They have been used successfully in the following environments: air with temperatures ranging from -235 to 450 ^{o}C, 10^{-7} torr to 8.5 MPa, water, seawater, steam, helium, hydrogen, methane, and nitrogen. One must ensure that in a corrosive environment the system's materials do not fail to perform their structural or sealing functions. In a normal environment there is generally no need to seal magnetic bearings; however, it is a good idea to seal “catcher” bearings, which come into play if the power fails.

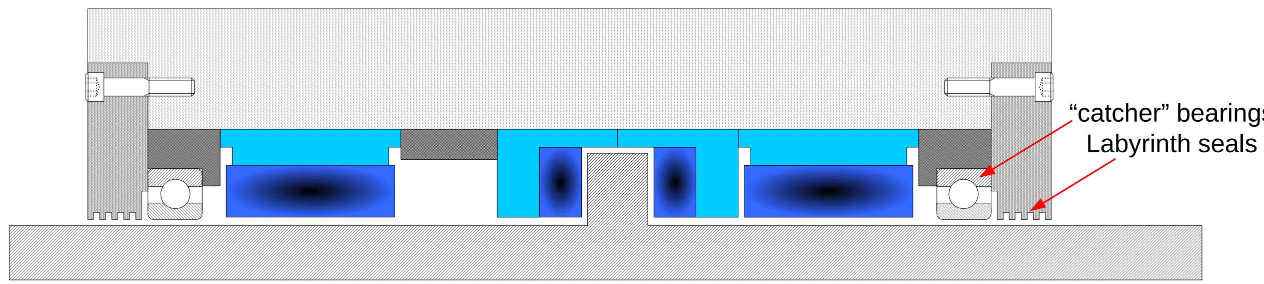
Magnetic bearings are much larger than the rolling element bearings they can replace, and they need large electronics support cabinets. In addition, they are generally custom designed for the application and are not available off-the-shelf except for some pre engineered spindle assemblies. It is thus not surprising that magnetic bearings are probably the most expensive type of bearing one can use; however, for the problems they solve, effective system cost can be low compared to design solutions that use other bearings. Examples include supporting pump shafts in natural gas transmission lines, and for the support of high-speed turbine rotors in turbo molecular ultra high vacuum pumps. These pumps use super high speed turbines to expel most molecules from a closed system.

1. See A. Slocum, Precision Machine Design, Chapter 9, SME 1995. Also see B. V. Jayawant, Electromagnetic Levitation and Suspension Techniques, Edward Arnold Publishers, Ltd., London, 1984.
2. Earnshaw, S., Trans Camb. Phil. Soc. 7, 97-112 (1842. Also see http://www.hfml.ru.nl/finger-tip.html and http://www.hfml.ru.nl/levitation-possible.html

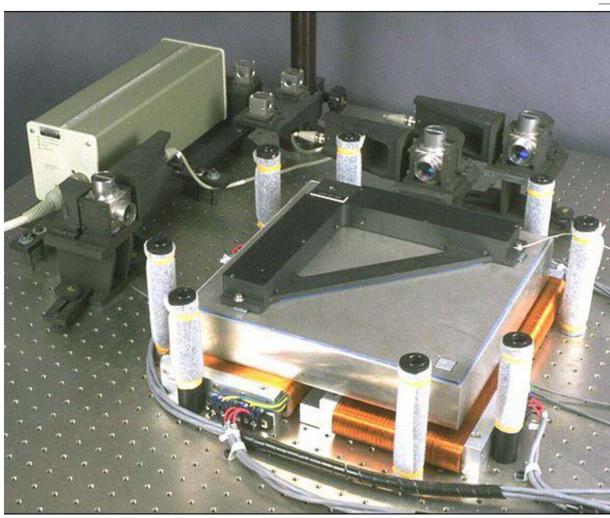
3. decibels (dB) = $10\mathrm{Log}_{10}$(value/ref. value); hence -20 db reduction = $10^{(-20/10)}$ ref. value; or a factor of 10 for every 10 decibels. For a nice definition of decibels (db) and some physical insight, see http://www.webster-dictionary.org/definition/decibel.

Non-Contact Bearings: *Magnetic*

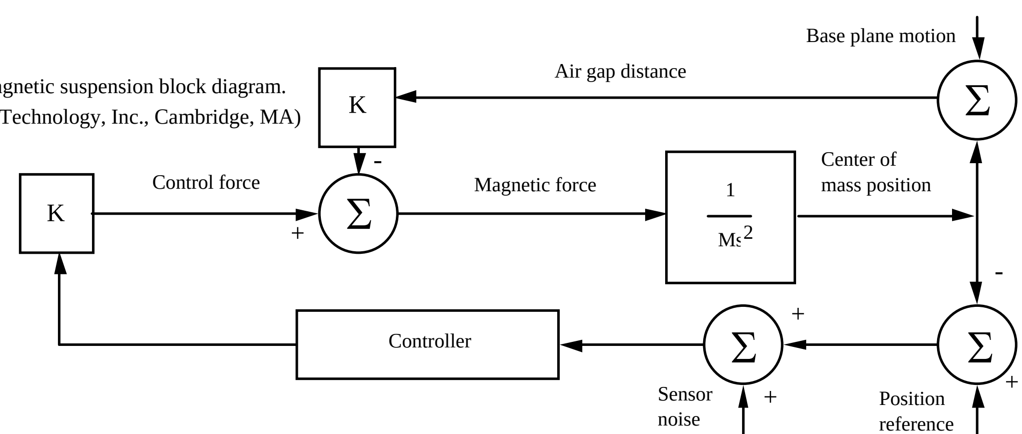
- Magnetic bearings have NO mechanical contact with the supported component
  - No speed limits: heat is only generated by shearing of the air between the coils and component
- Linear, rotary, or combined motions can be supported
  - Mechanical *catcher* bearings are also often used in case the power fails
- Sophisticated position feedback measurement and control systems are required
- First-order estimates of load capacity can be made by assuming a maximum "bearing pressure" (see page 7-10) on the order of 0.5 atm.



Magnetic bearing levitated stage from Prof. David Trumper's lab: http://web.mit.edu/pmc/www/Projects/Planar/planar.html



Basic magnetic suspension block diagram. (SatCon Technology, Inc., Cambridge, MA)



Preload

Bearings arranged in an opposing manner are usually required to support a bidirectional load. However, because a bearing also has typically has very high stiffness, unless all the surfaces are perfect, there must be some space or compliance between the bearings and the support surfaces. Otherwise as the motion occurs, by $F=k\delta$, the bearings will oppose each other and significant forces can result. If the stiffnesses are carefully selected, good performance can be obtained.

Assume that a set of bearings that opposes each other is loaded such that there is no gap between either bearing and the bearing rail. The bearings are said to be preloaded. In this case, even though the individual bearings can only support compressive loads, together they can support a load in either direction. In addition, each bearing has a stiffness associated with it, and the bearings are preloaded with an initial displacement. This means that the stiffness for each bearing is effectively bi-directional. A free-body diagram (draw it!) for the system shows that for a displacement δ on a preloaded bearing system:

$$F_{\text{load}} - \left(F_{\text{preload}} + k_{\text{upper pad}}\delta\right) + \left(F_{\text{preload}} - k_{\text{lower pad}}\delta\right) = 0$$

$$F_{\text{load}} = \delta\left(k_{\text{upper pad}} + k_{\text{lower pad}}\right)$$

$$k_{\text{total}} = k_{\text{upper pad}} + k_{\text{lower pad}}$$

This equation reveals the secret of preloading a bearing system in the presence of surface irregularities that might otherwise cause it to bind and jam: One of the bearings should be a low stiffness bearing that can accommodate variations in the rail thickness Δt, and the other bearing should be a high stiffness bearing that resists the primary load in the system. This will result in a small local force on the carriage of only $k_{low}\Delta t$. On the other hand, if one attempted to impose a displacement Δ on the entire carriage, the force required would be $(k_{low} + k_{high})\Delta$. This is true as long as the force acting in the direction of unloading the low stiffness bearing is less than the preload force.

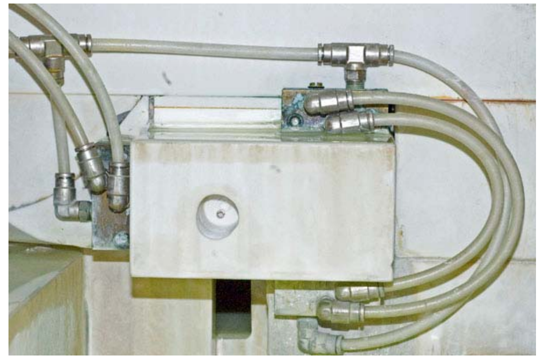
In the design and manufacture of modular rolling element bearings, quality control of the elements is so good, and the friction coefficient is so low, that all the effective bearing elements' spring constants can be high. For sliding contact bearings, the coefficient of friction is much higher, and thus it is important to use the principle of a low and high stiffness bearing. Recall the illustration of the boxway on page 10-5. The *keeper plates* on the bottom of the carriage act like flexural bearings (springs) to keep the bottom bearings preloaded against the rail with a low stiffness compared to the bearings on the top between the carriage and the rail.

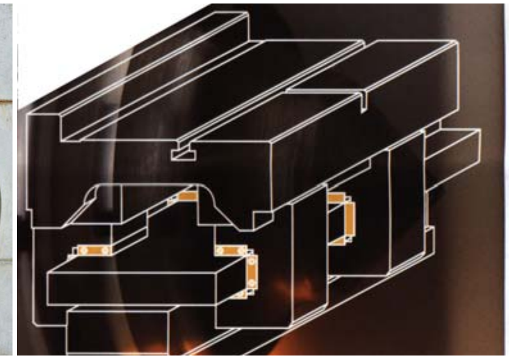
The same sort of system can be used to preload air bearings. Many large precision machines use spring-preloaded air bearings. Examples include coordinate measuring machines for measuring parts for quality control, and high speed machines for semiconductor manufacturing and circuit board drilling. The soft spring allows one air bearing pad to move up and down relative to the carriage to accommodate variations in bearing rail thickness, while a rigidly mounted opposed pad helps to support the weight of the carriage and the primary applied load.

Preloading of a shaft supported by sliding contact bearings (i.e., bushings) is more difficult. But, a shaft and a bore can be more easily made with high precision, so that the clearance between the bearing and the shaft is only on the order of 0.01-0.1 mm. Assume a spacing of 5 shaft diameters for the bearings that support the shaft, and the load on the shaft is applied at a distance less than 3 shaft diameters from the front bearing. The sine error (see page 3-11) at the point of load application due to the shaft tilting in the bearing bores will only be at about 1 + 3/5 of the bearing clearance.

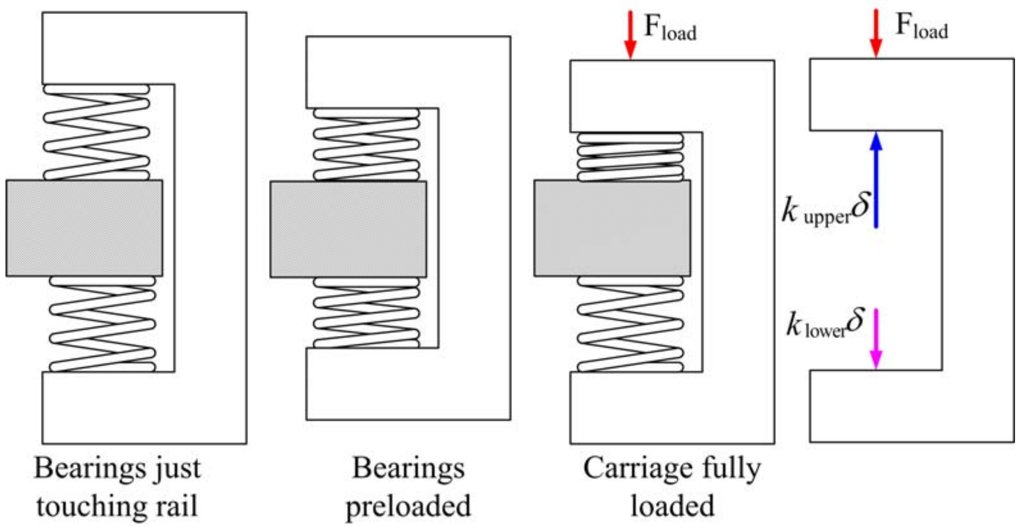
How might this preload principle be used in a robot design contest? For the motion of an axis where accuracy is required, or in the guidance of a projectile to obtain greater accuracy, a piece of sheet metal can act as a flexural element to preload one of a set of sliding contact bearings. Identify critical bearing components in your machine where you might need a high accuracy or repeatability of motion. Can these bearings either be preloaded, or spaced to give the desired level of accuracy or repeatability.?

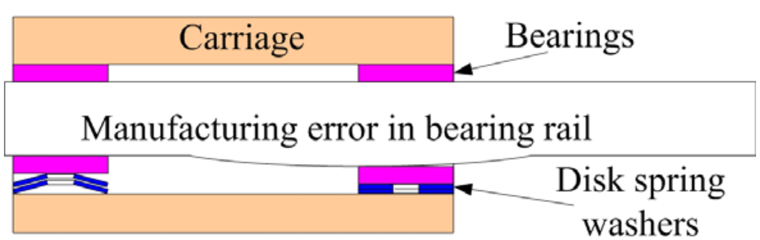
Preload





- Preload allows for bi-directional loading
  - If not careful, leads to over-constraint
- Preload maximizes stiffness
- Preload deflection is small, so preload can be lost by manufacturing error or wear
  - Preload loss via wear is avoided with the use of spring loaded preload systems
- Spring preloading allows dimension variations without a large change in preload force
  - Use springs or deformation of the bearings and/or structure





Preload: *Rolling Elements*

Rolling element bearings need to eliminate internal clearance, sometimes referred to as *backlash*, in order to increase the accuracy, stiffness and shock load capacity of the bearing. However, preload also causes the rolling elements to flatten slightly and thus effectively roll on different diameters. This shears the lubricating fluid and generates heat. For a rolling element diameter D and Hertzian contact zone of width d, the amount of slip can be estimated from:

$$\frac{2d}{D} = \sin\theta \approx \theta \qquad \delta = \frac{D}{2}(1-\cos\theta) \approx \frac{D\theta^2}{4} \quad \Rightarrow \quad \delta = \frac{d^2}{D}$$

$$\%_{\text{slip}} = \frac{D-2\delta}{D}\times 100 = \left(1-2\left(\frac{d}{D}\right)^2\right)\times 100$$

With the Hertz contact equations on page 9-12, assume a circular zone where $c = d$, to show that the radius of the contact zone is proportional to the contact pressure. Operating at a preload level that is a substantial proportion of the maximum load capacity generates excessive heat by differential slip. Fortunately, it only takes a little preload to achieve good performance:

- *Heavy preload* is typically 5% of the maximum bearing load and is used when the system is subject to heavy shock loads. These bearings are not to be run at high speeds because heat generation will be fairly high.
- *Medium preload* is typically about 3% of the maximum bearing load and is used when the system is subject to moderate shock loads.
- *Light preload* is typically about 1% of the maximum bearing load and is used when the system is only lightly loaded, but it is to be run at its maximum speed for extended periods.
- *Clearance* is specified for low cost bearings or for bearings which are known to be mounted where misalignment is likely to be encountered. However, bearings with clearance are easily damaged by shock loads.

Preload fully loads all the rolling elements so that they have a finite Hertzian contact zone and thus provide stiffness and resistance to load from all the rolling elements. A thin film of lubricant in the Hertzian contact zones helps to act like a damper and prevent damage to the bearing. A bearing without preload only has one rolling element in contact, and an impact load causes the very small wedge of fluid directly beneath the contact point to be driven into the raceway like a penetrating projectile.

Bearings can either be preloaded with the use of oversize rolling elements, by forcing one bearing against another, or by forcing radial expansion or contraction of the inner or outer races respectively. Deep groove bearings, four-point contact bearings, double-row, and self-aligning bearings are most often preloaded with the use of oversize rolling elements.

Angular contact and tapered roller bearings are manufactured with axial offsets between the inner and outer races. A pair are then preloaded by forcing them axially together so there is no axial clearance between the outer or inner races. When the outer rings are forced together, a *face-to-face* mounting is achieved. The lines of force all intersect at the center of the system, which means that there is one instant center of rotation and that the bearing set has essentially no moment resistance. This makes this combination particularly well-suited where there might be some misalignment between sets of bearings. On the other hand, imagine what happens when the bearings get hot. The inner ring gets hot faster because the outer ring has a big housing in which to dissipate heat. Axial and radial expansion of the inner rings causes balls to be compressed. This bearing mounting is thermally unstable at high speeds.

Applying reciprocity, if the bearings are mounted so the inner rings are forced together to preload the bearings, then a *back-to-back* mounting is achieved. Here the instant centers are located outside the bearings and they can support large moments. On the other hand, they are sensitive to misalignment when pairs of sets are used to support a shaft. Now imagine what happens when they get hot: The compression of the balls caused by the shaft and inner rings radially expanding is relieved by axial expansion. Hence the bearings are thermally stable and are referred to as a *thermocentric* design.

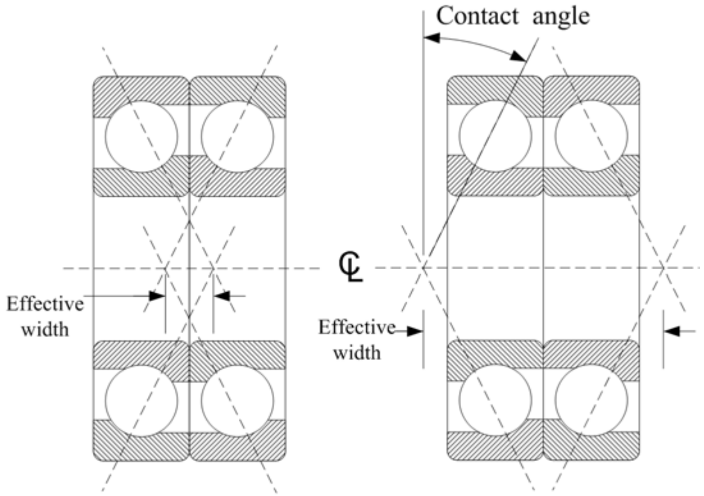
Cylindrical roller bearings are often preloaded by expanding or contracting the inner or outer rings respectively. This also means that a bearing can be inadvertently preloaded by pressing it into a bore or onto a shaft!

How are you using rolling element bearings in your machine, and will your bearings require you to preload them, or can you use bearings preloaded with negative internal clearance? What types of bearings do you have?

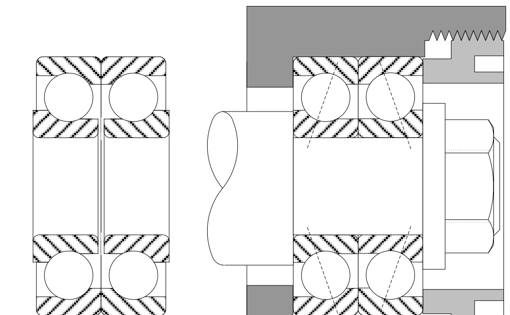
Preload: *Rolling Elements*

To increase stiffness and resistance to impact loads, rolling element bearings can be "preloaded"

- All the rolling elements must be under load, often one element loaded against another
- When preloaded, even the rolling elements "in tension" act as springs to provide stiffness
- Clearance may be required to allow for load deformations (see ISO5753 or ABMA 20)

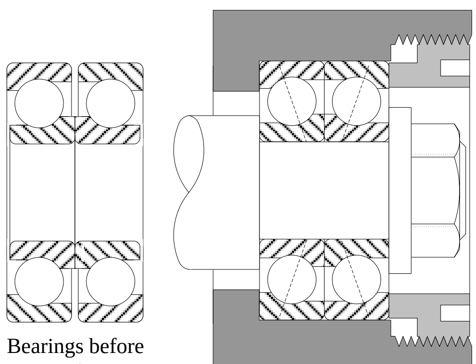


Face-to-face (DF) Back-to-back (DB)



Bearings before mounting (inner ring axial clearance exaggerated)

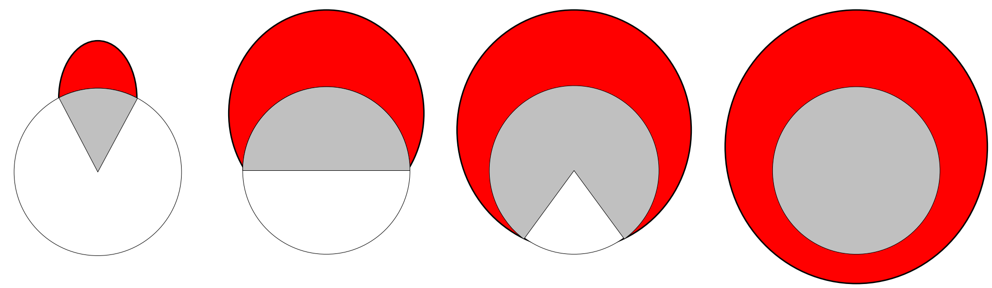
Back-to-back mounting after inner rings are clamped together



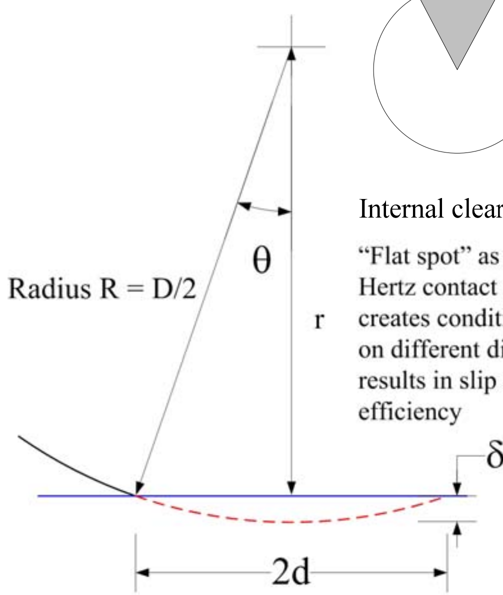
Bearings before mounting (inner ring axial clearance exaggerated)

face-to-face mounting after outer rings are clamped together

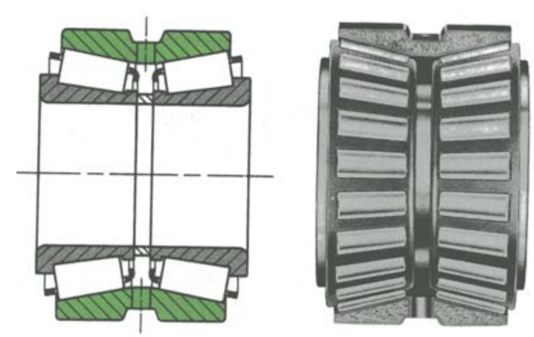
Load distribution on rolling elements due to radial load applied to bearings with various preload conditions



Internal clearance Zero clearance Light preload Medium preload



"Flat spot" as a result of Hertz contact deformation creates condition of rolling on different diameters which results in slip and loss of efficiency



Timken double row tapered roller bearings (back-to-back preload)

Mounting

How a bearing is mounted in a structure has as much to do with system quality and performance as the proper selection of the bearing and how the bearing supports the moving component. All mechanical elements have finite stiffness, and when forced to accept a displacement, δ, a force will result on the element because $F = k\delta$. Such forces are unavoidable, because no system is perfect. On the other hand, if they are accounted for in the consideration of the total effective loads on the system, then a bearing can be selected that can handle the loads and deliver the desired life and performance.

Now expand the idea of $F = k\delta$ into six dimensions: Every machine element has *six degrees of freedom*, and each degree of freedom has to be constrained or allowed to be free according to the functional requirements of the system. Most novice designers err by not properly identifying the degrees of freedom. The second most common error is to improperly add constraints to constrain the degrees of freedom, or to add too many constraints. A designer can learn to instinctively properly constrain a system with astute consideration of the FUNdaMENTAL principles of Topic 3[1], and by sketching a set of coordinate axes on each major component while checking off how each degree of freedom is accounted for.

In general, it is easier to make a machine's structure less compliant (more stiff) than a bearing. However, Maudslay's Maxim behooves us to ask the question; why are we spending resources to make a structure extremely stiff, only to mount a relatively compliant bearing on it? If the goal is to create a very stiff system, since a structure is generally less expansive to make than a bearing, it would make sense to make the structure to be several times less compliant than the bearing. Beyond that, one reaches a point of diminishing returns.

Typically the bearing stiffness, $k_{bearing}$, and the structure stiffness, $k_{structure}$, act in series. When a force is applied, each deflects by a proportionate amount, The system stiffness is the force divided by the total deflection:

1. It is one thing to read about the issues involved, and it is another thing to really have them ingrained in your thoughts. Warning, this is a nag: To be a very good designer, one must have one's neurons preloaded with the FUNdaMENTAL principles (see Topic 3) and consider them whenever designing.

$$\delta_{\text{bearing}} = \frac{F}{k_{\text{bearing}}} \qquad \delta_{\text{structure}} = \frac{F}{k_{\text{structure}}} \qquad \delta_{\text{total}} = F\left(\frac{1}{k_{\text{structure}}} + \frac{1}{k_{\text{bearing}}}\right)$$

$$F = \frac{\delta_{\text{total}}}{\left(\frac{1}{k_{\text{structure}}} + \frac{1}{k_{\text{bearing}}}\right)} \Rightarrow k_{\text{system}} = \frac{1}{\left(\frac{1}{k_{\text{structure}}} + \frac{1}{k_{\text{bearing}}}\right)}$$

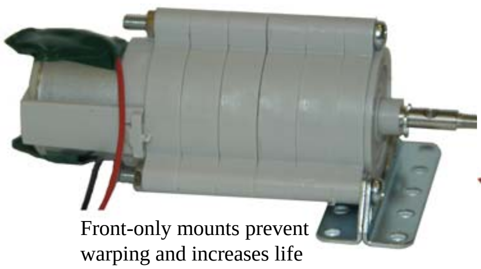
This relation holds true for linear and angular displacements and stiffnesses. The graph shows the effect of relative bearing and structure stiffness. At one extreme, it is not uncommon for a bearing to be stiffer than a structure when the bearing must have a very long life. This would also enable the bearing to better tolerate some over constraint. At the other extreme, where total system stiffness is to be maximized, such as in machine tools, Saint-Venant's principle seems to come into effect (surprise!), where if the structure is more than 3-5 times stiffer the bearing, not much more gain is achieved.

The expression for total system stiffness can also be used to estimate the loads placed on the bearings by misalignment of the bearings and the structure. These loads must be added to the functional loads that the bearings were intended to support (see *Bearing_stiffness_alignment.xls*).

$$F_{\text{misalignment}} = k_{\text{system}}\delta_{\text{misalignment}}$$

The process of going from a concept to detailed drawings is a critical phase for creating the mountings by which bearings will be held. It is at this phase that structure is defined, and enough structure must be provided to support the bearing either rigidly ($k_{structure} > k_{bearing}$) or compliantly ($k_{structure} < k_{bearing}$). Either case is acceptable, as long as the design engineer is aware of what they are doing, and has considered the effects with regard to the functional requirements that the system is supposed to meet.

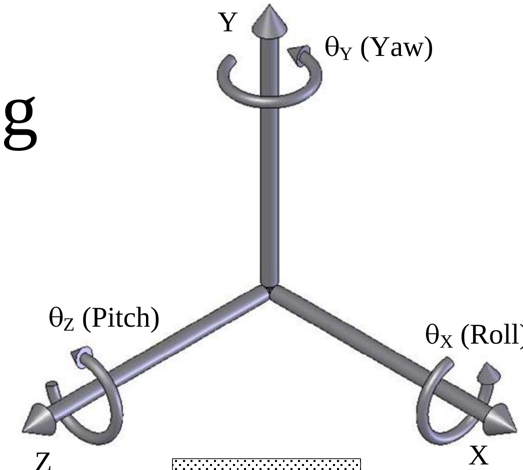
Review your expected use of bearings and where they need to be mounted in your structure. What are the implied stiffness or misalignment requirements of the system's functional requirements, and how might the mounting be designed to better meet these goals?



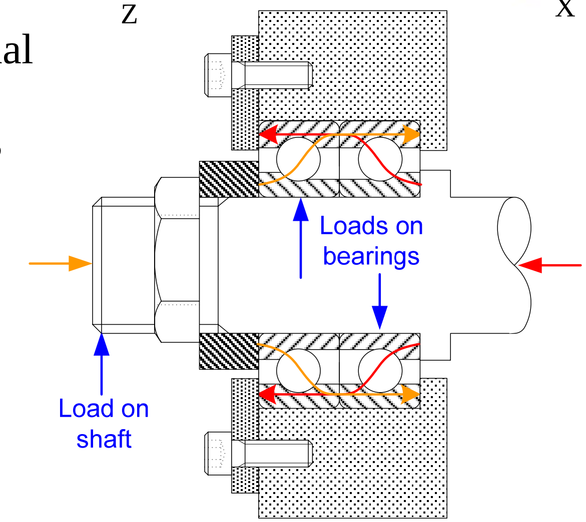
Front-only mounts prevent warping and increases life and efficiency

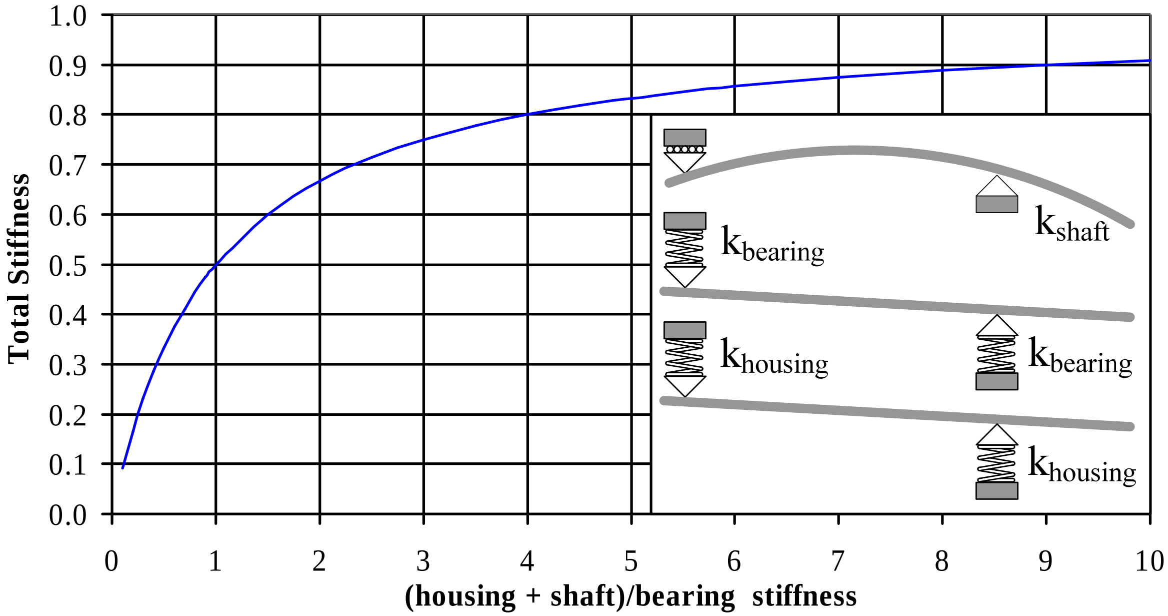
Will front & back mounts lead to warping of the gearbox?

Mounting



- Do a sensitivity analysis and base design decisions on analysis or experiments
- See ABMA 7 for ball, cylindrical, and spherical contact bearings; and ABMA 19.1 and ABMA 19.2 which cover fitting practices for typical applications
- *Exact Constraint Design:* The number constraint points should be equal to the number of degrees of freedom to be constrained
  - ***If deformations occur in your machine, will the bearings be overloaded?***
- *Elastic Averaging Design:* Total stiffness achieved by making sure many individual points (small spring elements) are all in contact





Mounting: *System Stiffness*

Bearings support applied loads and constrain components to move with a desired accuracy. However, the stiffer the bearing, the greater the indirect load on the bearing caused by misalignment. Even though balls contact a raceway at a small angle, e.g., 20 degrees, when axially loaded, all the balls share the load as compared to just a few balls when the bearing is radially loaded. Hence for most radial bearings, the axial load capacity is about equal to the radial load capacity, and only the radial load capacity is typically provided. The exception is for ballscrew support bearings, where the contact angle can be 50 degrees and the axial load rating can be several times higher, but in this case it is given by the manufacturer.

Often bearing catalogs do not provide radial stiffness data because it depends so much on the preload and stiffness of the mounting assembly. With conservative Hertz stress assumptions at bearing maximum load, estimates for radial and moment stiffness can be made. Assuming at the maximum static load F_{static} the strain is $\varepsilon = E/\sigma$, which is typically 0.5%, and that the strain takes place across the characteristic dimension of the bearing at this point, *(OD-ID)/2*, the estimated radial stiffness of the bearing is:

$$F = k\delta \Rightarrow k_{radial} = \frac{2F_{static}}{\varepsilon(OD - ID)}$$

What about moment (angular or tilt) stiffness? In general, a single rotary motion rolling element bearing, for example, is NOT meant to be loaded with a moment. Moments are to be withstood by multiple bearings, such as in a back-to-back configuration, where given the total radial stiffness k_{total} of a a pair of bearings spaced apart by $L_{bearing}$:

$$k_{angular} = \frac{k_{total} L_{bearing}^2}{4}$$

When estimating bearing loads due to angular misalignment, one can use the radial stiffness with basic bearing dimensions to estimate angular stiffness. Given a linear or rotary bearing system of characteristic length L *([OD+ID]/2)* with lateral stiffness $k = k_{lateral}$ (or K_{radial}) uniformly distributed over the length, if the system rotates an amount ϕ about its center of stiffness, then at a distance r from the center, the displacement will be $r\phi$. The differential stiffness at this point is kdr/L; hence:

$$dF = \frac{kdr}{L} r\phi \qquad dM = \frac{k\phi r^2 dr}{L} \qquad M = 2\frac{k\phi}{L}\int_0^{L/2} r^2 dr = \frac{kL^2}{12}\phi$$

$$\Rightarrow k_{angular} = \frac{k_{lateral} L^2}{12} \qquad \Rightarrow k_{angular} = \frac{k_{radial}}{12}\left(\frac{OD + ID}{2}\right)^2$$

Given the estimated angular stiffness $k_{angular}$ and an imposed angular misalignment ϕ, the resulting moment on the system is $M = k_{angular}\phi$. But what is the equivalent lateral (radial) force component that must be considered for bearing load/life calculations? A linear deflection profile was assumed, and hence a linear load profile can also be assumed:

$$dF = P_{max} r dr \qquad F = \int_o^R P_{max} r dr = \frac{P_{max} R^2}{2} \qquad R = \frac{1}{2}\frac{(OD + ID)}{2}$$

$$dM = rdF \qquad M = 2\int_o^R P_{max} r^2 dr = \frac{2P_{max} R^3}{3}$$

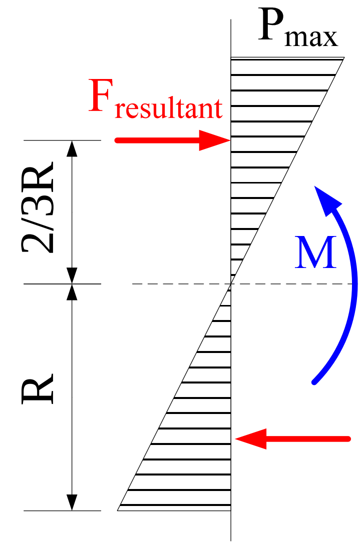
$$P_{max} = \frac{3M}{2R^3} \qquad F = \frac{3M}{4R} \quad \Rightarrow F_{misalign} = \frac{3M}{OD + ID} \Rightarrow \frac{\phi k_{radial}(OD + ID)}{16}$$

For a deep groove or angular contact bearing, the misalignment load P_{max} and applied axial load F_{axial} are added to the radial load F_{radial} when determining the total radial load on the bearing.

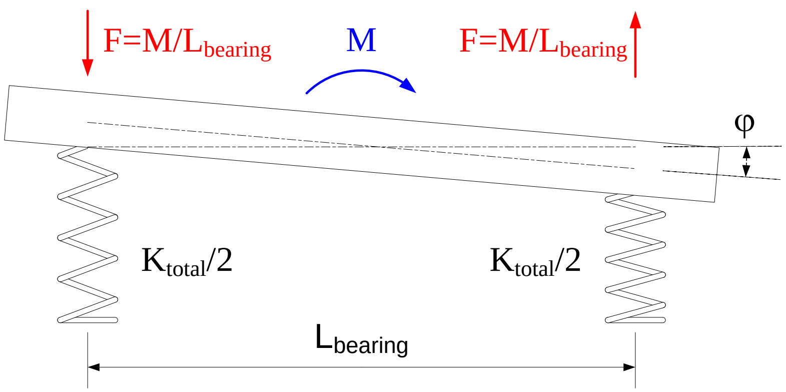
These equations allow the angular stiffness of the bearings to be estimated given just the size of the bearings and their load capacity. See *Bearing_stiffness_alignment*. Of course if the manufacturer provides the radial and/or angular stiffness for the bearings, use them.

Review your bearing mountings for proper constraint and robustness to misalignment errors. When bolts are tightened, will components deflect or deform and impose angular or lateral displacements on the bearings? Can they withstand these deformations?

Mounting: *System Stiffness*

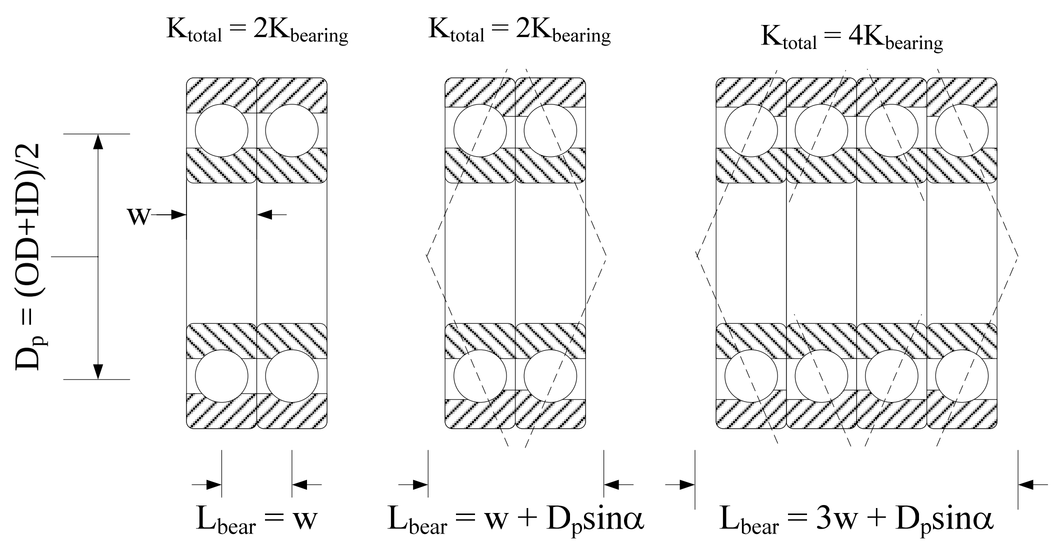


- Angular (tilt) stiffness can be estimated from lateral stiffness and bearing spacing
  - Angular misalignments are the most troublesome
  - Small misalignments can create very large bearing loads in stiff systems
- Springs modeling the system components are loaded by misalignment displacements
  - The resulting forces are added to the applied loads for life calculations



$$F = \frac{M}{L_{bearing}} \qquad \delta = \frac{F}{k_{total}/2} = \frac{2M}{k_{total} L_{bearing}}$$

$$\varphi = \frac{\delta}{L_{bearing}/2} = \frac{4M}{k_{total} L_{bearing}^2} \Rightarrow k_{angular} = \frac{k_{total} L_{bearing}^2}{4}$$



Deep Groove Pair

Angular Contact Back-to-back Duplex pair

Angular Contact Back-to-Back Quad set

Mounting: *Stiffness by Finite Element Analysis*

Solid models are often used to create the layout for a machine, and then more and more detail is added. Usually only the critical elements are included, and these include rectangular representations of bearings, both linear and rotary. These rectangular representations define the boundary volume for the bearing and include the precision surfaces against which the bearings are seated, and against which other components are located. The structure of the machine is also modeled with just enough detail to define the critical boundaries of the different modules, but with enough detail to capture the critical structural parameters that enable finite element analysis to be used to predict stresses and deflections. But what about the bearings? At first, initial analysis of the loads on the bearings blocks is done to make sure forces are not exceeded and the required life is achieved as is discusssed previously.

The different components in a solid model assembly can each have a material assigned to them. For linear bearings, for example, the rail would typically be steel, and in reality the block (or truck) would also be steel and there would be rolling elements between them. However, as discussed in Topic 9, Hertz contact associated with rolling elements is a nonlinear effect, and to model each rolling element and account for statistical variation in rolling element diameter and raceway profile would be immensely expensive and probably not very useful. However, the stiffness of bearings is usually provided by the manufacturer for the various preload conditions.

However, if one assigns steel to the materials of the bearing rail and block and then runs a finite element analysis, the results will be off by more than an order or magnitude. Engineers have typically built a second model for doing finite element analysis where they leave out the bearing block and then add spring elements. However, this complicates the analysis process, and the spring elements do not capture the roll, pitch, and yaw stiffness of the bearing. These stiffness values are not usually provided by bearing manufacturers.

A good approximate solution is to assign a virtual modulus of elasticity to the bearing block, such that when it is loaded, it deflects the same amount as the real bearing. The process for doing this is shown, where the spreadsheet segment shows the iterative process required. In general, a good starting guess for ball bearings is that the block modulus will be about 1/100 that of steel. Note that with the outside dimensions of the rail cross section modeled, for back-to-back bearing configurations, this also gives a reasonable estimate of the roll stiffness of the bearing. Also note that a single truck for pitch and yaw stiffness is less frequently used, usually two trucks on a single rail would be used for a low cost high speed efficient shuttle.

The dynamic radial load and roll moment loads can be used with the radial stiffness values from the bearing catalog to estimate the roll stiffness. The logic is as follows:

- The radial load capacity and radial stiffness are governed by two sets of Hertz contacts (left side and right side)
- The roll moment capacity is governed by the two sets of Hertz contacts (left side and right side) acting as a force couple a distance L apart.
- The roll stiffness is thus a function of the Hertzian contact springs also acting a distance L apart (see page 10-22).

This leads to the nice result that is easy to remember for estimating the roll stiffness of a linear guide from catalog data that is commonly available:

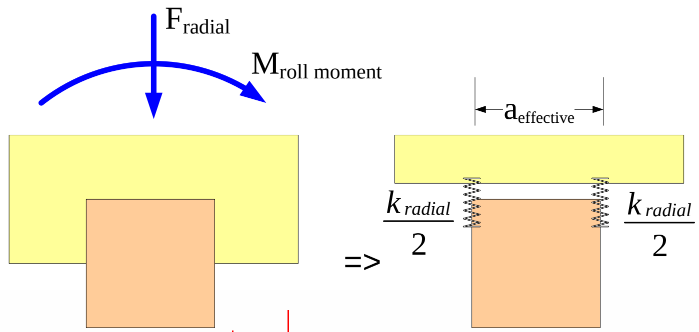
$$a_{effective} = \frac{2M_{\text{roll moment max}}}{F_{\text{radial max}}}$$

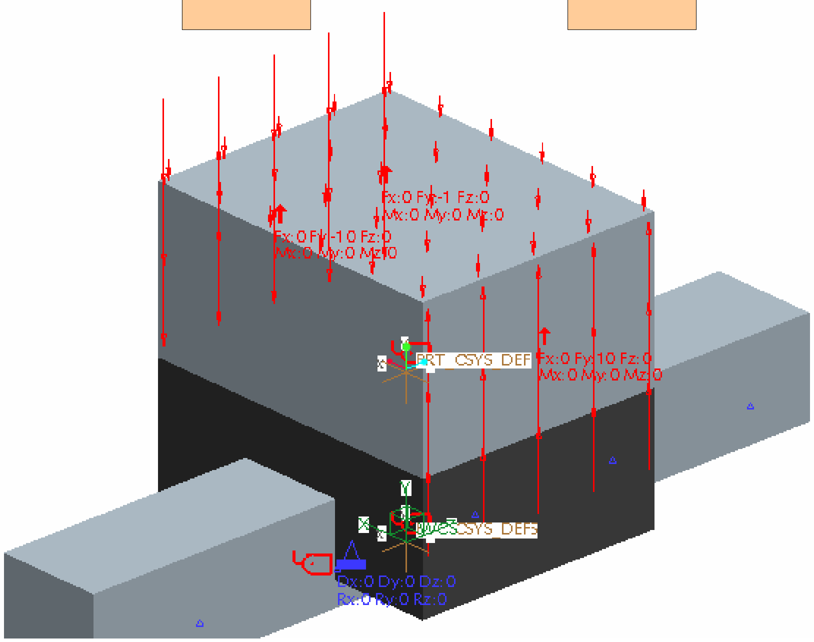
$$k_{roll} = \left(\frac{M_{\text{roll moment max}}}{F_{\text{radial max}}}\right)^2 k_{radial}$$

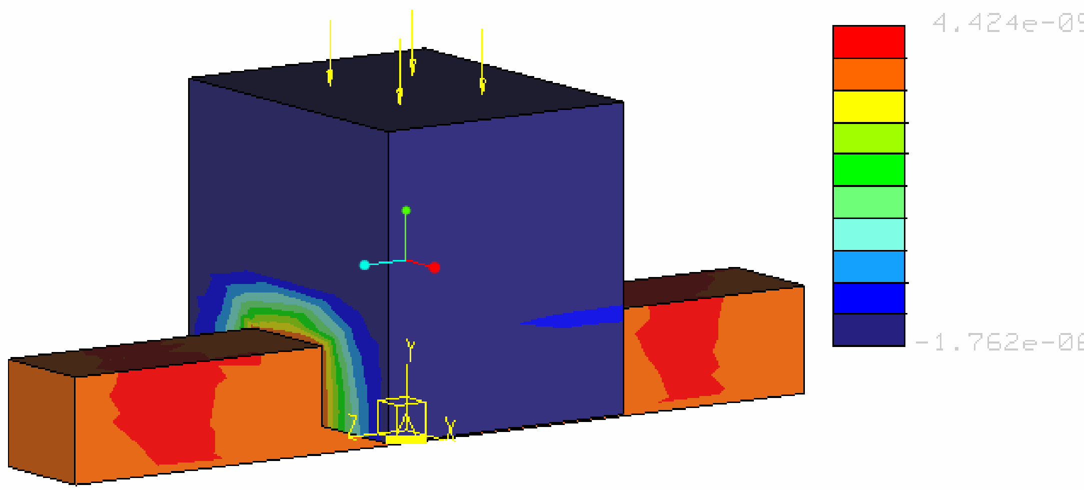
If you have a solid model program with finite element capability, create a model of a simple linear motion axis supported by one bearing rail and two bearing blocks and compare its stiffness to a system with two rails and four blocks. How far can loads be cantilevered in terms of characteristic dimensions for the one rail and two rail designs? How well does Saint Venant's principle apply?

Mounting: *Stiffness by Finite Element Analysis*

- A system's stiffness can be accurately predicted using finite element analysis
  - The bearing elements are assigned a scaled modulus of elasticity so they have the spring constant provided by the catalog
    - This enables a solid model of the machine to be built and then analyzed without having to add linear spring elements at nodes
    - This also captures the angular stiffness of the bearing







Applied load (N) F	1			
Catalog stiffness (N/micron) Kcat	578			
Deflection (mm) dcat	1.73E-06			
	Initial est	1st pass	2nd pass	Final
Bearing block modulus (N/mm^2)	10000	2404	1972	1915
FEA predicted deflection (mm) dfea	4.16E-07	1.42E-06	1.68E-06	1.73E-06
FEA predicted stiffness (N/micron)	2404	705	595	578
Scale factor required (dcat/dfea)	4.16	1.22	1.03	

Mounting: *Misalignment*

To determine how misalignments loads bearings, one must also consider the stiffness of the mounting structures, the supported component, and the mechanical interfaces. For the bearings, the mounting structure, and the mechanical interfaces between the bearings and other elements, assume that the mounting structure stiffness is equal to γ_s times the bearing stiffness, and that the interface (joint) stiffness is structure is γ_i times the bearing stiffness. The total stiffness is:

$$k_{\text{bearing, mount, interface}} = \frac{1}{\frac{1}{k_{\text{bearing}}} + \frac{1}{k_{\text{mounting structure}}} + \frac{1}{k_{\text{interface}}}} = \frac{k_{\text{bearing}}}{1 + \frac{1}{\gamma_s} + \frac{1}{\gamma_i}}$$

Should the mounting structure be infinitely stiffer than the bearings or should the interface stiffness be many times more compliant? If the bearing set is wide and firmly clamped to the component, then good moment stiffness likely exists. If a single ball bearing supports a shaft, then the moment interface will be poor. For a balanced design, $\gamma_s = \gamma_i = 1$.

The effect of bearing misalignments is manifested in two ways for both linear and rotary motion systems: First, there is the potential for relative angular misalignment ϕ of the bearing axes of motion (rotation). For example, the geometric centers of the bearing bores may lie on the ideal axis of rotation of the supported component, but they may be tilted. *Thus even though the design intent may have been to realize a simply supported beam that supports no moments, in reality, the "simply supported beam supported by bearings" acts as a free-free beam with a moment applied to each end by the tilted bearing axes.* The relative slope α between the two ends of the beam of length L_{beam}, modulus E_{beam}, and moment of inertia I_{beam} is:

$$\alpha = \frac{M L_{beam}}{E_{beam} I_{beam}} \quad \Rightarrow k_{\text{beam angular}} = \frac{E_{beam} I_{beam}}{L_{beam}}$$

All the angular springs now act in series, so the moment $M_{misalignment}$ generated by the misalignment ϕ between the bearing axes is equally felt by each of the springs. Since $M = k_{angular}\phi$:

$$M_{misalignment} = \frac{\phi}{\frac{1}{k_{\text{bearing, mount, interface}}} + \frac{1}{k_{\text{beam angular}}}}$$

In the second case, there is a displacement δ between the axes, but no tilt. If the bearings were not rigidly connected to the component, to the first order, the displacement could be modeled as causing a misalignment $\phi = \delta / L_{beam}$ and then proceed as above. If there is any reasonable moment connection between the bearings and the component, then the beam ends are guided. For a system such as a shaft supported by bearings, a conservative estimate for the misalignment moment imposed on each bearing set is:

$$\frac{\delta}{2} = \frac{F\left(L_{beam}/2\right)^3}{3 E_{beam} I_{beam}} \qquad \Rightarrow F = \frac{12 E_{beam} I_{beam} \delta}{L_{beam}^3}$$

$$M_{misalignment} = F\frac{L_{beam}}{2} = \frac{6 E_{beam} I_{beam} \delta}{L_{beam}^2}$$

When there is tilt and displacement in the "simply supported" model, the net angle is the tilt plus the displacement divided by the spacing between the bearing sets. For a guided beam, the moment from the expression for the tilt ϕ is added to the moment from the expression for the displacement δ. With the assumption that axial and radial loads have the same effect on a ball bearing, the added radial load on each of the ball bearing sets that supports the component is as derived previously:

$$F_{\text{ball bearing added radial load}} = \frac{3 M_{misalignment}}{OD + ID}$$

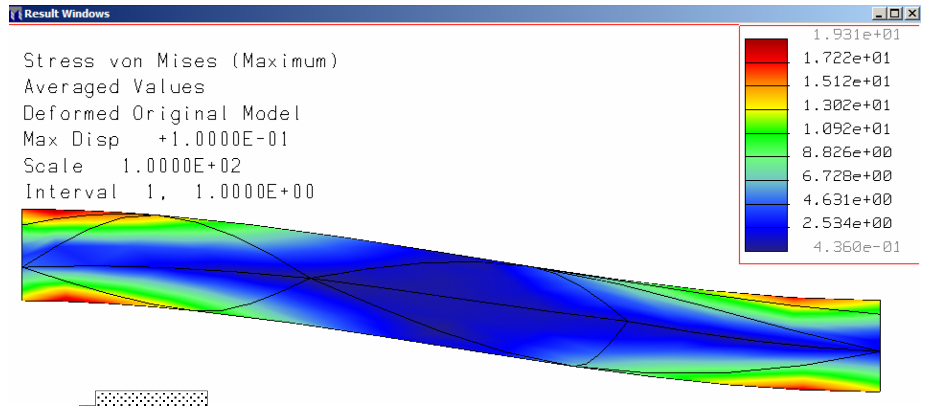
When the detailed design is complete, finite element analysis can be used to check it. Experiment with *Bearing_stiffness_alignment.xls.* Note the difference between a shaft that is actually truly guided at the ends (case 3) and a shaft where the ends are held by the bearing angular compliance.

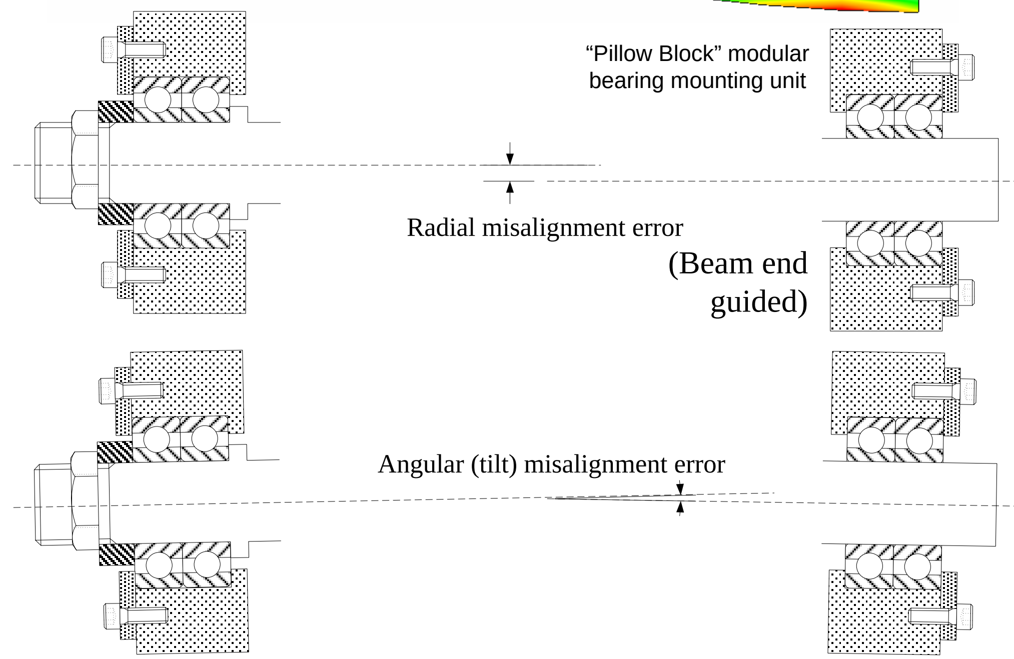
Mounting: *Misalignment*

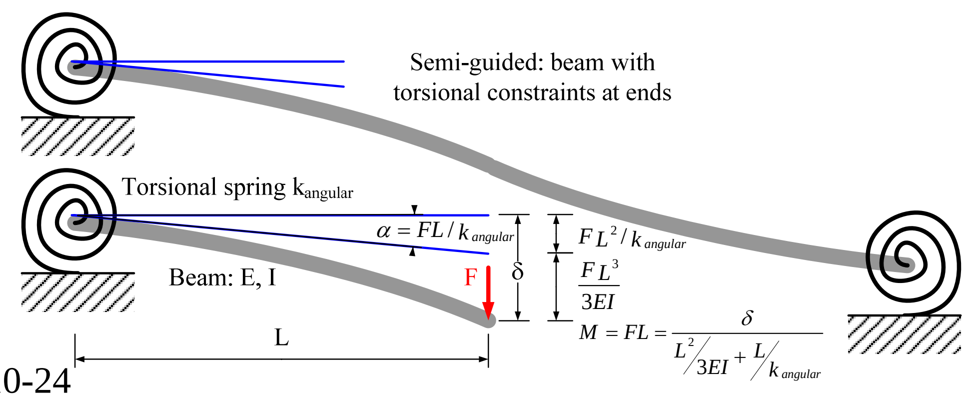
- Radial and angular misalignment errors can be major contributors to total bearing loading!
  - Springs-in-series models can be used to determine bearing loads caused by misalignment displacements, ala F = kδ
    - See the spreadsheet:
      - *Bearing_stiffness_alignment.xls*

Case 1, simply supported beam (typically Nbear = Nbear2 = 1)	
Resulting moment, Mresultss (N-m)	0.360
Resulting radial forces due to misalignment	
First bearing set (N)	30
Second bearing set	30
Case 2, beam ends guided (zero slope) with bearing angular compliance	
Resulting moment, Mresultbeg (N-m)	0.529
Resulting radial forces due to misalignment	
First bearing set (N)	44
Second bearing set	44
Case 3, beam ends guided (zero slope) with no bearing angular compliance	
Resulting moment, Mresultbeg (N-m)	12.0
Resulting radial forces due to misalignment	
First bearing set (N)	997
Second bearing set	997

Misalignment (displacement) delta only	
Both ends guided	
Force at ends, F (N)	40.6
Moment at ends, M (N-mm)	2031
Stress at ends (N/mm^2)	20.7
Cantilevered	
Force at ends, Fc (N)	10.2
Moment at base, Mc (N-mm)	1015
Stress at base (N/mm^2)	10.3







Mounting: *Centers of Action*[1]

The centers of action, stiffness, mass, and friction are critically important in the design of bearing mounts, and in determining how the supported component will behave under load. Recalling the lesson of Abbe, ideally the three centers coincide, and even more ideally, the forces applied to the system will be aligned with this common center. If this can be done, then only translational deflections will occur. Avoiding angular deflections will minimize Abbe errors which is critical to maximizing system performance.

The center of stiffness is often at the geometric center of a group of bearings, but in more complex loading situations it can be found from a calculation similar to that for finding the center of mass when the location of the reference coordinate system is arbitrary:

$$x_{\text{center of stiffness}} = \frac{\sum x_i k_i}{\sum k_i} \quad y_{\text{center of stiffness}} = \frac{\sum y_i k_i}{\sum k_i} \quad z_{\text{center of stiffness}} = \frac{\sum z_i k_i}{\sum k_i}$$

Compare the designs for the machine tool shown, where the column moves in and out of the page on the machine's Z axis (which is aligned with the spindle axis of rotation). The spindle, which holds the tool, also moves vertically on the Y axis. The workpiece would be held on an X-axis that moves left to right in front of the column. One design places the column directly on top of linear bearings, and a single actuator (not shown) can be attached midway between the bearings so in one plane it drives the system through its X center of mass. This is a low-cost design. However, the actuator is located far away from the Y center of mass location. In addition, the spindle is often located at the midpoint shown, so most of the time, the actuator force required for machining is creating a moment that wants to tilt the column back. This can of course be accommodated by adding more structure to the design; however, as just discussed, the total stiffness will not be more than that of the bearings, so instead the solution really is to use bigger bearings. This increases cost...

The other machine locates the bearings on the tops of risers so the bearings are located in the plane of the axis' center-of-mass. This adds some structure cost, but a smaller bearing can be used. However, in order to actuate the column through its center of mass, two actuators, such as ballscrews and their motors and sensors, must be used which does add cost. The design does achieve the goal of having the centers of mass, friction, and stiffness all being coincident, *and* nominally in-line with the cutting forces. Machines with this type of configuration are among the most accurate.

In robot design contests, extending axes and projectile launchers are often used, and for these systems, making the centers of action, and applied loads, coincident can greatly increase performance. Designing the machine to have these attributes from the beginning is best, as opposed to hastily throwing together a design, finding out it does not work well, and then attempting to salvage the machine.

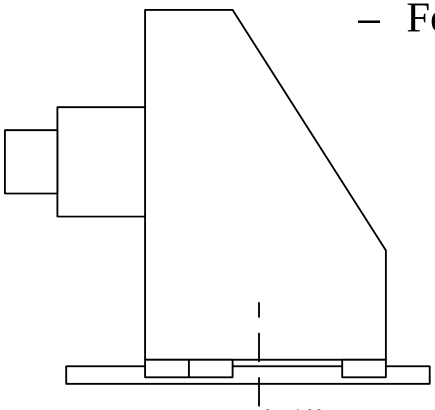
With respect to rotary motion systems, as previously discussed, the centers of stiffness play a particularly important role with respect to system performance. For example, the centers of stiffness of a pair of bearings can be brought together with a face-to-face mounting so they have a high radial and low moment stiffness which maximizes misalignment capability. On the other hand, they can be spaced far-apart so that their combined effect is a center of stiffness which has high radial and high moment stiffness, good thermal performance, but poor misalignment capability. Reciprocity can be an awesome asset tool or an awful adversary.

Start with a stick figure or sketch of your design that shows the overall structure, each component, and clearly labeled interfaces and axes of motion. For each interface, axis of motion, and major structural element, identify where the center of stiffness is likely to be, and think about the ramifications. Can you obtain better performance by changing the configuration to move the centers of stiffness to other locations? Play the game of reciprocity. Wherever you have located the centers of stiffness, ask yourself what you would have to do to move it to essentially an opposite position. Then ponder which location is truly best, and for what logical reason?

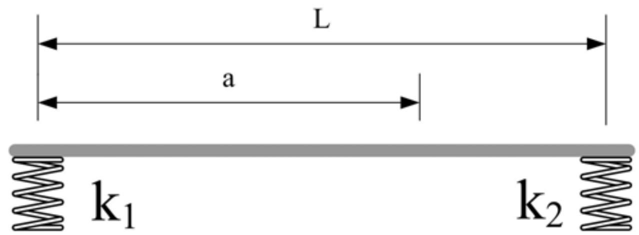
1. Make sure to review page 3-35

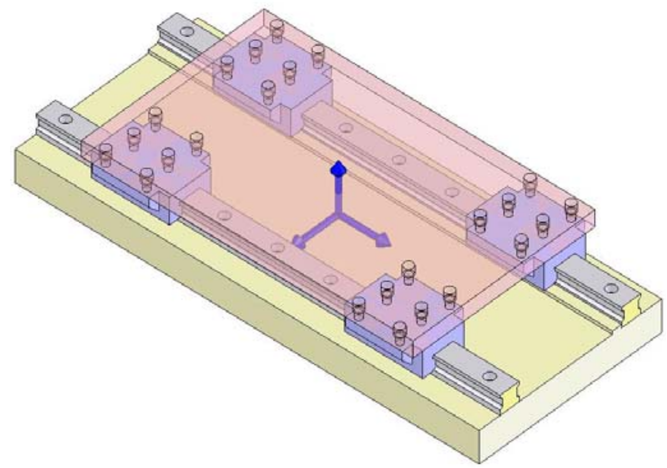
Mounting: *Centers of Action*

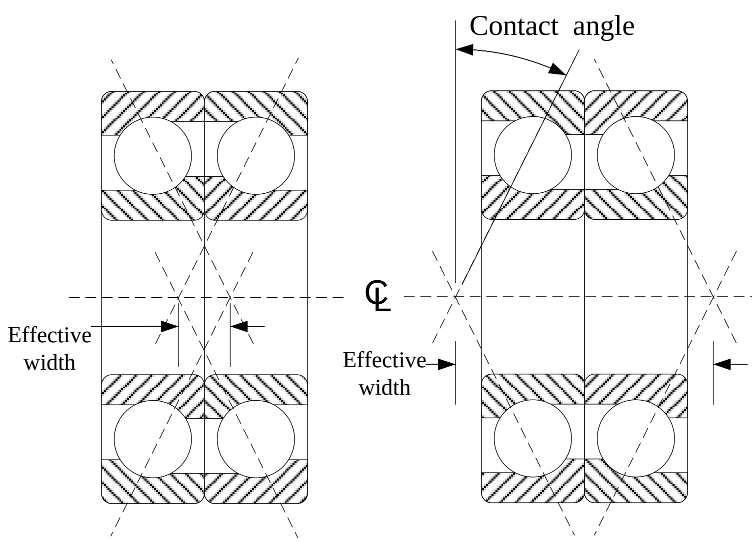
- A body behaves as if all its mass is concentrated at its ***center of mass***
- A body supported by bearings, acts about its ***center of stiffness*** *(there can be several in an axis…)*
  - The point at which when a force is applied to an axis, no angular motion occurs
    - The point about which angular motion occurs when forces are applied elsewhere
      - Found using a center-of-mass type of calculation (K is substituted for M)



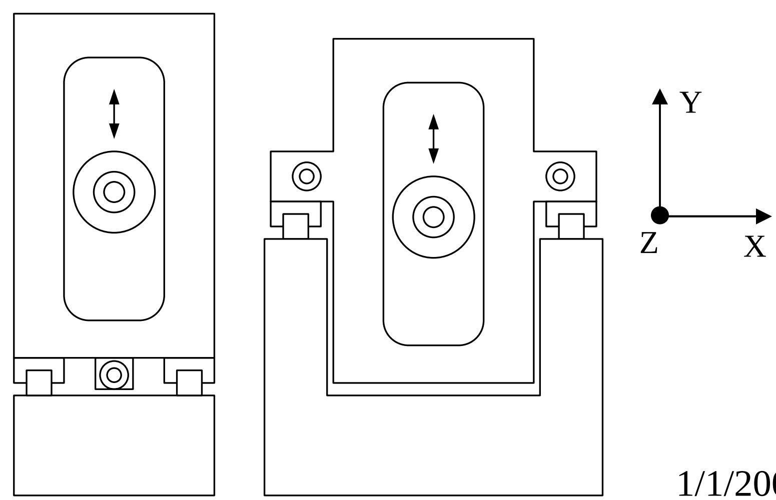
Center of stiffness







Face-to-face Back-to-back



Mounting: *Saint-Venant*

Once the centers of action have been used to focus the designer's attention on where to place components, Saint-Venant's principle can be used to initially identify where constraints might be placed to achieve a robust design. Saint-Venant's principle is a guideline that can help a designer to initially sketch out a machine and its components without having to do much in the way of analysis. However, before a design advances beyond the detailing stage, analysis usually needs to be done to confirm/optimize the ideas that have been sketched.

Consider how Saint-Venant's principle combines with for example, Abbe's principle. Can you see how angular motions can be amplified to help (or hurt) the mounting of pairs or bearings or rails? Sometimes all that is needed is a different view of a problem![1]

EVERY element has six degrees of freedom. Assuming that a bearing allows one degree of freedom of motion, the mounting must thus properly constrain five other degrees of freedom. "Properly" means either rigidly or to allow for error motions or structural deformations. A bearing's mount must therefore let move what needs to move, and constrain that which is intended to be constrained. *Do not take any degree of freedom for granted.* For example, a car's drive shaft has universal joints at the ends and a spline (linear sliding joint) to allow for length changes and flexing of the car body and suspension. In other designs, shafts that bend a lot have their bearings in spherical mounts, or they use spherical ball or roller bearings.

Consider a typical rotary motion application where a wheel is to be supported and driven by a shaft. The wheel can be press-fit onto the shaft to constrain all six degrees of freedom (this was well-covered in Topic 5). The center of stiffness of the wheel-to-shaft connection is at the center of the connection. This center should be less than 3-5 shaft diameters away from the center of stiffness of the closest bearing support. How far away should the second bearing support for the shaft be? There are two issues: If the second support is too close, then the radial load on the front bearing will be excessive. If the second support is too far away, then the deflection of the shaft will be excessive. The spreadsheet, *Bearings_simply_supported_shaft.xls*, can be used to analyze the design of the bearing supports, including the load/life and clearance tolerance issues:

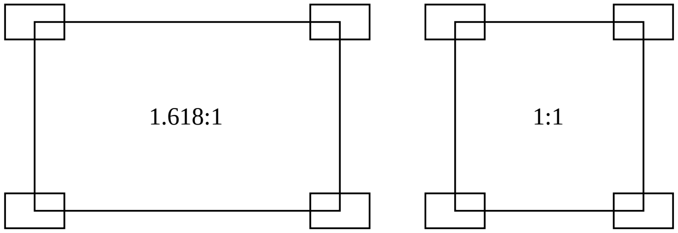
$$\alpha = \frac{1}{EI}\left(\frac{-Fx^2}{2} + \frac{-F_A\langle x-a\rangle^2}{2} + C_3\right) \quad \delta = \frac{1}{EI}\left(\frac{-Fx^3}{6} + \frac{-F_A\langle x-a\rangle^3}{6} + C_3x + C_4\right)$$

$$C_3 = \frac{F\left(L^3-a^3\right)}{6\left(L-a\right)} - \frac{F_A\left(L-a\right)^2}{6} \qquad C_4 = -C_3a + \frac{Fa^3}{6}$$

What about linear bearings? Experience with drawers provides most people with a physical feel for the effect of improperly spaced bearings. Sometimes a drawer must be pulled with both hands, so the pull forces can be adjusted to prevent jamming. How do drawers with rolling sliders minimize jamming even though they often seem to violate Saint-Venant's principle? They use low friction bearings on each side of the drawer, where each bearing supports five degrees of freedom. If the force is applied in the middle at point A, it is applied at the system center of friction, mass and stiffness, and Saint-Venant's principle does not blindly apply, and the carriage will move even if good bearings where not used. If the force is applied at point B, the fact that the right bearing's length is five times it width, means it very nicely obeys Saint-Venant's principle, and it can guide a carriage with a force applied at several times its length away. The bearing on the left supports five degrees of freedom and does not want to twist or jamb either, so it will go along for the slide. However, the bearings are overconstrained, so unless the rails are carefully aligned and the bearings are low friction, the product of the misalignment (m) and the bearing stiffness (N/m) and the coefficient of friction could create an unacceptable misalignment induced drag force F (N). Hence this particular system is stable. If, on the other hand, the bearings were not overconstrained, but the bearing spacing/length ratio was as large as shown, then the system would be neutrally stable for loads applied at A, and unstable for loads applied at point B.

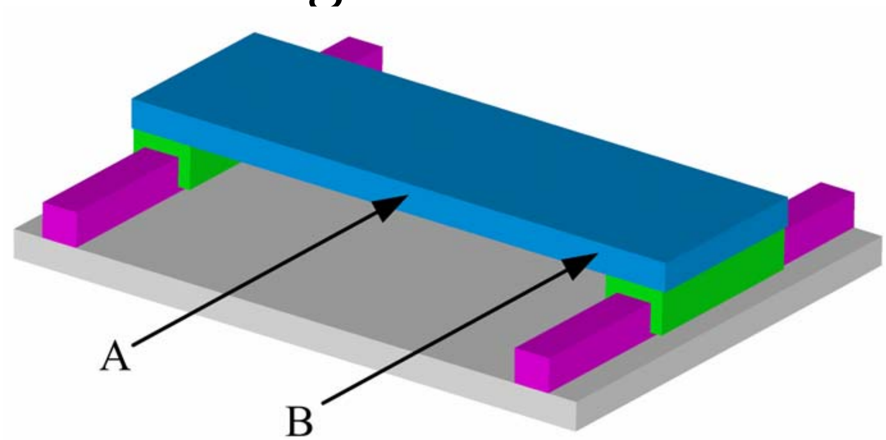
Experiment with the spreadsheets such as *Bearings_linear_spacing.* Is a bearing spacing of 3-5 shaft diameters "optimal"? What about bearing-shaft diametrical clearance? Is it needed to accommodate shaft deformations or misalignment between the holes being bored for the bearings? Since the length (width) of the bearings is a fraction of the outer diameter, does Saint-Venant work for you to help accommodate misalignment?

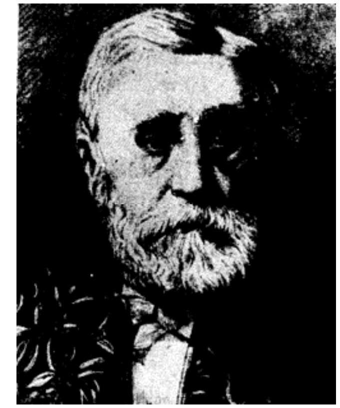
1. See for example US patents 5,176,454 and 4,637,738 (expired)



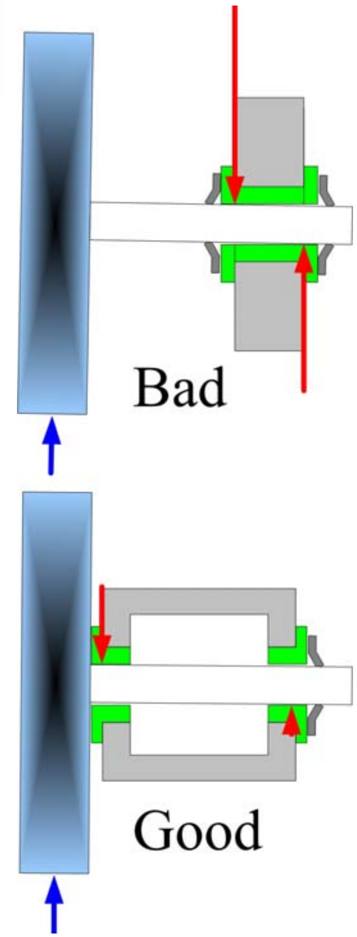
Barré de Saint-Venant 1797-1886

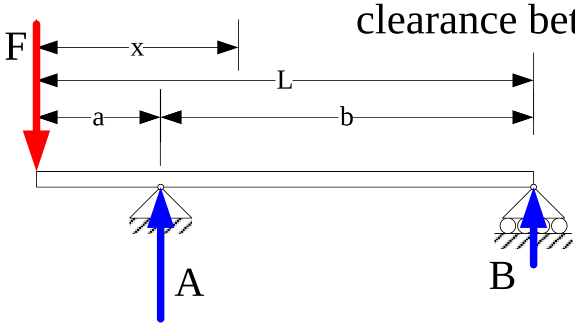
Mounting: *Saint-Venant*



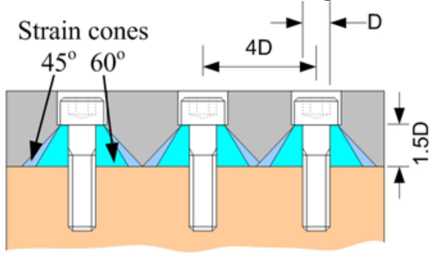


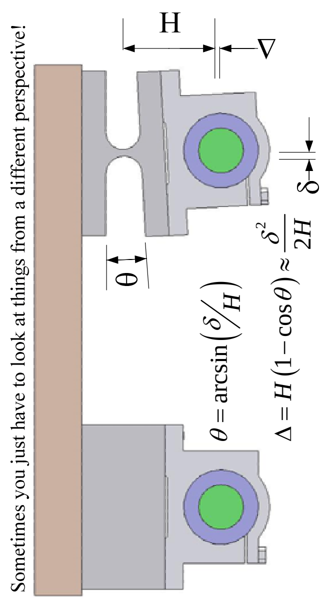
- St. Venant: Linear Bearings:
  - L/D>1
  - 1.6:1 very good
  - 3:1 as good as it gets
- St. Venant: Rotary Bearings:
  - $L_{shaft}/L_{bearing\ spacing} < 1$ and the shaft can be cantilevered
  - $L_{shaft}/L_{bearing\ spacing} > 3\text{-}5$ and the slope from shaft bending might overload the bearings, so provide adequate clearance
    - – A shaft should not have to bend to remove all clearance between it and the bearing bore!

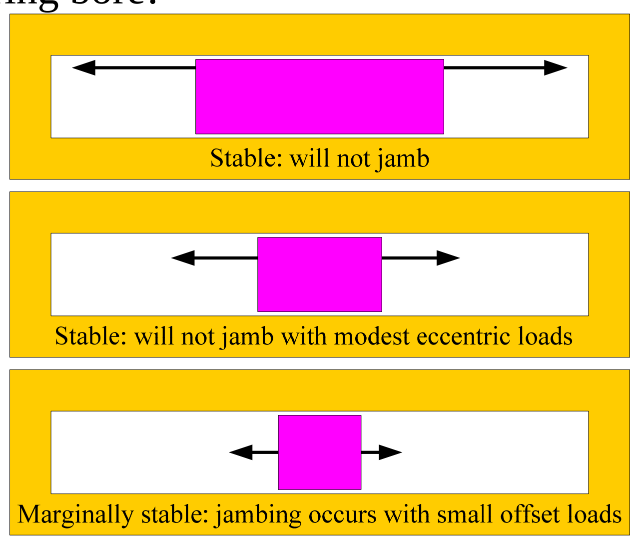




See *Bearings_simply_supported_shaft.xls* to determine if shaft deflection will edge load bearings







Mounting: *Rotary Motion*

From a combinatoric approach, there are two different practical ways sliding contact rotary bearings can be configured. For each one, make sure to judiciously apply Saint-Venant's principle and if possible the principle of *Exact Constraint Design:*

Rotating shaft, fixed component: This configuration is used when the shaft transmits torque to the component. It is also preferred even when the shaft is used to merely support the component. The reason is simple: The component is larger than the shaft, and in order to minimize the amount of shaft overhang to avoid problems anticipated by Saint-Venant, it is easier to support the shaft at two spaced-apart points. Also small bearing clearances will not be amplified (Abbe's principle). An interference-fit between the component and the shaft requires fewer shaft diameters width of the component to constrain angular motions and maintain quality.

- The shaft needs to be radially constrained by bearing support points that are 3-5 shaft diameters apart to prevent tilting of the shaft when radial loads are applied. See *Bearings_simply_supported_shaft.xls*. This can be accomplished by two radial bearings which constrains four degrees of freedom (two translational and two angular degrees of freedom). The axial degree of freedom can be constrained by machined shoulders, collars, clips, or threaded nuts on the shaft as shown in the figures.
- The bearings themselves need to be constrained with respect to the support structure. Remember, nothing is ever perfectly aligned, nor perfectly finished, and in a sense, everything acts like a screw; therefore, rotary shaft motion will act on the "screw" and cause axial forces that can push out bearings that are not axially restrained. The figures illustrates some of the many ways that bearings can be axially constrained, including using flanged bearings with both radial and axial features.
- The object which the shaft supports must similarly be properly constrained to the shaft. Since this object is typically many times the diameter of the shaft, particular care must be taken to constrain the object with respect to its perpendicularity. If the object tilts on the shaft, it is likely that the intended function of the object, be it a wheel, gear, or pulley, will be adversely affected.

Fixed shaft, rotating component: This configuration is commonly used when a rotating member is not driven by torque delivered through the shaft; AND there is limited structural space for supporting the shaft; AND the component diameter is less than 5 shaft diameters in diameter. This latter point is particularly important because the supported rotating component must be supported by effective radial bearing points that are ideally at least three shaft diameters apart. It would also imply that the cantilevered portion of the shaft is significantly longer than the portion of the shaft that is used to constrain it to the structure. Can you see how the problem starts to compound itself? When the component is relatively small, such as an idler gear (a small gear used to transfer power between two other gears), and the shaft is held by a tight press-fit or axially preloaded, then the system can perform admirably.

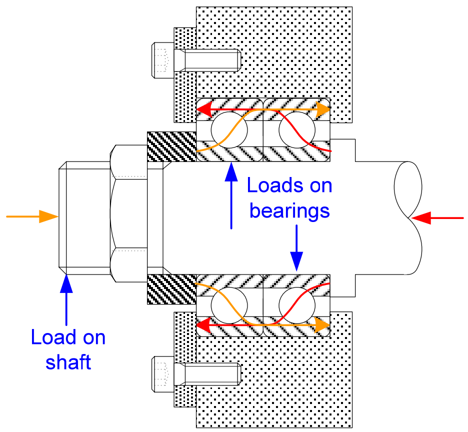
- The shaft should be radially constrained by support points that are 3-5 shaft diameters apart to prevent tilting of the shaft when radial loads are applied. These can be discrete points, or the shaft can be pressed into a hole that is several shaft diameters in length. Alternatively the shaft can be made from a shoulder bolt. Can you visualize how this can restrain all six degrees of freedom of the shaft?
- The bearings between the component and the shaft are defined and mounted as in the previous case, where one imagines the "shaft" is the structure, and the component that is to be allowed to rotate as the "shaft". A back-to-back bearing arrangement will support moment loads.

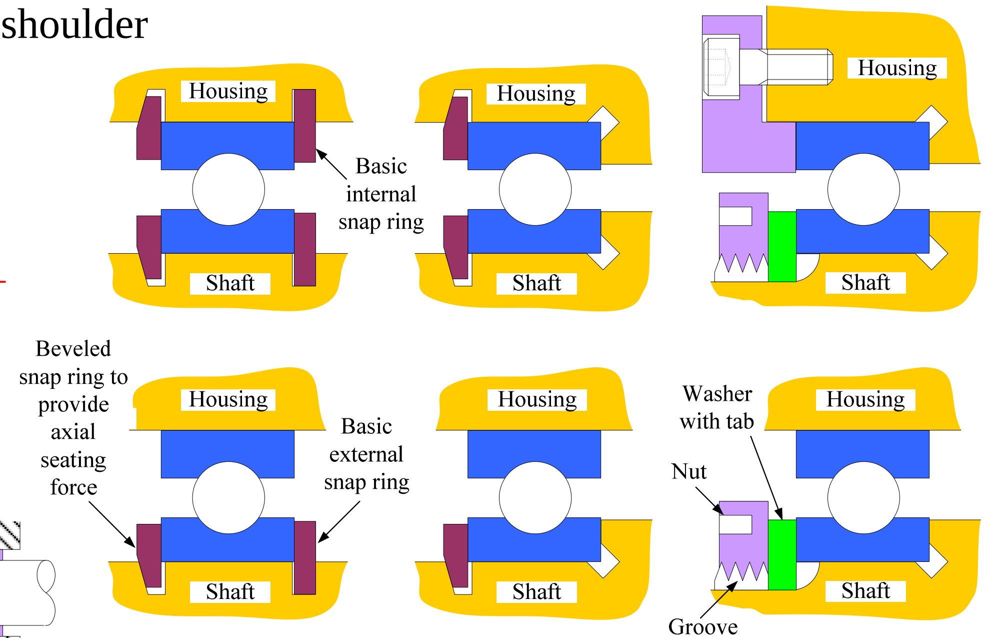
As an example, examine the cutaway view of the shaft supported by flange bearings in a *pillow block* (a modular bearing housing): Axial forces on the shaft flow from the shaft into the snap-ring, to the inner-ring, though the balls, to the outer ring, through the flange, and into the housing. The mirror image (reciprocity!) handles forces in the opposite direction. With respect to a component held by the shaft, these components usually have a hub that is used to give angular stability (constraint) to the component, and the hub can be located on the outside. A key (see page 5-27), transmits torque. See the gear-on-motor shaft mounting illustrated on page 7-20.

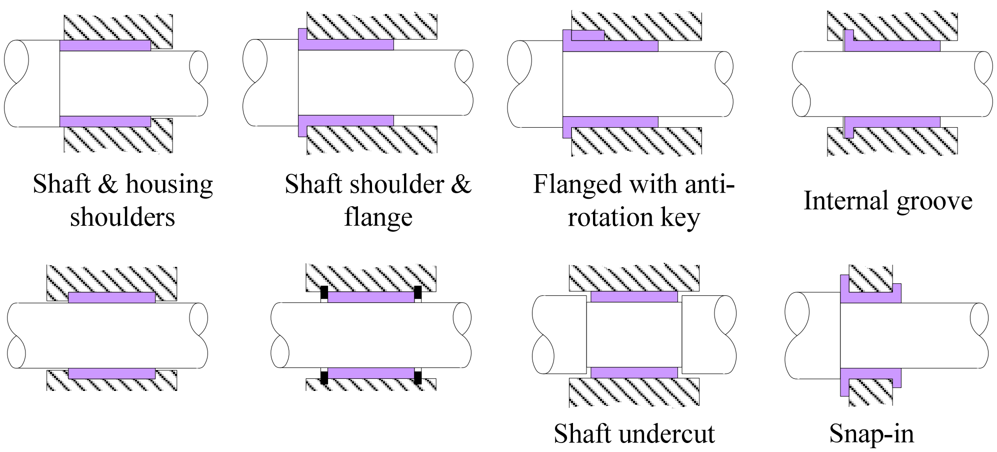
Are your shafts and components properly constrained? Review each one with the above discussion in mind. Apply reciprocity as needed! What about coupling the actuator to the rotary motion axis? See pages 5-29 & 5-30.

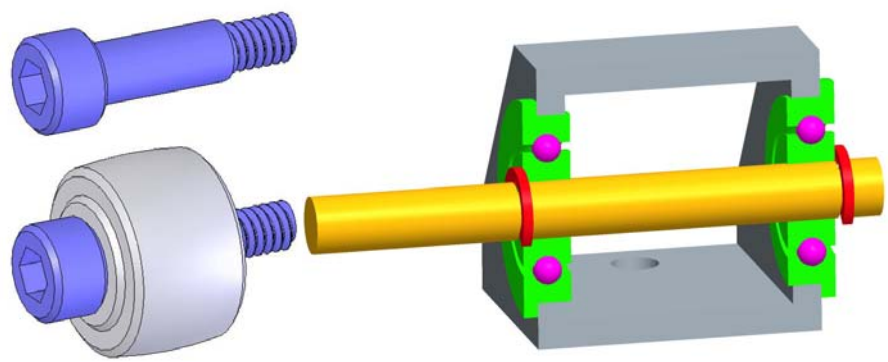
Mounting: *Rotary Motion*

- Every rotary motion axis has one large degree of freedom, and five small error motions
- 5 degrees of freedom are typically constrained with one thrust bearing and two radial bearings
  - Axial constraint obtained by use of e-clip, push nut, snap-ring, threaded nut, or shoulder









Mounting: *"Ball Bearings"*

There are three Functional Requirements for any mounting of ball bearings, and they also apply to most other bearing types as well:

- Support the component to restrain the required degrees of freedom
- Support the bearings so that they are ideally properly constrained, and in the presence of structural deformations and manufacturing errors, the product of the imposed deflections and the bearing compliance is an acceptable value.
- Support the bearings so when heat is generated by the rotating system, thermal expansion is either tolerated, or accommodated by a back-to-back arrangement to yield a thermocentric design (see page 10-20).

To achieve these FRs, when supporting a shaft, one set of bearings usually is used to withstand radial and axial loads and a second set of bearings usually is used to withstand only radial loads. The second set must not be axially overconstrained to allow for thermal growth. Methods for dealing with the problem of axial thermal growth include:

- Constrain the bearings on one end of the shaft to the shaft and the housing so they are thermocentric, and let the bearing(s) on the other end of the shaft be free to slide in the bore. This is the most common, and least expensive method, but it can lead to decreased radial stiffness and accuracy if the fit is too loose or early bearing failure if the fit is too tight.
- Preload the bearings on both ends of the shaft against each other to yield a thermocentric design. When the bearings must be spaced far apart to support a longer shaft, this can be a very difficult configuration to design and requires significant modeling and may also require angular contact bearings with a steeper angle.
- Use a hydraulic device to maintain a fixed preload on the bearings . This is an effective, but more expensive, method.
- Constrain the bearings on one end of the shaft to the shaft and the housing so they are thermocentric, and mount the bearing(s) on the other end of the shaft into an intermediate component that can axially float by means of a diaphragm flexure or a die-set bushing. This is an effective, but more expensive, method.

Thermal expansion becomes more serious as the DN number increases beyond a few thousand. Despite their low coefficients of friction, rolling element bearings can generate significant amounts of heat when the components they support move at an appreciable velocity. A rolling element bearing's frictional properties will also generally change with load and speed, which also affects the rate of heat generation. The interdependence of this relation makes modeling bearing thermal performance difficult but not impossible using computer aided design packages. These programs are available from bearing suppliers and can help answer such questions as how much oil (or grease) and what type should be used? Must the oil be cooled? What will be the steady state operating temperature?

Moment loads are resisted by using pairs of bearings that are spaced suitably far apart. If radial load-carrying capability is to be increased by using more than one bearing at one end of a shaft, match ground bearings must be used because of the likelihood that slight differences in unmatched bearings will lead to unequal load sharing with failure of one bearing and then the next. *The bearings must be purchased as a matched set and installed with their alignment marks properly aligned.* Matched sets of bearings with 2, 3, and 4 back-to-back bearings are known as duplex (DB), triplex (DBD), and quadruplex (DBB) sets. If high radial load capability is needed and space is limited, another alternative is to use a double-row bearing. A double-row bearing is essentially two ball bearings that share common inner and outer races and a retainer. A big advantage of using a double-row bearing is that the balls can be one-half pitch apart, which can reduce sinusoidal variation in radial stiffness as the balls roll by up to 70%.[1]

Ball bearings are not difficult to mount if done correctly Designers get into trouble when the incorrectly mount bearings and the design works at low speeds and loads, but then fail when the designer tries to use the same type of mounting at a higher speed. Be careful!

Review any designs you have that use ball bearings and ask yourself how the design would perform at high speeds or loads? Make sure that for every bearing design decision you can point to a calculation or decision justification in your designer's notebook. Take nothing for granted!

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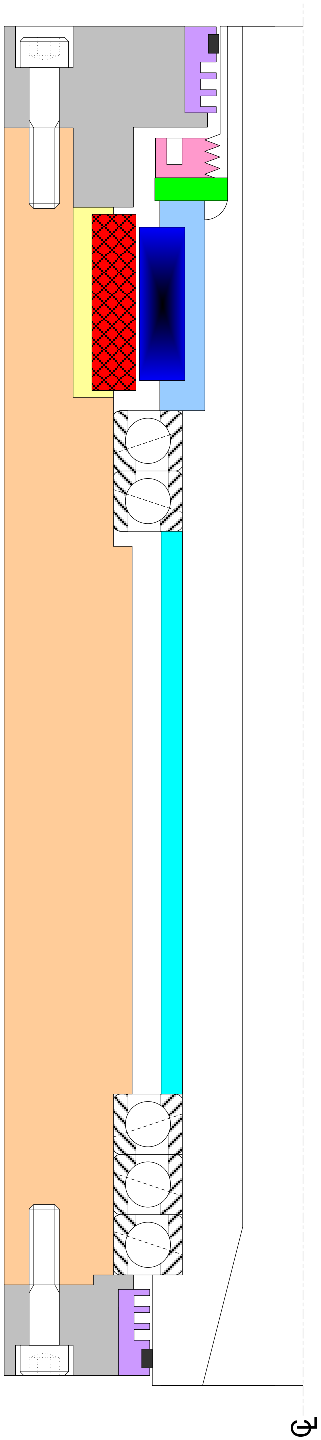
1. M. Weck, Handbook of Machine Tools, Vol. 2, John Wiley & Sons, New York, 1984, p. 187.

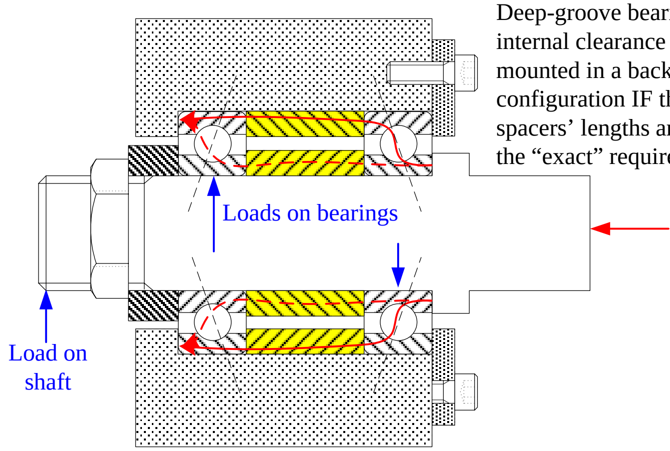


Angular contact ball bearing spindle with integral frameless brushless DC motor

Mounting: "*Ball Bearings*"

- *Ball* bearings' *inner races* are mounted on a *shaft*, and the *outer races* fit in a *bore*
  - All bearings generate heat when they rotate
  - Thermal growth can cause overconstraint and overloading
  - A spindle's rotating shaft gets hotter faster than the housing
    - Back-to-back mounting balances radial and axial thermal expansion to maintain constant preload (*thermocentric* design)
- Multiple bearings can be used to achieve required load capacity & stiffness
- Angular contact bearings can be mounted in a *thermocentric* configuration
- Deep groove bearings can be mounted with one fixed and one floating to achieve good <u>*low*</u> speed performance (DN<1000)

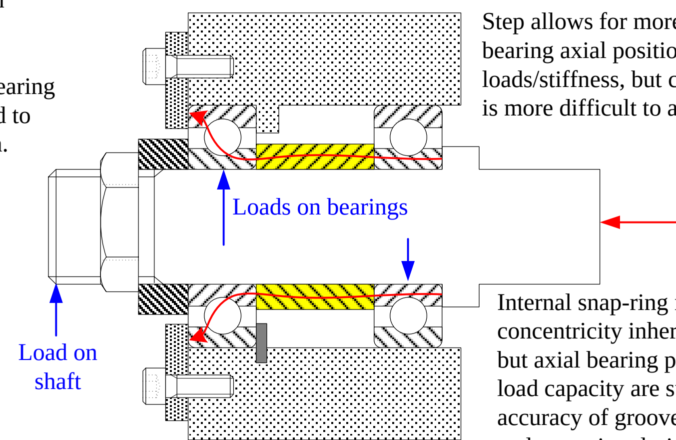
Thermocentric configuration



Deep-groove bearings with internal clearance can be mounted in a back-to-back configuration IF the two bearing spacers' lengths are ground to the "exact" required length.

Back-to-back preload:
axial load capacity = axial load capacity of one bearing
axial Stiffness = 2 X axial stiffness of one bearing

Floating configuration



Step allows for more accurate bearing axial position and higher loads/stiffness, but concentricity is more difficult to achieve

Internal snap-ring makes concentricity inherently good, but axial bearing position and load capacity are subject to the accuracy of groove machining and snap-ring design

Deep-groove preload by oversize balls
axial load capacity = axial load capacity of one bearing
axial Stiffness = 2 X axial stiffness of one bearing

Mounting: *Linear Motion*

Mounting linear motion bearings requires much of the same thought process as mounting rotary bearings: Careful attention must be paid to managing constraints and wisely applying Saint-Venant's principle.

For a linear motion system with intended motion in the X-direction, constraints need to be applied in the Y and Z directions. A second Y direction constraint constrains rotation about the Z-Axis (pitch). A third Y direction constraint constrains rotation about the X-Axis (roll). A second Z-constraint constrains rotation about the Y-Axis (yaw). Together, these five constraints can be realized by bearings, such as small pads of plastic bearing material, that ride on an appropriate rail system.

Can more than five contact points (constraints) be used? Will they necessarily over constrain the system, and if so, how can this be managed? If clearance was provided so they did not over constrain the system, why would a designer want to add them? Would Maudslay's Maxim not dictate asking why are they there, and if for no reason, get rid of them? The answer lies in the functional requirements for the system. If the FRs call for the system to be able to resist large external loads applied from any direction at a significant distance from the center of stiffness, then an elastically averaged system (over constrained) of bearings can be considered to provide greater tipping resistance the way a five-legged chair does as discussed on page 3-28. Preload can then be applied using compliant members to achieve a stable robust design even though it is "overconstrained".

Another significant challenge in the managing of constraints is establishing parallelism between the rails. *Horizontal parallelism* is a divergence of the rails in a plane, and it causes the greatest misalignment forces on bearings. *Vertical parallelism* is a twist of the rails. Can you see the difference in the figures? For a small parallelism error of θ degrees, as one moves along the rail a distance *x*, the initial distance *w* between the rails changes more for horizontal parallelism errors than for vertical parallelism errors. The distances between the rails for the two parallelism cases are:

$$L_h = w + x\sin\theta \qquad L_v = \sqrt{w^2 + x^2\sin^2\theta^2}$$

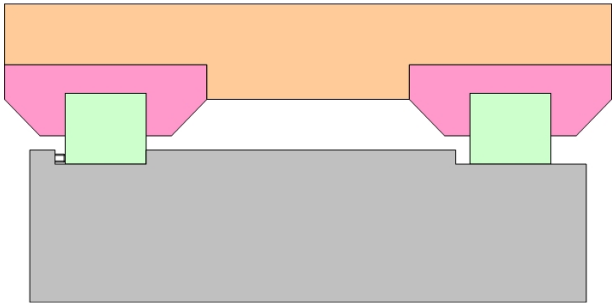
Vertical parallelism errors are less common, because it is relatively easy to establish a flat plane. They are also less problematic as shown by the analysis. In both cases, the bearings must have internal bearing-to-rail clearance to accommodate the angular divergence error between the rails. They must have enough compliance to allow for the divergence without overloading the bearings. In addition, finite angular stiffness in the bearings prevents them from being overloaded as they are bolted to the flat carriage. Recall that face-to-face bearings can accommodate more angular misalignment.

To help minimize horizontal parallelism errors, precision reference edges can be machined into the surface on which rectangular rails are to be mounted. Another technique is to mount a reference rail to the surface, where it is made to be straight either with a reference edge or with a temporary external reference. The bearings are then mounted to the carriage, against reference edges. The second rail, is then placed on the surface, and the bearings on the carriage are used to establish its distance and hence also parallelism to the master rail. As the carriage is moved along, the second bearing rail's bolts are tightened. This is referred to as *zippering* the second rail. Be careful to not over constrain the system of rails and carriages in the plane of motion. In general, only one reference edge on the bed and one reference edge on the carriage should be used.

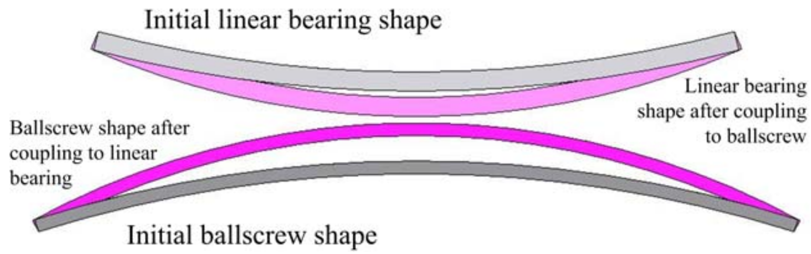
Round rails simply supported at their ends are easier to align, because the mountings can be machined as a matched set: The holes in the mounts and the carriage can all be made at the same time using the same fixture, so the center distances for the rails and the bearings will be "exact". In addition, because the rails are simply supported, their own compliance acts like a coupling.

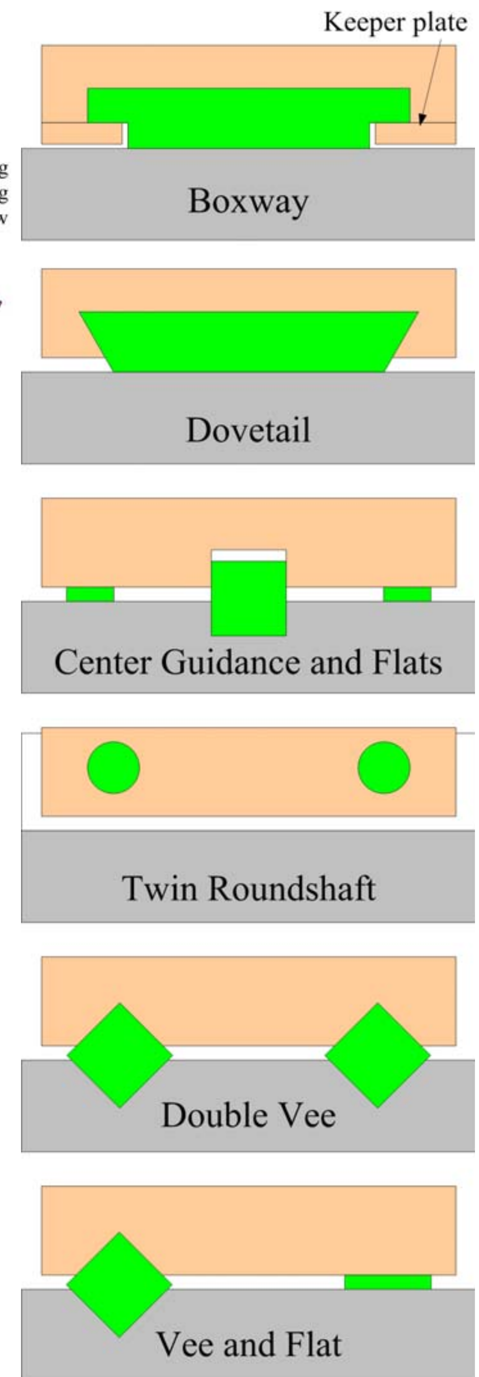
For a fluid or air bearing system, a carriage often has at least 6 bearing pads to constrain five degrees of freedom: one on each corner of a rectangular carriage, for vertical, pitch, and roll guidance, and 2 pads for horizontal and yaw guidance. This means that the system is over constrained, but because of the fluid layer, a high degree of *elastic averaging* occurs as the fluid film accommodates surface irregularities.

Look at all the different rail types of rail mountings and assess which one meets your functional requirements for the least cost. Experiment with Bearing_linear.xls. What about coupling the actuator to the linear motion axis? See page 5-29.

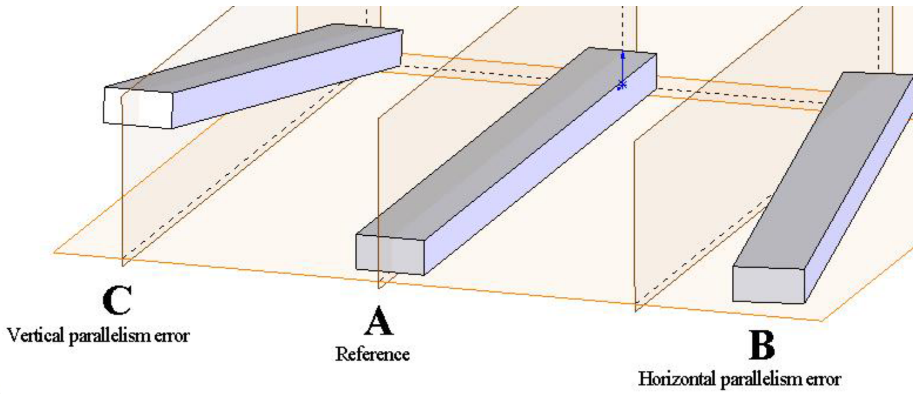


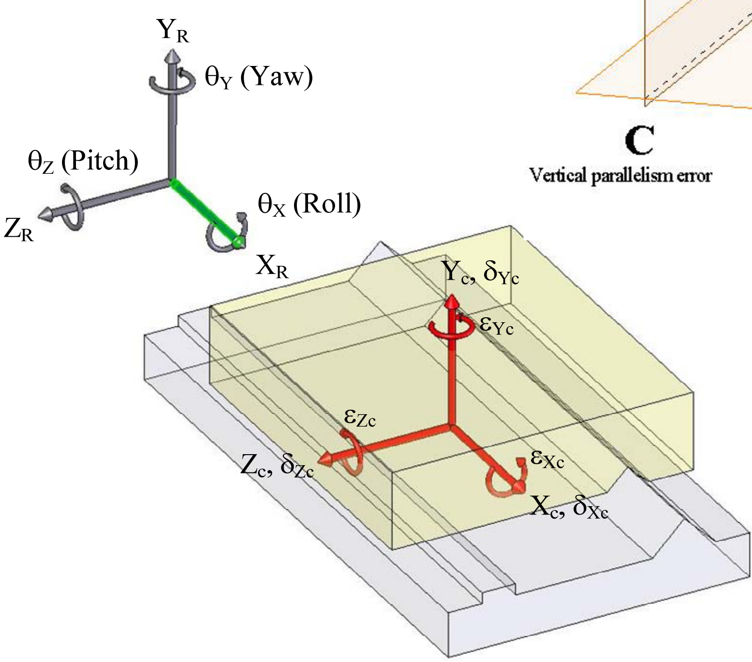
Mounting: *Linear Motion*

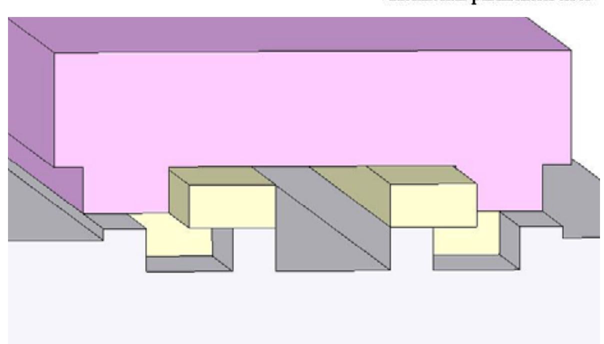




- Linear motion axes usually have one large degree of freedom, 5 degrees of freedom constrained, and five small error motions
  - Typical preloaded machine tool carriages have pairs of preloaded bearing pads in vertical and horizontal directions at each of 4 corners







Example: *Multiple Linear Guide Carriages*

It is very common to have four bearing carriages supporting an a moving axis at each of its four corners. This system is overconstrained, but as discussed on page 10-13, with proper installation technique, the effects of overconstraint can be minimized. Linear guide catalogs, however, often state that in order for the carriages to be thought of as independent units, they must be placed several carriage lengths apart. If the structure to which the rails is bolted is very stiff as well as the structure bolted to the carriages, however, to be conservative it is a good idea to model the system as a set of springs with a net stiffness that acts at the center of stiffness.

The center of stiffness is determined as described on page 3-27. The net linear stiffness along an axis at the center of stiffness is just the sum of the linear (radial) stiffnesses of all the bearings parallel to the axis. Using the method described on page 10-22, it can be shown that the net moment stiffness contribution of all the linear springs of stiffness k_i each a distance r_i from the center of stiffness is the product of their distance squared and the linear stiffness:

$$k_{\text{linear j}} = \sum_{i=1,N} k_{i,j}$$

$$k_{\text{angular j}} = \sum_{i=1,N} k_{i,j} r_{i,j}^2$$

The location of the center of stiffness is thus first found, and the above relations are used to find the net linear and angular stiffnesses of the system about the center of stiffness. Forces and moments can then be applied to determine the linear and rotary deflection of the system.

How can this model be used to determine the load on a bearing block that is misaligned by an assumed amount? After determining the stiffness of the system about the center of stiffness, one could say that the equivalent linear stiffness of the system acts in series with the single bearing block: the two springs are separated by the misalignment gap and then the springs are attached and allowed to come to a new equilibrium position, and the force in the bearing block "spring" is the product of the bearing block stiffness and the equilibrium stiffness. Each bearing block in the system would also be loaded by the product of the system displacement and the system center of stiffness value distributed between the bearing blocks. This gets to be an accounting nightmare, and a conservative estimate is that the misalignment force is equal to the product of the misalignment displacement and the bearing block stiffness.

It is important to remember the principle of harmony in design: if the bearing has stiffness K, why make the structure which supports it, and the structure which it supports have stiffness K/10? Assuming each element has a stiffness on the order of the bearing, one could assume that the effective bearing stiffness to be used in this preliminary model is therefore K/3. This also means that given a misalignment δ, the misalignment force will be δK/3.

The spreadsheet *Bearings_linear_carriages.xls* embodies these models and can be used during the conceptual design phase, even before solid models or FEA are called up, to help answer such design concept questions as:

What happens to bearing forces and deflections if there are very large deadweight loads applied on one end of the carriage?

- What happens when there are large overhanging loads
- Will adding a 3rd bearing rail and set of carriages help
- Where should the rail and carriages be added?
- Will the added carriages too fight each other and reduce life?

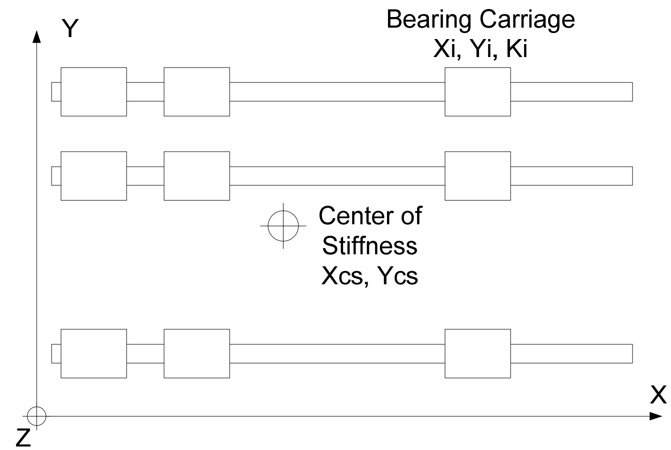
Once the what if scenarios are completed, the design engineers have a much better picture of what the machine can look like, and they select a concept which can then be solid modeled. Finite element analysis can be used to predict the performance, such as stiffness, of the machine.

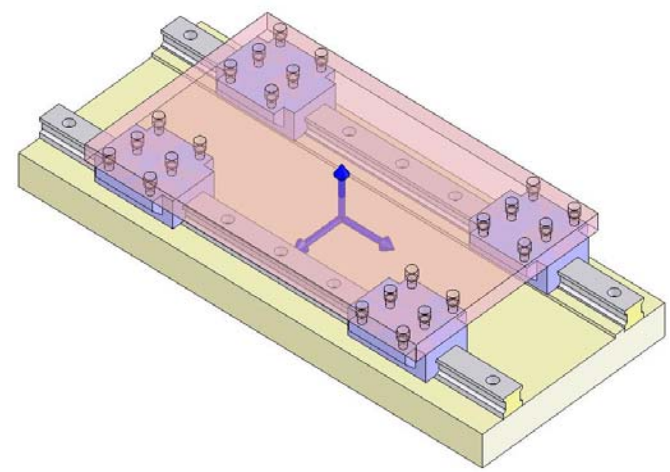
Experiment with *Bearings_linear_carriages.xls* and determine the stiffness of a 4 block and a 6 block supported machine tool table. What assumptions did you make regarding entering in not the actual catalog stiffness of the bearing block, but a lower value to account for the stiffness of the bed and table... How does the stiffness compare to an FEA predicted stiffness?

Example: Multiple Linear Guide Carriages

- Four bearing carriages supporting an axis are common
- What if there are very large loads focused on one end of the carriage?
  - E.g., overhanging load
- Will adding a 3rd set of carriages help?
  - Will the added carriages too fight each other and reduce life?

Bearings_linear_carriages.xls				
To determine forces on each of 4 learn bearing carriages centered about a coordinate system				
Written by Alex Slocum. Last modified 10/9/2006 by Alex Slocum				
Enters numbers in **BOLD,** Results in **RED. NOTE: BE CONSISTENT WITH UNITS**				
Assumes supported structure is much stiffer than bearing carriages, and misalignment loads are conservatively estimated to be product bearing carriage stiffness (N/micron) and misalignment (microns)				
Location of center of stiffness				
Xcs	**0**			
Ycs	**0**	Stiffness contribution from rows		
Actuator stiffness Kxact (assume actuator placed at Ycs) (N/micron)	**1000**	Row 1	Row 2	Row 3
Net Y radial stiffness Kycs (N/micron)	**4000**	2000	0	2000
Net Z radial stiffness Kzcs (N/micron)	**4000**	2000	0	2000
Net roll (KθX) stiffness Kroll (N-m/microrad)	**4000**	2000	0	2000
Net pitch (KθY) stiffness Kpitch (N-m/microrad)	**4000**	2000	0	2000
Net yaw (KqZ) stiffness Kyaw (N-m/microrad)	**2828**	1414	0	1414
Resultant forces and moments at center of stiffness		Resultant deflections at center of stiffness		
Fx (N)	**0**	dxcs (micron)	**0.000**	
Fy (N)	**0**	dycs (micron)	**0.000**	
Fz (N)	**100**	dzcs (micron)	**0.025**	





Example: *Ballscrew Nut Coupling to a Linear Motion Carriage*

The situation described on the facing page is typical and is sometimes referred to as "peeling the onion" because it seems like you are never given enough information or when you start to look at a problem in more detail, you keep peeling away layers. Fortunately, as astute engineers comfortable with FUNdaMENTAL Principles, you can use the basics to assess the situation.

First off all, you recognize this as a springs problem, and the solution for the mounting of the ballnut at the end or in the middle of the carriage will be the same. The solution is similar to the bolted joint discussion on page 9-9 or the bearing preload discussion on page 10-19: The carriage represented is represented as a spring k_1 whose end is the misalignment distance δ away from a spring k_2 that represents the ballscrew system. When they are bolted together, both springs deflect by δ_1 and δ_2 respectively. The stiffer spring will deflect more, and the sum of the deflections of the two springs will equal the total misalignment distance:

$$\delta_{\text{misalignment}} = \delta_1 + \delta_2$$

Since the springs are in series, the forces in each spring will be the same, and since F=kδ:

$$k_1\delta_1 = k_2\delta_2$$

Solving for the deflections and subsequent force in the springs:

$$\delta_2 = \frac{k_1\delta_{\text{misalignment}}}{k_1 + k_2} \qquad \delta_1 = \frac{k_2\delta_{\text{misalignment}}}{k_1 + k_2}$$

$$F = \frac{(k_1k_2)\delta_{\text{misalignment}}}{k_1 + k_2}$$

We can now investigate the two different mounting scenarios. If the ballscrew nut is attached at one end of the carriage, the nut will be close to the support bearing and the screwshaft stiffness will be so large that it can be ignored. Hence k_2 will be due to the radial stiffness of the ballscrew support bearing and the ballscrew but in series. Since the radial stiffness of the ballscrew nut was not given, assume it is equal to that of the ballscrew support bearing. At one end of the carriage, the net radial stiffness will be equal to that of two of the bearing blocks.

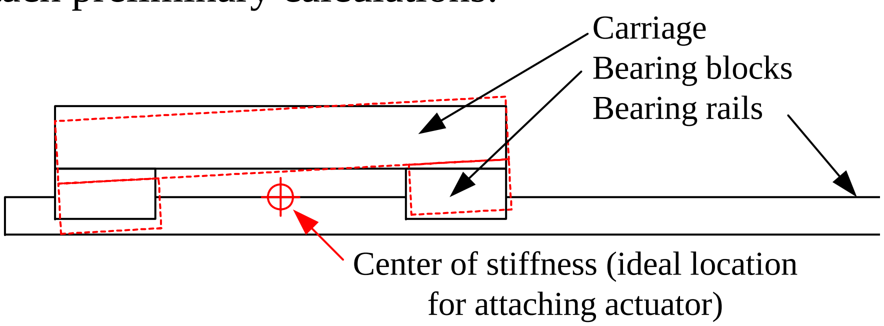
Where the nut is mounted in the middle of the carriage, when the carriage is once again all the way to one end, again, the ballscrew diameter is irrelevant. However, the carriage radial stiffness will be equal to that of four of the bearing blocks. The calculations show: The maximum error motion of the carriage thus occurs when the ballscrew nut is mounted at one end. Since the length of the carriage was not provided, but the length of the ballscrew was (2m) a reasonable assumption for the carriage length is 1m. If the carriage was designed using the Golden Rectangle as a guideline, it is 0.6m wide.

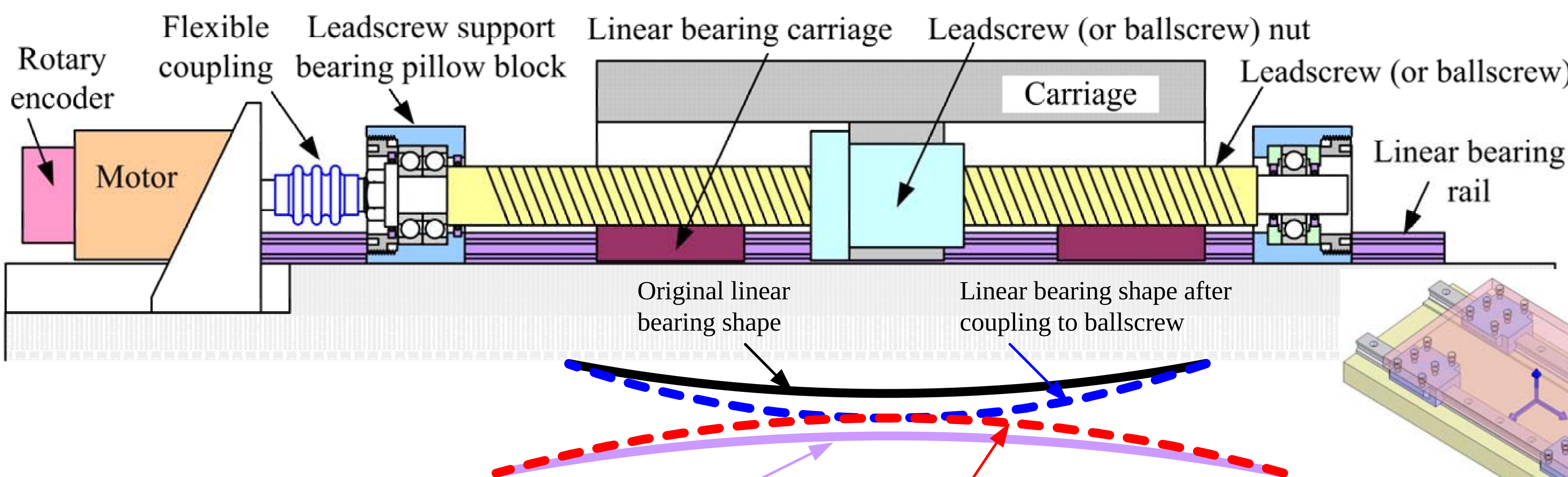
Kradial bearing block	**1000**	N/micron	
Kradial ballscrew bearing support block	**1000**	N/micron	
Assumed Kradial ballnut	**1000**	N/micron	
Net ballscrew system K (k2)	**500**	N/micron	
Misalignment ÷	**5**	microns	
Carriage length	**1**	m	
Ballscrew mounted at carriage	end	middle	
k1	**2000**	**4000**	N/micron
k2	**500**	**500**	N/micron
÷1	**1.0**	**0.6**	micron
÷⌡	**4.0**	**4.4**	micron
F	**2000**	**2222**	N
Force per bearing block	**1000**	**556**	N
pitch error	**1**	**0**	microradian

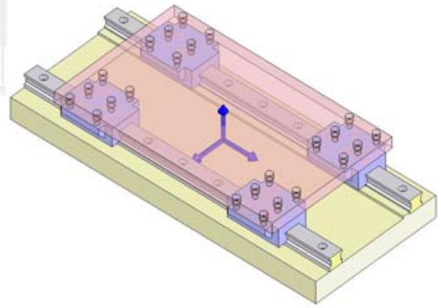
The analysis confirms good design practice: try to mount the nut in the center of the carriage, but if you cannot, you at least are aware of the actual consequences and could design accordingly. You can always start with *F=kx* and sum of the forces in your analysis and do a first order estimate. Remember to look for the dominant springs in the system. Sometimes you have too much (extraneous) or not enough information for a conservative first order analysis, so do not be afraid to create a rational model and make (and state) your assumptions.

Example: Ballscrew Nut Coupling to a Linear Motion Carriage

- In a design review, you see a very large diameter ballscrew bolted to a very stiff carriage supported by 4 linear guide bearings
- The team wonders should they mount the ballscrew nut near the center or at the end of the carriage?
- During coffee break, you sketch the two options and attach preliminary calculations:
- Parameters:
  - Kradial bearing block = 1000 N/micron
  - Kradial ballscrew bearing support block=1000 N/micron
  - Lballscrew = 2000 mm
  - Dballscrew = 75 mm
  - Dmisalignment = 5 microns
- What is the maximum potential misalignment load applied to the bearing blocks?
- What are the maximum potential error motions in the carriage?







Loads, Lube & Life

It is important to understand issues which affect the life of a bearing so it can be designed to last an acceptable period of time. If the bearing has been properly mounted and sealed, the applied loads and lubrication supplied are the two most important factors in the life of a bearing. As long as the maximum *static load* is never exceeded, permanent deformation will not cause premature failure. As long as the intended dynamic load is not continually exceeded, the bearing can keep operating for the intended time if proper lubrication is maintained.

Lubrication separates the bearing material from the raceway material. *Dynamic friction* is due to the shearing of a thin layer of lubricant. The lubricant also prevents small surface finish features (asperities) on the bearing and raceway from cold welding together. The lubricant also helps to reduce the static coefficient of friction, which will always be higher than the dynamic coefficient of friction. This is also true for lubricating fastener's threads. In fact, the functional requirement for life of many fasteners and joints can be met in part by preventing corrosion at their interfaces with other materials, and lubrication applied before assembly can help to meet this goal.

The coefficient of friction is easy to measure using a block sliding on an inclined plane, and this can also be used as a quick test to see if a lubricant is even needed, or to test for the effectiveness of lubricants.[1] Oil is a commonly used lubricant, and the viscosity specified depends on the bearing interface speed. Oil also usually requires a seal to keep it from leaking out. Grease is a soap that holds oil and releases it as it warms, although seals are still required to keep dirt out. lubricating should be done according to a maintenance schedule. Lubricants attract dirt, so less is usually better.

The surface roughness of bearing elements has an important influence on the effectiveness of the lubricant layer. If the surfaces have peaks, *positive skewness*, then the peaks will extend through the lubricant layer and file away the surfaces. If the surfaces have valleys, *negative skewness*, then the lubricant will be held in the valleys but will emerge as heat increases which reduces friction and heat until equilibrium is generally reached.

1. The field of *tribology* is dedicated to studying lubrication and wear.

Most bearings run without any sign of trouble, and then fail rapidly. How can this be so? Bearings are surprisingly robust, but when the lubrication fails, e.g., it leaks out, or a larger particle breaks free, the principle of *self-hurt* seems to take over: One particle breaks off another, and the two together break off two more... until the particles are like the sands of the sea and the machine comes to a screeching halt. Before failure, however, the machine may start to give subtle warning signs such as an increase in temperature, or increased vibration and sound, or the obvious puddle of oil on the floor.

An important aspect of bearing life is not just catastrophic failure, but functional failure: For example, if an important part of function is cleanliness, such as for equipment to be used in a semiconductor clean room, then a slow continual shedding of wear particles may not be acceptable. For such applications, the designer should carefully check manufacturer specifications, because many manufacturers have bearing lubricants for specialty applications.

Because there are always design trade-offs in choosing a bearing for a production machine, many factors must be considered simultaneously by the design engineer[2]: Speed and acceleration limits, range of motion, applied loads, required life, accuracy, repeatability, resolution, preload, stiffness, vibration and shock resistance, damping, friction, thermal performance, environmental sensitivity, sealability, size and configuration, weight, support equipment & maintenance requirements, material compatibility & cleanliness, mounting, availability, manufacturability, and cost.[3]

Most design contest machines only have to make a few runs on the contest table, BUT they undergo dozens and dozens of trial runs. Beware a false sense of security about what it means to engineer a bearing for long life. Many a contest machine has been known to fail in the final round because of poor bearing design. Create a spreadsheet, much like a budget, that lists all the bearings in your machine and their applied loads and resulting expected life. If done right, you can show your design when you interview for a job...

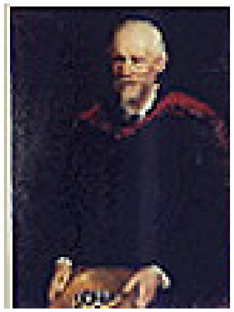
2. There can also be other factors to consider when seeking the answer to the question of bearing life, loading and lubrication, as many was 42!
3. For a detailed discussion of these factors with respect to different types of bearings, see A Slocum, Precision Machine Design, Chapter 8, SME Detroit MI, 1995, available from www.SME.org or www.amazon.com.

Loads, Lube & Life

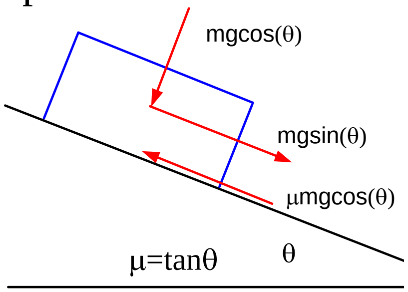
- Loading:
  - The maximum load is the load the bearing can withstand for short periods
  - Longer life is achieved with lower loads
- Lubrication: *Tribology* is the study of lubrication and wear
  - Separates the structural materials, and prevents chemical bonding
  - Allows for viscous shear of a fluid thereby reducing material wear
  - Oil is common ; Grease is soap that holds oil and releases it as it warms
  - Lubricants attract dirt, so less is typically better, and use seals if possible
  - Surface finish is critical!
- Oil impregnated bearings release lubricant as they get warm
- Some materials are inherently lubricious and function “dry”

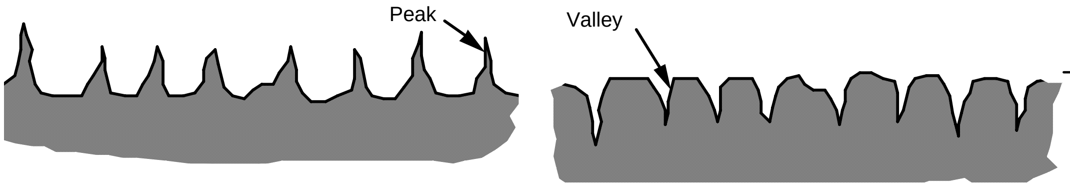
Claude Louis Marie Henri Navier (1785-1836)

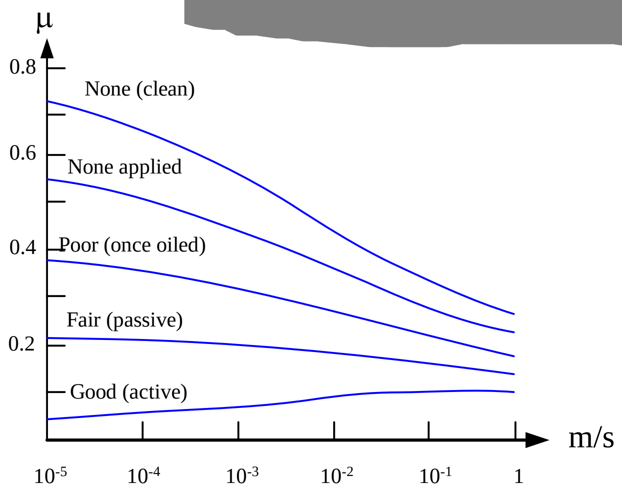
George Gabriel Stokes (1819-1903)

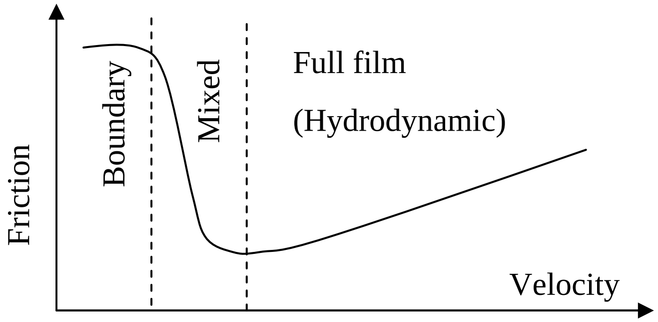


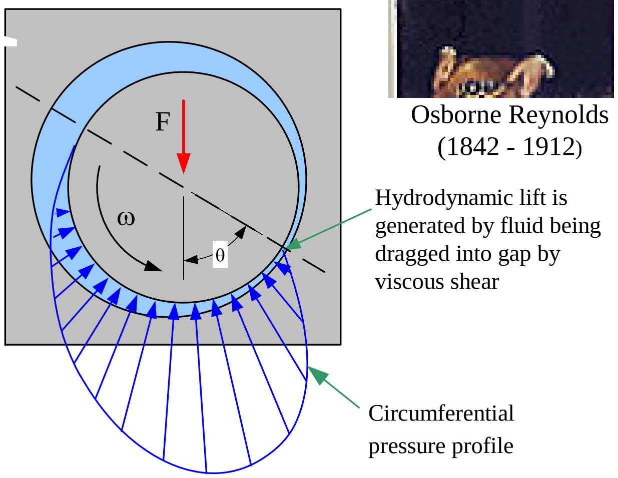
Osborne Reynolds (1842 - 1912)











Loads, Lube & Life: *Sliding Contact*

There are many different theories on sliding contact (*plain*) bearing wear. The first method to prevent wear is to make sure the surfaces all have negative skewness and are well lubricated. Secondly, the materials should be of a different type, preferably with one being harder than the other. In addition, the lower the contact pressure, the longer the life. It has also been postulated that small grooves in the bearing surface can be provided to collect wear debris before they can cause more damage. There are also certain design limits to not exceed, or a sliding contact bearing will rapidly wear. These include the maximum *contact pressure*, *surface speed*, and *PV value*.

Once the total load applied to each bearing is determined, the *contact pressure* can be obtained by dividing the load by the bearing area. The contact pressure usually has units of N/mm^2. For a shaft in a bushing, the area is just the product of the length and diameter of the bushing. The contact pressure must be below the maximum contact pressure, because higher pressures can cause the material to permanently deform. The *surface speed* is equal to the relative speed of the bearing and moving component. For rotary systems, convert from rpm to rad/s:

$$P_{contact} = \frac{F_{load}}{w_{width} L_{length}} \qquad V = V_{linear} \qquad V = \frac{\pi D_{shaft} \omega_{rpm}}{60}$$

The product of the contact pressure and the relative velocity between the bearing and the rail (shaft) is called the *PV value*. PV_{max} depends on the ability of the system to transfer heat away from the bearing, where h_{shaft} and $h_{bearing}$ are thermal conductivities, T is temperature, t is the bearing thickness, and μ is the coefficient of friction:

$$PV_{\text{max}} = \frac{\pi h_{\text{shaft}} h_{\text{bearing}} \left(T_{\text{bearing max}} - T_{\text{ambient}} \right)}{2t\mu}$$

The wear rate δ_{wear} in micrometers of wear per kilometer of travel (μm/km) is a function of the contact pressure and temperature. Often a simple linear relation with slope β (μm/(km-Pa) can suffice for ambient applications. For some materials, the wear rate varies significantly with contact pressure, but then the relation can be broken up to be piece wise linear. δ_{wear} also depends on the contact pressure and can be inferred from manufacturers' wear-rate graphs. For a typical sliding contact (plain) polymer bearing, P_{max} = 40 MPa, V_{max} = 1.5 m/s, PV_{max} = 3500 Pa-m/s, μ = 0.2, and β = 5E-7 μm/(km-Pa).

$$\delta_{wear} = \beta P_{pressure}$$

In addition to loads imposed by the process that the bearing is intended to support, as discussed for bearings in general, significant loads may be developed by deflections of the structure acting on the stiffness of the bearings ($F = k\delta$). Ideally structural deformations are accommodated by clearances between the bearings and the supported load.

Another difficult issue is edge loading of a bushing on a shaft. When a shaft is supported by two bushings that are held rigidly in a structure, the shaft imposes higher loads at the ends of the bushings as it deforms. The ends thus become like Hertzian line contacts. From $F = kx$, a solution is to make the radial stiffness of the bushing lower near the ends of the bushing[1]. This could be accomplished using compliant features in the structure that supports the bushing. A simpler approach if acceptable, is to just provide enough clearance space to accommodate the deflections.

For a robot design contest, it may be feasible to oil lubricate surfaces without using seals, because the oil flows and coats better than grease. Porous bronze bushings release just enough lubricant, which even allows them to be used in photocopiers. DU™ and DX™ bearings are made from layers of different materials to also have the same self-lubricating effect, although they do appreciate being greased. Another strategy is to use an inherently lubricious bearing that can function "dry" such as Teflon, Delrin, UHMWP (ultra high molecular weight plastic) and sometimes Nylon.

Evaluate your machine's bearings' maximum contact pressures and PV-values to make sure they are well below maximum allowable values. See *Bearings_sliding_simply_supported_shaft.xls*. Try to systematically visualize each element in the system as a soft piece of rubber: How does its resulting deformation affect clearance and bearing loads?

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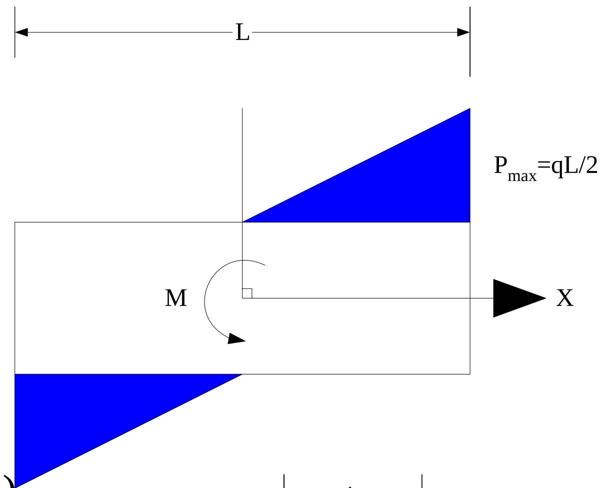
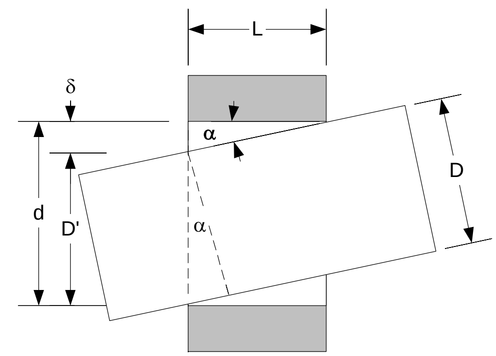
1. This concept was proposed by Prof. Samir Nayfeh at MIT

Loads, Lube & Life: *Sliding Contact*

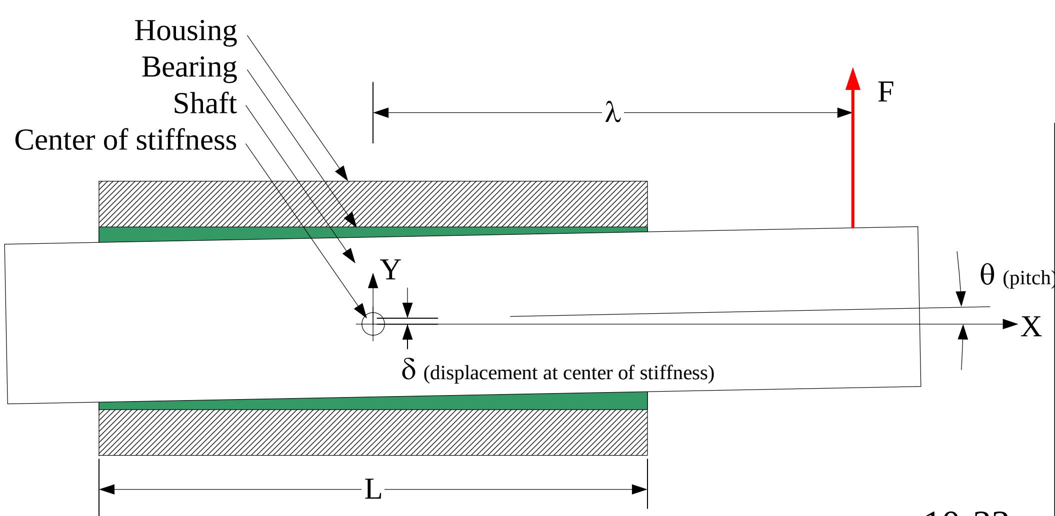


- The PV value is the product of the pressure and the velocity
- Sliding contact bearings have a maximum allowable pressure and a maximum PV value`
  - The allowable product of pressure (F/(D*L)) and velocity
  - The stiffness of the bearing
  - E.g., a Delrin bearing used as a bushing (Nylon has ½ these values)



Maximum Pressure (N/mm^2, psi)	140	19,895
PV continuous (N/mm^2-mm/s, psi-ips)	1800	9,791
PV short periods (N/mm^2-mm/s, psi-ips)	3500	19,581
Compressive Modulus (GPa, psi)	4	579,710

  - http://www.dupont.com/enggpolymers/americas/products/deldata.html
    - Perform calculations and if in doubt, do a *Bench Level Experiment*



Bearing_Sliding_Pitch_PV.xls	
To determine bearing contact pressure in a slider loaded by a moment	
By Alex Slocum 3/8/98. Last modified 5/2/04 by Alex Slocum	
Enters numbers in **BOLD,** Results in **RED**	
Total slider length, L (mm)	**25**
Slider contact width, w (mm)	**10**
Pivot point height above center of stiffness, h (mm)	**25**
Force, F (N)	**100**
Max PV continuous, PVC (N/mm^2-mm/s)	**1800**
Moment, M (N-mm)	**2500**
Maximum contact pressure, qmax (N/mm^2)	**2.400**
Speed, v (mm/sec)	**50**
PV (N/mm^2-mm/sec)	**120**

Loads, Lube & Life: *Rolling Contact Rotary*[1]

Rolling element rotary bearings have an L_{10} life which is defined as the allowable load for a given number of revolutions where only 10% of the bearings will fail. The basic load life equations for rotary motion bearings are:

$$F_e = K_\omega K_r F_r + K_\omega K_A Y_1 F_A$$

$$L_a = a_1 a_2 a_3 \left(\frac{f_B C}{F_e} \right)^\lambda$$

- L_a: millions of revolutions.
- a_1: 1 @ 90%, 0.62 @ 95%, 0.53 @ 96%, 0.44 @ 97%, 0.33 @ 98%, 0.21 @ 99%
- a_2: materials factor, which is typically 3.0 for steel bearings.
- a_3: lubrication factor, which typically is 1.0 for oil mist. Grease and oil are generally equivalent at low speeds, and oil is preferred at high speeds.
- C: basic dynamic load rating.
- f_b: Dynamic load rating factor for number of adjacently mounted. bearings: f_b = (number of adjacently mounted bearings)$^{0.7}$
- F_e: the applied equivalent radial load, determined by bearing type.
- λ: 3 for balls and 10/3 for rollers.
- K_w: rotation factor = 1 for rotating inner ring and 2 for a rotating outer ring.
- K_r: radial load factor = 1 for normal duty, 3 for continuous heavy duty (shock) applications.
- K_A: axial load factor = 1 for normal duty, 3 for continuous heavy duty (shock) applications.
- Y_1: For many bearings, because so many more balls support thrust than radial loads, even though the contact angle is relatively shallow, the thrust load is topically directly added to the radial load to arrive at an equivalent radial load for the bearing. If this is not the case, then the factor Y1 must be obtained from the bearing manufacturer.

1. See ISO 281 or ABMA 9 for ball bearing and ABMA 11 for roller bearing life issues Also see *The Torrington Company, Service Catalog, 1988*, and BJ Hamrock & D Dowson, Ball Bearing Lubrication, John Wiley & Sons, NY

Precision life is about 90% of the L_{10} life, because when bearings start to fail, they do so very rapidly as a small bit breaks off which creates a cascading effect. Contrast this to sliding contact bearings which more typically gradually wear away.

One of the primary reasons for selecting rolling element bearings, is their low coefficients of friction, which is why they are often referred to as *antifriction* bearings. Typically μ = 0.01 - 0.005 with balls having less friction than rollers because the rollers' ends invariably make some sliding contact with the raceway sides. However, this is predicated on essentially true rolling contact. Even when a rolling element is placed between two planes, it does not undergo true rolling contact because of the Hertz contact deformation. The deflection and contact width can be calculated, and slip is proportional to the difference between the true diameter and the diameter at the contact point. As discussed on page 10-20, the amount of slip and hence the efficiency can be predicted if the contact zone width can be measured or predicted. Note that the contact width *d* can sometimes be measured by coloring the surface with a dry-erase marker and then sliding the contacts sideways.

In a four-point contact bearing, the Hertz contact zones are inclined with respect to the axis of rotation, and thus the difference in diameters across the width of the contact zone is much larger, and can lead to tens of percent slip. This does, however, increase damping which can help some servo mechanisms subject to large external disturbance forces. Further applying reciprocity to the "problem" of slip, an interesting mechanism results that makes use of differential slip present in a four-point contact ball bearing. The "Backlash free rotationally adjustable mount in the nature of a transmission" described in US patent 5,435,651 uses this effect along with a *tractive fluid* lubricant which thickens under very high pressure to increase traction at the contact interface.

Many bearings come pre-lubricated for life. Although lubrication is critical, just as critical is keeping the bearings clean and dry using seals and/or shields. Too many bearings fail when dirt or water are forced past the seals by pressurized air or water that is being used to clean a machine!

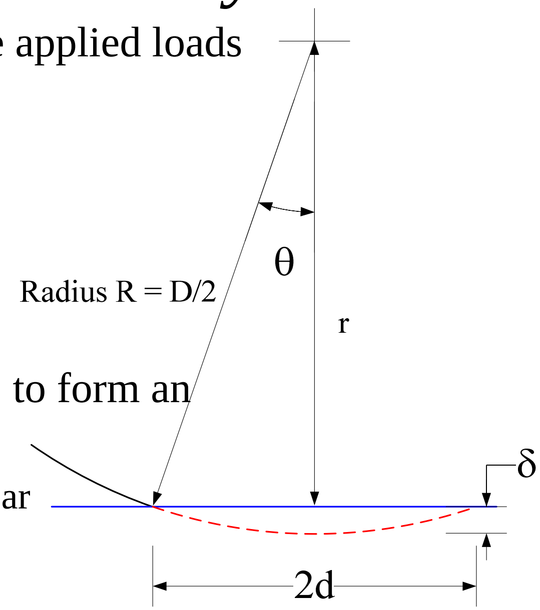
How long will your rolling element bearings last? Are the primary life issues applied loads, misalignment, or environmental effects? What are your countermeasures?

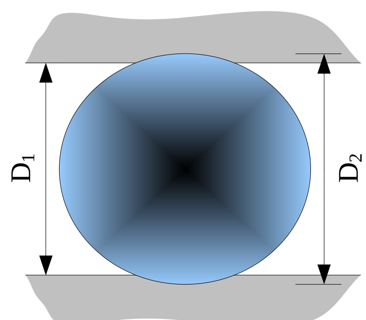
Loads, Lube & Life: *Rolling Contact Rotary*

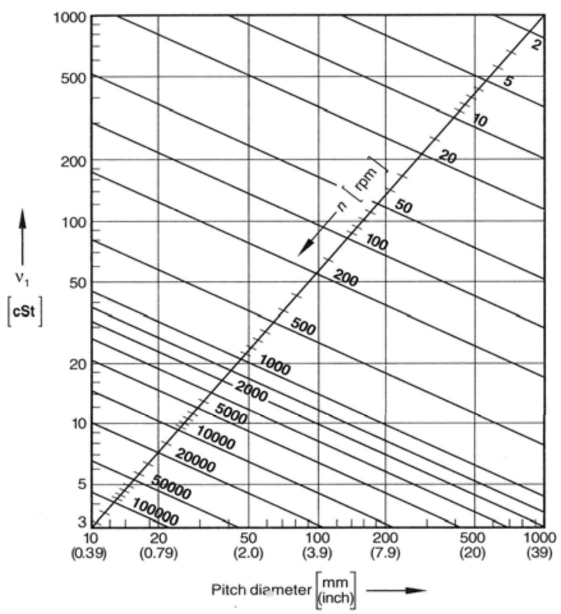
- Forces from the product of misalignments and stiffness must be added to the applied loads

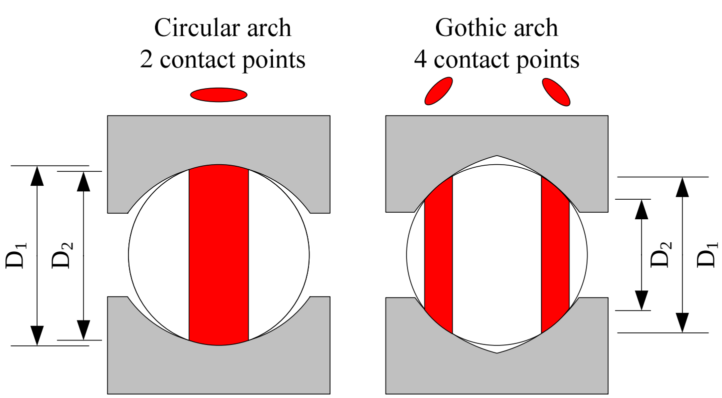
$$L_{10\text{ ball bearings}} = \left(\frac{C_0}{P}\right)^3 \quad L_{10\text{ roller bearings}} = \left(\frac{C_0}{P}\right)^{10/3}$$

  - L_{10} is the number of revolutions after which 10% of a set of bearings fail
    - See standards: ISO 281 or ABMA 9 (balls) or ABMA 11 (rollers)
    - Assumes lubrication prevents metal-to-metal contact: Cleanliness is critical!
- A rolling bearing pulls in lubricant, whose viscosity increases with pressure, to form an *elastohydrodynamic* lubrication layer between the ball and the race
  - The EHD layer accommodates differential slip, but generates heat via viscous shear
- Rolling contacts have friction because the elements deform under load and cause rolling across different effective diameters (slip)
  - The rolling contact interface geometry also plays a significant role

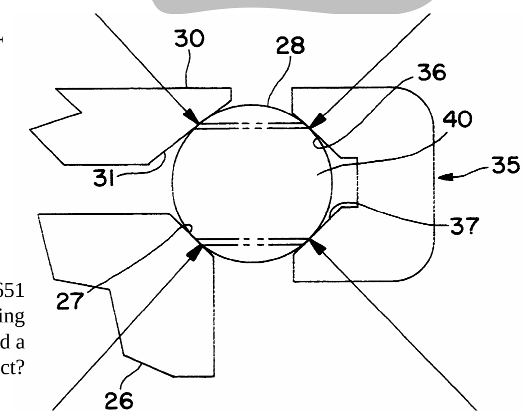








$$\%\text{ slip} = \frac{D_1 - D_2}{D_1} \times 100$$



Check out US Patent 5,435,651 (www.uspto.gov). How does it use rolling slip to create differential motion and a transmission effect?

Loads, Lube & Life: *Rolling Contact Linear*

A typical linear motion carriage has a rectangular footprint and one linear bearing mounted near each corner. For an accurate analysis of load capacity and stiffness, a finite element model is needed that includes characteristics of the bed and carriage. However, before one can build a finite element model, one should use engineering calculations to initially size members and bearings. For the purposes of finding approximate bearing reaction forces, it can be assumed that the carriage and structure the rails are mounted to behave like a rigid body.

Consider the general case shown. There are three forces, F_X, F_Y, and F_Z, which can act anywhere in space with respect to the carriage coordinate system. Forces of the F_X type can include cutting forces, actuator forces, center-of-mass acceleration forces, and forces from axes stacked on top of the carriage. Forces of the F_Y and F_Z type can include cutting forces and forces from other axes stacked on top of the carriage. Gravity can act in any direction, depending on the machine configuration. In order to estimate the magnitudes of the resultant forces on the bearings, two assumptions must be made: 1) moment stiffness of the bearings is insignificant, 2) forces are distributed in relation to the bearings' proximity to them.

With these assumptions, careful scrutiny of the geometry represented by the figure, and some algebra one can find the effect of the generic forces on each of the four bearing carriages. With superposition, the net forces on the bearing carriages caused by a number of generic forces applied at different points can be determined.

X direction forces create a moment that cause Y and Z direction forces in the bearing carriages in the form of couples where the total force couple is evenly distributed between bearings at respective ends:

$$F_{1Y,F_X} = F_{2Y,F_X} = \frac{Y_{FX}F_X}{2(X_1 - X_4)} \qquad F_{3Y,F_X} = F_{4Y,F_X} = \frac{-Y_{FX}F_X}{2(X_1 - X_4)}$$

$$F_{1Z,F_X} = F_{2Z,F_X} = \frac{Z_{FX}F_X}{2(X_1 - X_4)} \qquad F_{3Z,F_X} = F_{4Z,F_X} = \frac{-Z_{FX}F_X}{2(X_1 - X_4)}$$

Y direction forces cause Y direction forces in the bearing carriages that are assumed to be proportional to the bearing carriages' relative XZ location with respect to the point-of-force application. First consider how the relative X position affects the distribution of forces between bearing carriages 1 and 2, and carriages 3 and 4. Lumping carriages 1 and 2 together and carriages 3 and 4 together, the relative X positions of the carriages yield the following allocation of forces:

$$F_{1Y+2Y,F_Y} = -F_Y\frac{(X_4 - X_{FY})}{(X_4 - X_1)} \qquad F_{3Y+4Y,F_Y} = F_Y\frac{(X_1 - X_{FY})}{(X_4 - X_1)}$$

The distribution of forces between carriages 1 and 2 depends on their Z location relative to the force (similarly for carriages 3 and 4):

$$F_{1Y,F_Y} = -F_Y\frac{(X_4 - X_{FY})(Z_2 - Z_{FY})}{(X_4 - X_1)(Z_2 - Z_1)} \qquad F_{2Y,F_Y} = F_Y\frac{(X_4 - X_{FY})(Z_1 - Z_{FY})}{(X_4 - X_1)(Z_2 - Z_1)}$$

$$F_{3Y,F_Y} = -F_Y\frac{(X_1 - X_{FY})(Z_4 - Z_{FY})}{(X_4 - X_1)(Z_3 - Z_4)} \qquad F_{4Y,F_Y} = F_Y\frac{(X_1 - X_{FY})(Z_3 - Z_{FY})}{(X_4 - X_1)(Z_3 - Z_4)}$$

A Z direction force with an X axis offset causes Z direction forces, which are assumed to be evenly distributed between pairs acting as couples:

$$F_{1Z,F_Z} = F_{2Z,F_Z} = -F_Z\frac{(X_4 - X_{FZ})}{2(X_4 - X_1)} \quad F_{3Z,F_Z} = F_{4Z,F_Z} = F_Z\frac{(X_1 - X_{FZ})}{2(X_4 - X_1)}$$

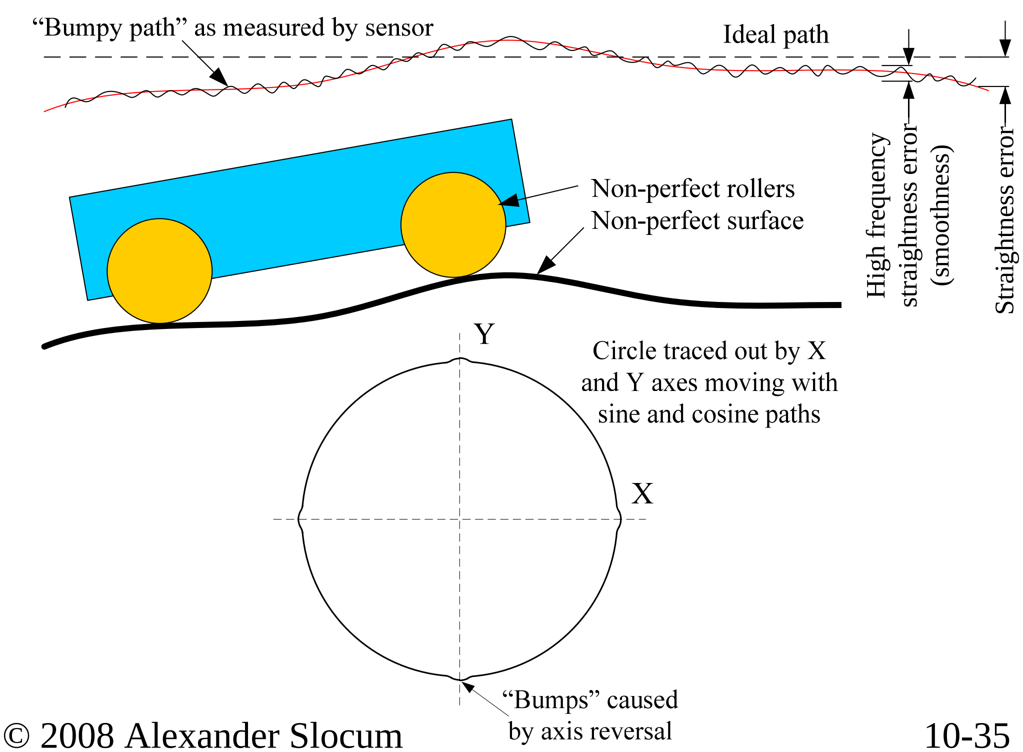
A Z direction force with a Y axis offset causes Y direction forces, with bearing carriages 1 and 4 acting as a couple with carriages 2 and 3:

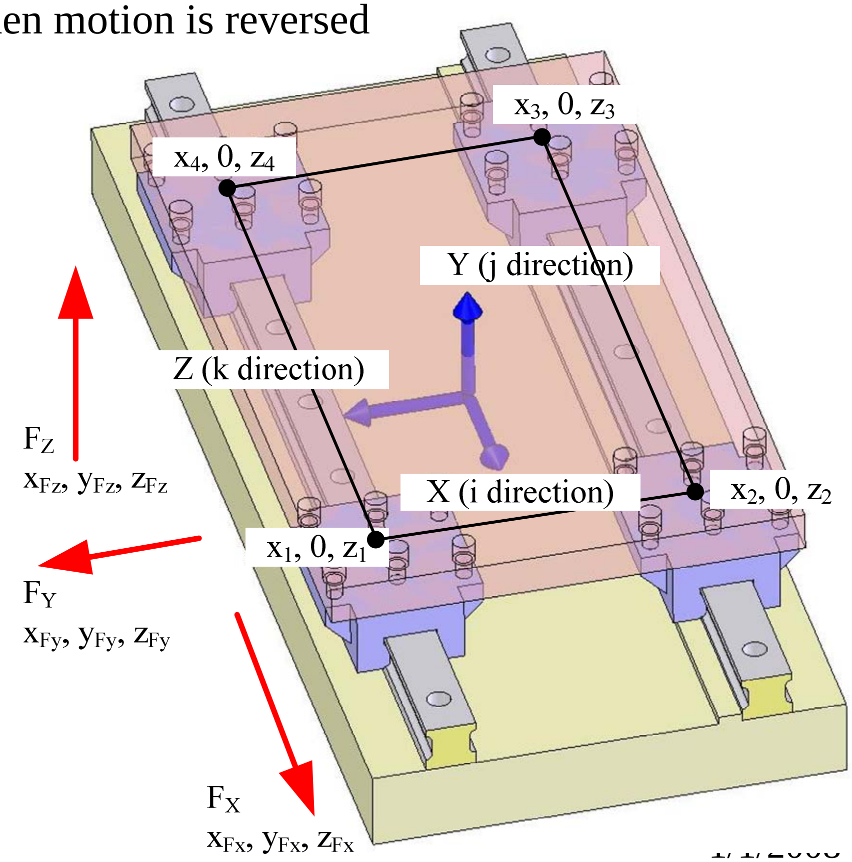
$$F_{1Y,F_Z} = F_{4Y,F_Z} = \frac{Y_{FZ}F_Z}{2(Z_1 - Z_2)} \qquad F_{2Y,F_Z} = F_{3Y,F_Z} = \frac{-Y_{FZ}F_Z}{2(Z_1 - Z_2)}$$

See *Bearings_linear_forces.xls!*

Loads, Lube & Life: *Rolling Contact Linear*

- Linear motion rolling element bearing life is affected by many of the same issues as rotary motion rolling element bearings
  - Linear bearings are harder to seal because the seals must slide over exposed surfaces
  - Linear bearings do not generate as much heat as rotary motion bearings
- Systems are often overconstrained to provide moment stability
  - Forces from the product of misalignments and stiffness must be added to the assumed applied loads
  - Loads are assumed distributed in proportion to their location with respect to the bearings (*Bearings_linear_forces.xls*)
- Rolling contact friction causes hysteresis (hesitation) when motion is reversed
  - Imperfections also cause micro bumpiness





Loads, (no) Lube & Life: *Flexures*

The ability of a flexure to provide desired motion requires that the beams be long and thin; however, this limits the strength and stiffness in the out-of-plane direction. If the flexure is cut from a solid block so it is monolithic, then it can be made "over constrained" with little negative impact. For a four-bar flexure with N flexural elements of thickness t, depth (into page) w, and length L, with in-plane and out-of-plane deflections δ and γ respectively:

$$k_{in-plane} = \frac{NEwt^3}{L^3} \qquad k_{out-of-plane} = \frac{NEtw^3}{4L^3}$$

$$\sigma_{in-plane} = \frac{3Et\delta}{L^2} \qquad \sigma_{out-of-plane} = \frac{3Ew\gamma}{2L^2}$$

Another primary issue with flexures is the parasitic error motion which often accompanies the desired motion. For example, a four-bar flexure moves in the desired X direction, but it is accompanied by a Y direction error motion as was discussed on page 10-14. Using the principle of reciprocity, however, this effect can be mitigated with a folded flexure as shown in the purple figure. Note that the floating structures are free to move in a manner that allows the flexural beam elements to shorten as they bend. If this degree of freedom was not allowed, then the flexure would be 10x stiffer in the desired motion direction, and hence have 10x less motion capability.

The equations presented for flexure design predict the deflection and stress in the flexures, but how accurate are they? What is the effect of a rounded fillet at the base of the flexure on reducing stress concentrations, and how does it affect the maximum deflection? The images show a model that was input to Pro/MECHANICA for finite element analysis using the same parameters as used in the following spreadsheet. Indeed, the analytical results correlate very well with the finite element analysis:

	Analytical	**FEA**
delta	0.076	0.075
stress	6.29	6.37
deltap	5.71E-05	5.00E-05
thetap	-0.47	

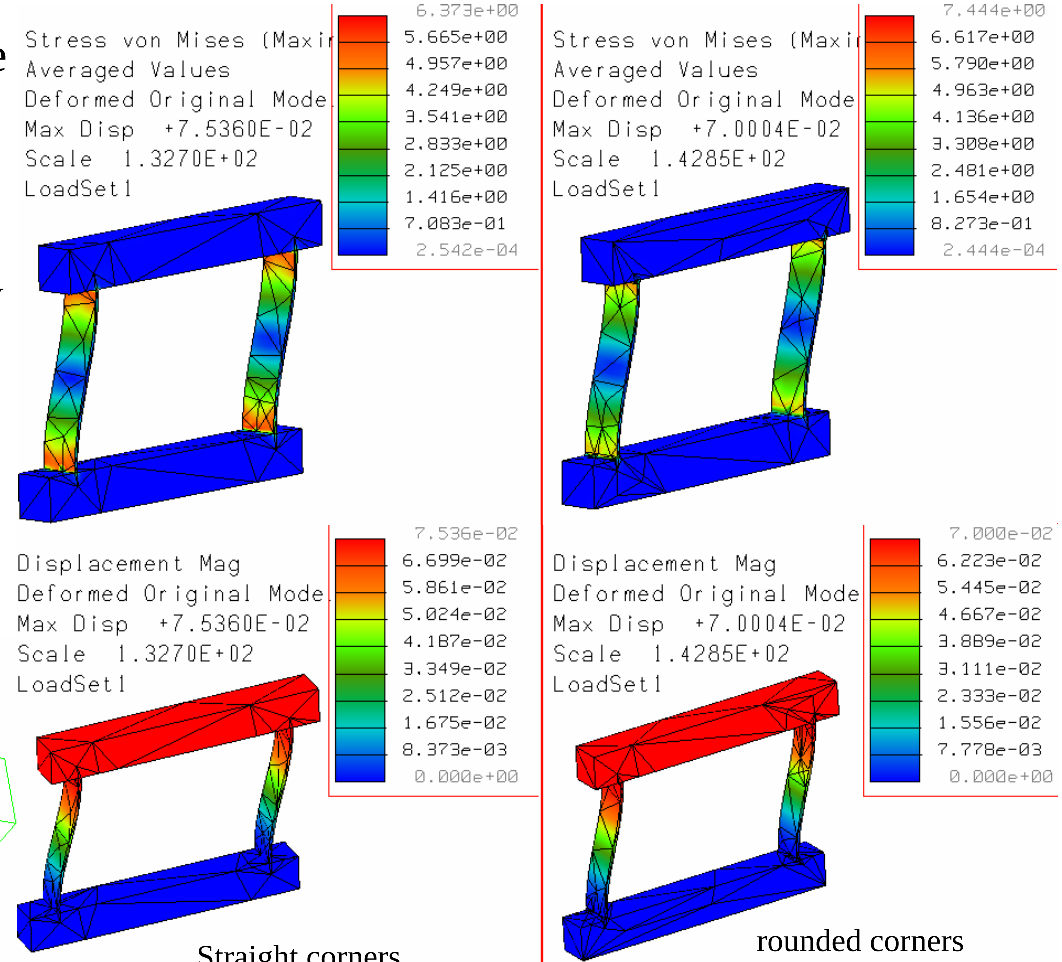
Flexture_4_ bar.xls		
To design 4-bar parallelogram flexures (more than 2 blades can be used)		
By Alex Slocum 4/25/04, last modified 10/6/04 by Alex Slocum		
Enter numbers in **BOLD,** results are in **RED**		
L (mm)	**50**	Length of blade
r_root	**1**	Stress concentration relief radius
a (mm)	**50**	Distance force applied from base
sigmax (N/mm^2)	**250**	maximium allowable stress
w (mm)	**12**	Width of blade
E (N/mm^2)	**6.89E+04**	Modulus of elasticity
t (mm)	**1.0**	Thickness of blade
b (mm)	**80.0**	Distance between blades
N	**2**	Number of blades (>=2)
F (N)	**2.0**	Total applied force
delta (mm)	**0.072**	Displacement from force
stress (N/mm^2)	**6.16**	Stress from force
Fmax (N)	**81.63265306**	Max total allowable force
deltamax (mm)	**2.90**	Max allowable deflection
Parasitic errors from applied force		
deltap (microns)	**0.052**	Vertical error motion
thetap (microrad)	**-0.475**	Pitch error motion
Stiffnesses		
kip (N/mm)	**28.1**	In-plane
koop (N/mm)	**506.3**	Out-of-plane

It is important to keep the stress in a precision flexure well below the elastic limit. Iron-based alloys have an endurance limit: If the maximum stress, including stress concentration effects, is less than half the maximum yield stress, the flexure will last indefinitely. Non-ferrous materials have no limit, but at half the yield strength, they can typically service for tens of millions of cycles. Plastics craze and fail, but plastic flexures are usually used in simple consumer products.

The spreadsheet *Flexure_hourglass.xls* (in Topic 4) helps with design of hourglass flexures (hinges) and *Flexure_4_bar.xls* and *Flexure_folded.xls* help with translational motion. Are flexures useful for trigger motions?

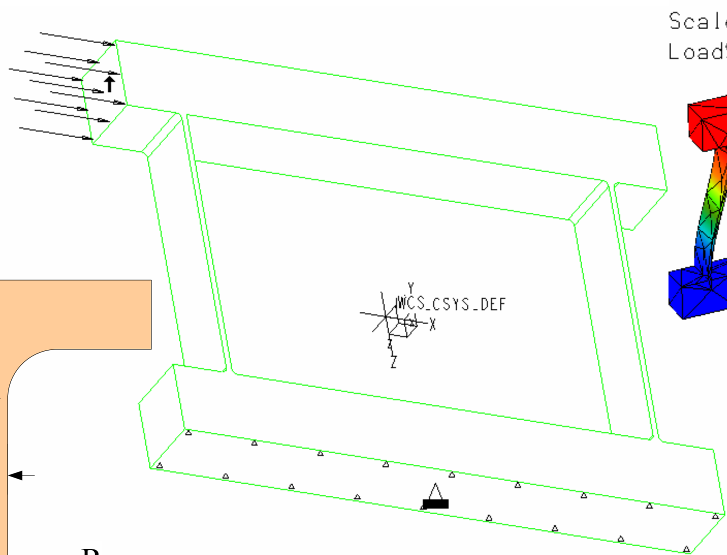
Loads, (no) Lube & Life : *Flexures*

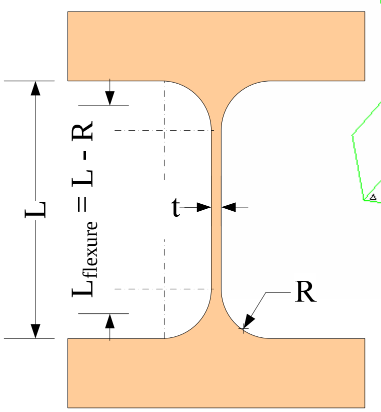
- To reduce stress, use rounds equal to the blade thickness
  - This reduces the effective blade length by one blade thickness
- Spreadsheets allow a designer to rapidly develop designs
  - Once a design is developed, it can be checked with Finite Element Analysis
  - For FEA to accurately predict stress concentrations, a fine mesh is required

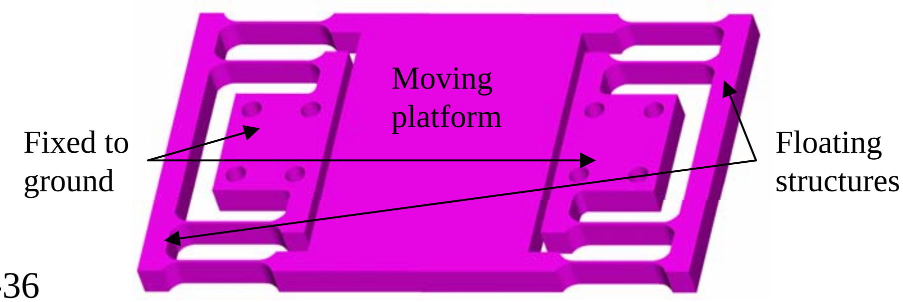


Straight corners

rounded corners (r = blade thickness)







Dynamic Seals[1]

The job of a dynamic seal is to keep lubricant in and foreign material out while a shaft rotates and/or translates[2]. Indeed, the most impressive bearing with the biggest load capacity can rapidly be destroyed by the introduction of just a little bit of dirt. There are two primary methods of protecting bearings: seals or shields integral with the bearing, and external seals or shields. Bearings with integral seals or shields offer basic protection and take up little space as shown, and are commonly found in many consumer products and industrial machines used in non-aggressive environments.

Contact-type seals make mechanical contact with the haft, and are used for both rotary and linear motion sealing. All contact-type dynamic seals generate heat, and thus they have speed limits on the order of 15-20 m/s. Non-contact type seals are used for high speed rotary motion applications, and use a labyrinth to create a long resistance path for foreign matter to follow before it can get to the bearing. By applying slight air pressure inside the bearing region, a small steady flow of air out of the bearing through the labyrinth further helps to keep out foreign material.

One of the most common forms of contact-type seals is the simple "O" ring. Recall that O-rings are relatively stiff and are primarily designed to seal against high pressures by using the *principle of self-help*: as the pressure is increased, the O-ring is forced against the sealing surfaces, so they are not commonly used to seal bearings that run at high speeds.

The simplest and most common contact-type seal used for sealing bearings is a simple *felt seal.* A *lip seal*, or *U-cup* seal uses a compliant rubber lip to contact the shaft, where the elasticity of the rubber keeps it in contact, but with low force. A coiled spring behind the lip creates an *energized* lip seal with better control of the contact pressure. The use of the spring allows a harder sealing material, such as PTFE (Teflon) to be used, which enables such seals to operate from high vacuum to hundreds of atmospheres pressure.

1. Also see page 7-28 for a discussion of seals in the context of hydraulic and pneumatic systems.
2. It has been said jokingly that fish do not have propellers so as to minimize the potential for problems with seals!

Rotary motion of a shaft can act like a screw to pump foreign matter into the bearing. This effect can be reversed by placing small vane-like features on the surface of contact seals to have the effect of auguring out foreign matter. This principle of self-help extends to the use of shapes that act to fling materials off a rotating shaft, or to fling lubricant back into the bearing. Flingers are thus commonly used to help seal bearings used in wet environments.

For linear motion applications, wiper-type seals work effectively to keep foreign matter out. Some will always get in, however, and then it is only a matter of time to failure. In severe environments, a scraper can act like a plow to remove most of the debris from the rail. The wiper then does a final cleaning and it also seals in the lubricant.

Seals are often almost an afterthought to many designers, yet they are the first line of defense for protecting bearings. Because they are not themselves structural members and do not support loads, they are often relatively delicate compared to the bearings and structures they protect. It is thus important to design the structure that supports the bearing to also support and protect the seal. If a seal is impacted, its structure may bend and contact the shaft with greater than intended force which can cause excessive heat and seal failure. Many a spindle has failed when shaft deflections increase rotating seal pressure which generates excessive heat that causes the seal's elastomer to fail.

One common form of seal failure is from pressurized air or water that is used to clean off equipment. The pressurized fluid will push dirt and water through a seal and into the bearings. High pressure washers used to clean off engines may have caused far more harm than ever imagined. A shiny clean engine on the outside may have its bearings loaded with water and grit forced pass its seals!

Fortunately, there are many seal companies with many different types of seals just waiting to help. Just remember to consider seals near the beginning of the design phase!

For your robot contest machine, what are the primary sources of contaminants that could harm the bearings? How can you most easily protect the bearings? Are external seals really necessary, or can simple shielded bearings work? When should you select a sealed bearing? Will the extra friction torque of the seals too unacceptably affect your power budget?

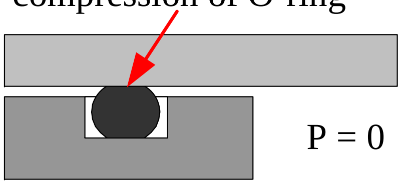
http://nmml.afsc.noaa.gov/education/pinnipeds/seals.htm

Folks, use good seals and keep the environment clean!

Dynamic Seals

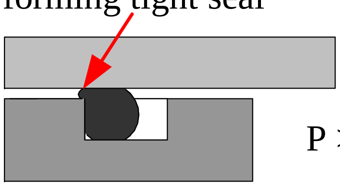
- Keeping dirt out of a bearing is of utmost importance for long bearing life
- The simplest seals are shields that keep dirt out using a labyrinth path
  - Internally supplied air pressure create a net outflow
- A mechanical contact seal is the best for low speed sealing, but it generates friction
- *Flingers* use centrifugal force to keep contaminants out and lubricants in

Initial installation compression of O-ring

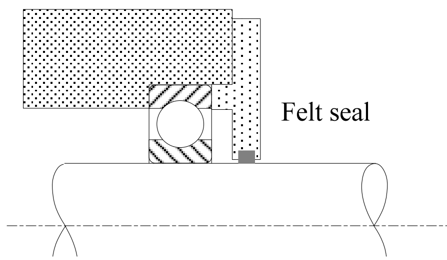


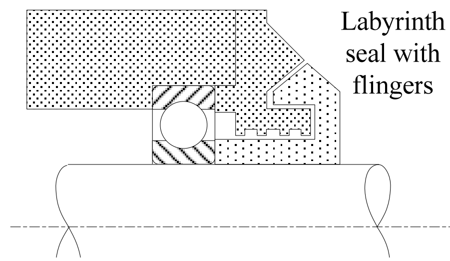
P = 0

"Extrusion" of O-ring forming tight seal



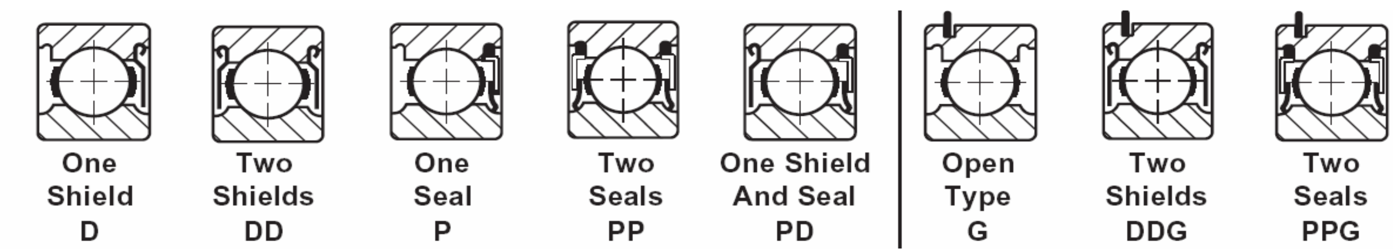
P >> 0





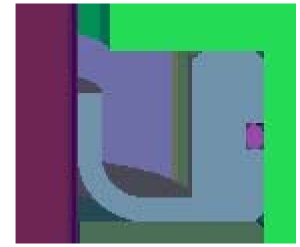
Integral shields and seals

Integral snap rings

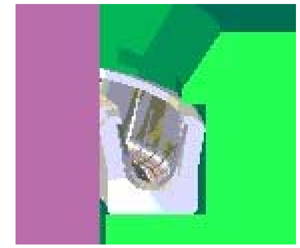


http://www.timken.com/industries/torrington/catalog/pdf/fafnir/radial.pdf

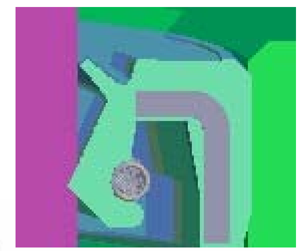
FlexiLip™



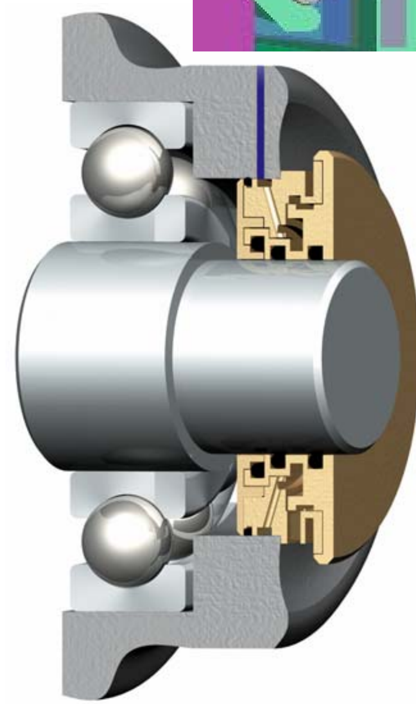
FlexiSeal™



Oil Seal



http://www.parker.com/sg/indexv4.asp



http://www.inpro-seal.com

Bearings in Systems[1]

Machine systems often consist of supporting structures, bearings that join together and allow relative motion between components, actuators to move the components, and sensors to determine positions of the components along with a control system. Whenever there is relative motion between components, a bearing is involved, and if there is mechanical contact in the bearing, then there is a chance of failure. because bearing materials are not perfectly homogenous, their failure is statistical in nature, which means that if there are many bearings in a system, then collectively the chance of failure is greater than for any single bearing.

The Weibull distribution[2] is often used in many fields including bearing failure analysis. For any single bearing:

$$\ln\left(\ln\frac{1}{\zeta_a}\right) = e\ln\frac{L}{A}$$

Where:

- ζ_a is the probability of a bearing surviving L million revolutions
- L is millions of revolutions
- e is the slope of the Weibull plot (dispersion coefficient or measure of scatter in bearing life data) and it is typically 10/9
- A is a coefficient, such that for bearings, when $\zeta_a = 1$, and $e = 10/9$, L = 0; A = 7.579.

Most bearing manufacturers quote a dynamic load rating C_o at which the L_{10} life is one million revolutions. This means that when subjected to a load of C_o, of a large number of single bearings being tested, 90% of them will last one million revolutions. The Weibull equations can be used to predict the number of revolutions that these bearings can be expected to operate with the same C_o applied load, but a different probability of survival. For a different load, the L10 life is scaled as described on page 10-33, and in summary:

$$L_{10\text{ ball bearings}} = \left(\frac{C_0}{P}\right)^3 \qquad L_{10\text{ roller bearings}} = \left(\frac{C_0}{P}\right)^{10/3}$$

Hence if the applied load P is 1/3rd the dynamic rated load, a ball bearing's expected L_{10} life will be nine million revolutions.

What if there are ten such bearings in a system where if one bearing fails the system shuts down? How many revolutions on average can each bearing rotate before the system shuts down? Weibull predicts:

$$L = \left(\frac{1}{\sum_{i=1}^{\text{N bearings}} \frac{1}{L_i^e}}\right)^{1/e}$$

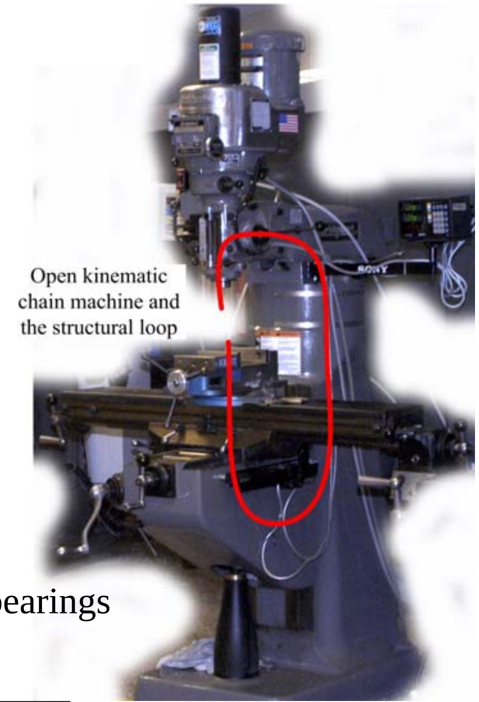
This is why very rugged high reliable machinery can sometimes appear to be way over-designed, because failure is not an option. On the other hand, this equation can also be used to predict when components and the system need to be inspected when it is important to minimize size, weight, fixed cost... The spreadsheet *Bearings_linear_forces.xls* shows some very dramatic results when more bearings are added to a system.

It is easy to have a creative idea, harder to make the idea work, and harder even still to make an idea work reliably. Bearings are often the most critical element in a system, because they are depended upon for accuracy and repeatability, not only from a geometry perspective, but from a life perspective. Carefully review your system design and check your designs and analysis to make sure you have paid attention to the FUNdaMENTALS of Design!

1. Also see page 3-21. A good general reference that discusses many of the topics in this chapter and more is Technology of Machine Tools, Vol. 5, Machine Tool Accuracy, Robert J. Hocken (ed.), Machine Tool Task Force. Available from U.S. Dept. of Commerce National Technical Information Service as Report UCRL-52960-5. There are numerous references on this subject, far too many to list here, but the Technology of Machine Tools provides a very good summary of the art and science of machine tool errors, as it was written by a task force of leading researchers.
2. See for example *http://www.barringer1.com/weibull_bio.htm* or Abernethy, Robert B. "The New Weibull Handbook," Fourth Edition, November 2000, published by the author. ISBN 0-9653062-1-6.

Bearings in Systems

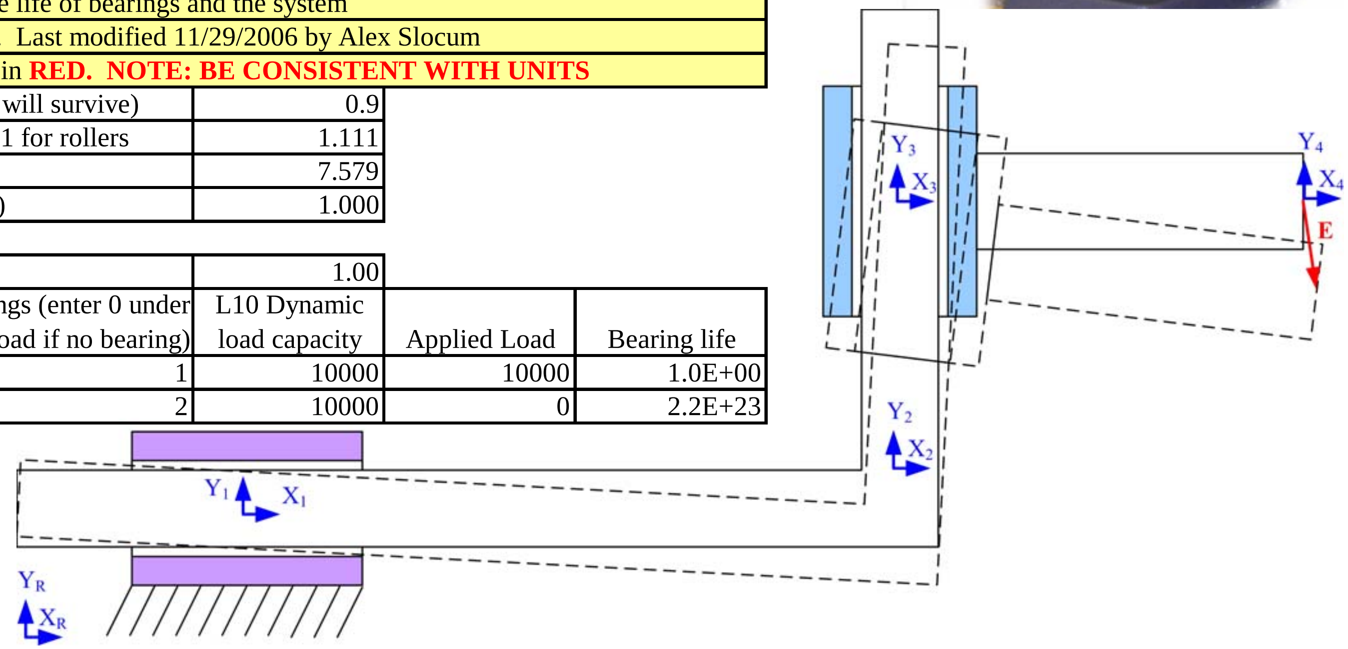
- Bearings are just interfaces between parts of a machine
  - They are critical for the supporting of loads
    - Load-life calculations for individual bearings can be combined to predict the life of the entire machine
      - Weibull distribution function is a good model
    - Most effort is usually put into load-life calculations
- Bearings are critical for the guidance of loads
  - The topic of Precision Machine Design assumes load-life calculations have been addressed, and focuses on the accuracy and repeatability of motion provided by the bearings



Bearing_life_Weibull.xls			
To determine life of bearings and the system			
Written by Alex Slocum. Last modified 11/29/2006 by Alex Slocum			
Enters numbers in **BOLD,** Results in **RED. NOTE: BE CONSISTENT WITH UNITS**			
Probability of survival, p (x10= % of bearings that will survive)	0.9		
Weibull dispersion coefficient, eps (10/9 for balls, 1 for rollers	1.111		
Constant, A	7.579		
Life (millions of revolutions) (L10 is when p = 0.9)	1.000		
System life (millions of revolutions)	1.00		
Bearing # in system containing many bearings (enter 0 under Applied Load if no bearing)	L10 Dynamic load capacity	Applied Load	Bearing life
1	10000	10000	1.0E+00
2	10000	0	2.2E+23

Waloddi Weibull

1887-1979



Topic 10 Study Questions

Which suggested answers are correct (there may be more than one, or none)? Can you suggest additional and/or better answers?

1. Preload is the process by which a bearing is first loaded to seat it, and then unloaded before it is put into operation:
   True
   False
2. Preload is typically obtained by applying a portion of its maximum load when it is installed using a mechanism that will maintain the preload over the life of the bearing:
   True
   False
3. Preload is often obtained by forcing one set of bearings against another:
   True
   False
4. Preload can be maintained more constant, even when temperature changes, by using a soft spring to load one set of bearings against another:
   True
   False
5. In general, bearings must always be preloaded by some means if they are to achieve best performance:
   True
   False
6. Bearings preloaded by a soft spring have low stiffness in the direction of the spring:
   True
   False
7. Simple models of stiffness and friction torque can help a designer make an intelligent decision as to what type of bearing to use:
   True
   False
8. Sliding contact bearings should *always* be designed with clearance to prevent them from binding:
   True
   False
9. Sliding contact bearings should be used whenever possible for linear or rotary motion to reduce costs:
   True
   False
10. Sliding contact bearings often provide for adequate load capacity and motion accuracy, so one should not always assume ball bearings are best:
    True
    False
11. Sliding contact bearings can be preloaded if the preload means spring constant is significantly lower than the primary load support path:
    True
    False
12. Jewel bearings are used primarily in watches and jewelry and rarely seen in machines or instruments:
    True
    False
13. Rolling element rotary motion bearings can better withstand impact loads if they are preloaded:
    True
    False
14. *Thermocentric* designs are those that balance thermal expansion of different elements to maintain near-constant preload:
    True
    False
15. A *back-to-back* arrangement of bearings has the lines of action through the bearing contact regions intersecting at points far away from the center of stiffness:
    True
    False
16. A *back-to-back* arrangement of bearings typically has a high degree of angular stiffness:
    True
    False

17. A *face-to-face* arrangement of bearings has the lines of action through the bearing contact regions intersecting at points close to the center of stiffness:
    True
    False
18. A *face-to-face* arrangement of bearings has a high degree of angular compliance:
    True
    False
19. Rolling element rotary motion bearings arranged in a *face-to-face* configuration are generally *insensitive* to thermal expansion:
    True
    False
20. Rolling element rotary motion bearings arranged in a *back-to-back* configuration are generally *sensitive* to thermal expansion:
    True
    False
21. To allow for thermal expansion of a high speed rotating shaft supported by bearings, you can:
    Constrain one set of bearings, at the end of the shaft nearest the load, to the bore and the shaft, and constrain the other set of bearings to the shaft and use adhesive to fix the outer ring to the bore
    Constrain one set of bearings, at the end of the shaft nearest the load, to the bore and the shaft, and constrain the other set of bearings to either the shaft or the bore, but let either the inner or outer ring slide
    Use springs to maintain preload while allowing for thermal expansion
    Exactly constrain the system, including providing for compliance or motion to accommodate thermal growth of the shaft
22. The preload on rolling element rotary motion bearings arranged in a back-to-back configuration generally does not change with temperature:
    True
    False
23. Preload reduces the sensitivity of rolling element bearing systems to shock and impact loads:
    True
    False
24. Extreme care should be taken when pressing or shrink-fitting a bearing onto a shaft because the strains could cause too much preload in the bearing:
    True
    False
25. Maintaining proper bearing preload is a significant challenge in designing bearing systems for high speed spindles:
    True
    False
26. High speed rolling element rotary motion bearings should be mounted with all bearings fully constrained to maximize elastic averaging:
    True
    False
27. High speed rolling element rotary motion bearings must always be mounted so the system is exactly constrained to maximize thermal stability:
    True
    False
28. Rolling element bearing supported spindles are generally used for modest precision high speed, applications, and there are a number of reputable suppliers, so buy before build if possible:
    True
    False
29. Roller-type (e.g., cylindrical, tapered, spherical) bearings are generally used for high load, moderate speed applications that also require more damping:
    True
    False
30. Roller-type (e.g., cylindrical, tapered, spherical) bearings are generally stiffer and have higher load capacity than ball bearings:

True
False

31. Cylindrical roller bearings only support radial loads and thus are typically used in conjunction with ball or tapered roller bearings:
    True
    False
32. Cylindrical roller bearings are in sensitive to misalignment when mounted in a back-to-back configuration:
    True
    False
33. Spherical roller bearings are generally used for high load, moderate speed applications that also require misalignment capability:
    True
    False
34. Tapered roller bearings can provide axial and radial load support:
    True
    False
35. *Flexural bearings* use elastic deformation of elements to allow for relative motion between elements:
    True
    False
36. *Flexural bearings* can use plastic deformation of elements to allow for relative motion between elements, but this is a rare application:
    True
    False
37. *Monolithic* flexural bearings are machined from a single piece of material:
    It is sometimes less costly than bolting pieces together
    Even though it may sometimes be more expensive, it ensures that all the flexural elements' dimensions are as close to ideal as possible, thereby helping to reduce parasitic error motions
38. The relative proportion between the size of a monolithic flexural bearing and its allowable range of motion is typically on the order of 20:1 :
    True
    False
39. *Clamped-flat-spring* flexural bearings are assembled from components because:
    To obtain the range of motion required in the space allotted, high aspect ration hardened spring steel flexural elements (*blades*) are required, and the cost to EDM the system from a hardened steel block can be prohibitive
    If the flexure is damaged, only the blade typically needs to be replaced
40. Flexural bearings are inherently preloaded:
    True
    False
41. Flexural bearings are often chosen for use because they are inherently preloaded and can have essentially infinite life:
    True
    False
42. Flexural bearings require specially formulated lubricants to avoid damaging the grain structure of high strength steels:
    True
    False
43. The relative proportion between the size of a clamped-flat-spring steel flexural bearing and its allowable range of motion is typically on the order of 10:1 :
    True
    False
44. The *dominant factors* that affect the relative size of flexural bearings and their allowable range of motion are:
    Material modulus of elasticity
    Material strength
    Thickness of the material
    Surface finish
45. The *dominant factors* that affect the life of flexural bearings are:
    Material strength
    Transition zone between the blade and the structure
    Applied loads
    Applied off-axis loads

46. Flexural bearings have poor inherent damping:
    True
    False
47. Flexural bearings can be damped by adhering a sheet of visco-elastic material to the side of the flexure or placing viscoelastic foam between the blades:
    True
    False
48. Parasitic error motions in flexural bearings are those errors in the desired motion which are caused by:
    Errors in dimensions
    Off-axis loads deforming the flexural elements
    Loads not being applied through the center of stiffness of the bearing
    Dust mites in the lab which feed on hair that falls off researcher's heads as they try to get their device to work
49. Thermal expansion errors can be problematic in flexural bearings because the large surface area to volume ratio of the flexural blade elements makes them more susceptible to temperature changes:
    True
    False
50. Thermal expansion errors in flexural bearings can be minimized by:
    Making the flexures from Invar low expansion steel
    Covering the flexures with an insulating material
    Keeping heat sources far away from the bearings
    Using symmetry in the design and shielding the flexural elements from air currents and heat sources
51. To avoid applying off-axis loads to a precision flexural bearing supported stage, a secondary stage is often used which is directly actuated, and then the secondary stage applied forces to the primary stage by means of a coupling such as a *wobble pin*:
    True
    False
52. Crossed-strip (blade) rotary motion flexural bearings provide nearly perfect limited range of rotary motion:
    True
    False
53. Bolt torques can induce residual stresses in clamped-flat-spring flexural bearing elements and cause parasitic error motions:
    True
    False
54. Fluid film bearings have high damping because squeeze film-damping dominates and it is proportional to viscosity*area/gap^3:
    True
    False
55. Hydrodynamic bearings are most often used when the loads are constant and act primarily from one direction:
    True
    False
56. Hydrodynamic bearings last longer the fewer times they are stopped and started:
    True
    False
57. Hydrodynamic bearing design is well understood and therefore hydrodynamic bearings can be deterministically designed:
    True
    False
58. Primary issues in hydrodynamic bearing performance include manufacturing tolerances and lubrication:
    True
    False
59. Aerostatic (externally pressurized) bearings are only good for high speed precision applications and thus are generally to be avoided:
    True
    False
60. Primary challenges in designing high speed aerostatic or aerodynamic bearings include maintaining gaps of about 10 microns under external load and viscous shear heating induced thermal expansion:
    True
    False

61. Aerostatic bearings supported spindles are generally used for precision, high speed, and/or ultra high cleanliness applications, and there are a number of reputable suppliers, so buy before build if possible:
    - True
    - False
62. The air supply for aerostatic bearings:
    - Can be from the shop air source as long as it goes through a filter & dryer and is then properly lubricated
    - Must be filtered to better than 3 microns and dried to less than 100 ppm moisture
    - Should only come from pressurized cylinders filled by a licensed supplier
63. Hydrostatic (externally pressurized) bearings are only good for low speed precision applications and thus are generally to be avoided:
    - True
    - False
64. The main advantage of hydrostatic (externally pressurized) bearings over aerostatic bearings is that they can operate at higher pressures and thus can carry greater loads in a smaller space:
    - True
    - False
65. A disadvantage of hydrostatic bearings is that they typically operate at higher pressures and thus they can suffer from pressure-induced expansion of the structure which can increase the effective bearing gap:
    - True
    - False
66. Primary challenges in designing hydrostatic bearings include:
    - Maintaining gaps of about 25-50 microns
    - Viscous shear heating induced thermal expansion
    - Stable fluid supply and return system
    - Temperature control of the fluid being pumped
67. Hydrostatic bearing spindles are generally used for precision modest speed applications, and there are a number of reputable suppliers, so buy before build if possible:
    - True
    - False
68. The flow through an externally pressurized fluid film bearing depends on:
    - The gap cubed
    - The viscosity and pressure of the fluid
    - The restrictor and bearing pocket geometry
69. The damping in a fluid film bearing depends on:
    - The gap cubed
    - The viscosity of the fluid
    - The pressure of the fluid
    - The restrictor type
    - The pocket geometry
70. The damping in a rolling element bearing depends on:
    - The number and shape of the rolling elements
    - The preload
    - The lubricant viscosity
71. Aerostatic bearings are poorly damped because the viscosity of air is so low:
    - True
    - False
72. Aerostatic bearings can be well damped because even though the viscosity of air is so low, the bearing gaps are very small:
    - True
    - False
73. Dynamic instability issues in aerostatic bearings can be caused by:
    - Improper restrictor design
    - Improper pressurization
    - Too large a bearing gap
74. Magnetic bearings can usually be stabilized by using permanent magnets thereby avoiding the need for a costly feedback control system:
    - True
    - False
75. Magnetic bearings are most often used for ultra high speed and/or high precision applications with modest varying loads:
    - True

False

76. Magnetic bearings are often used for ultra high reliability applications such as natural gas compressor stations or turbo molecular vacuum pumps:

    True

    False

77. Magnetic bearing design requires extensive expertise in mechanical electrical, and control system engineering, and thus such bearings are probably best purchased unless one is able to undertake a substantial R&D effort:

    True

    False

78. Magnetic bearings can run essentially forever if the appropriate electronic backup systems are provided:

    True

    False

79. Magnetic bearings can only be used for rotary motion systems:

    True

    False

80. Permanent magnets can help reduce required coil size and hence power consumption of magnetic bearings:

    True

    False

81. Because the load is supported by magnetic fields, there are no viscous shear losses in magnetic bearings, and hence the total system power is far lower than with any other type of bearing:

    True

    False

82. Linear motion systems generally are not subject to large thermal strains, so elastic averaging can thus sometimes be used to increase rolling element bearing performance:

    True

    False

83. A linear motion carriage should be supported by three rolling element bearing blocks whenever possible to maintain kinematic design principles:

    True

    False

84. A linear motion carriage is typically supported by four bearing blocks to achieve better load support and to achieve some degree of elastic averaging:

    True

    False

85. Mounting design for linear motion system rolling element bearings depends on:

    Properly choosing the closest design in the catalog and doing what they recommend

    Expected bearing rail misalignment, the stiffness of the bearings and the structure and the resulting portion of the bearing load capacity that will be used up by misalignment tolerances

    The ability of the manufacturer to adjust the system as it is built

    Elastic averaging can be relied upon to enable the bearings to achieve running equilibrium

86. Facing forward, *pitch*, *yaw*, and *roll* are angular error motions whereby:

    Pitch is associated with tilting your head forward

    Roll is associated with tilting your head side-to-side

    Yaw is associated with turning your head from side-to-side

87. *Horizontal parallelism* errors between bearing rails can be described as diverging bearing rails:

    True

    False

88. *Vertical parallelism* errors between bearing rails can be described as twist between bearing rails, and it induces roll motion in the carriage:

    True

    False

89. The overloading effect of vertical parallelism errors between bearing rails on bearings can be minimized by maximizing the roll compliance of individual bearings:

    True

    False

90. Cylindrical rolling elements’ life is proportional to r_{roller}^(10/3):

True
False

91. Cylindrical rolling elements' life is proportional to $(F_{\text{dynamic allowable load}}/F_{\text{applied load}})$ ^(10/3):
    True
    False
92. Spherical rolling elements' life is proportional to the r_{ball}^3:
    True
    False
93. Spherical rolling elements' life is proportional to $(F_{\text{dynamic allowable load}}/F_{\text{applied load}})$ ^3:
    True
    False
94. Modular rolling element linear motion bearings with *face-to-face* arrangements of bearings generally have large roll compliance:
    True
    False
95. Modular rolling element linear motion bearings with *face-to-face* arrangements of bearings achieve large system roll stiffness for a carriage by being used in pairs that are spaced several characteristic dimensions apart:
    True
    False
96. Modular rolling element linear motion bearings with *back-to-back* arrangements of bearings have large roll stiffness:
    True
    False
97. Modular rolling element linear motion bearings with *back-to-back* arrangements of bearings achieve large roll stiffness for a carriage by being used in pairs that are spaced several characteristic dimensions apart:
    True
    False
98. Modular rolling element linear motion bearings with *back-to-back* arrangements of bearings can be used in systems with vertical parallelism errors only by any of the following alone or in combination:
    Determining the amount of roll error and making sure that the resulting moment induced on the bearings, considering the bearings' and the structure's stiffness, is a small fraction of the total load capacity of the bearings
    Incorporating a flexural bearing to locally increase the roll compliance of the bearing
    Utilizing an external lubrication system to minimize friction
99. Horizontal parallelism errors between linear motion bearing rails are best accommodated by:
    Using modular bearings with *face-to-face* arrangements of rolling elements
    Using modular bearings with *back-to-back* arrangements of rolling elements
    Calculating compliances of the structure, carriage, and bearings and determining what the resulting bearing loads will be, and then adjusting the allowable tolerance, or by adding horizontal compliance to one set of bearings associated with one of the bearing rails
    Doing whatever the catalog recommends as a typical solution
100. It makes sense that a linear motion bearing which has high roll compliance should also have high radial compliance:
    True
    False
101. It makes sense that a linear motion bearing which has high roll stiffness should also have high lateral stiffness:
    True
    False
102. An actuator should be attached to a linear motion carriage:
    At the outer edge so it is available for easy access during manufacturing and for maintenance
    At the center of stiffness to minimize parasitic error motions
    It depends on the application
    With bolt torque sufficient to ensure elastic compliance
103. Very high speed linear motion recirculating ball bearing systems can be subject to recirculating ball erosion of plastic end cap return paths:

True
False

104. Very high speed linear motion recirculating ball bearing systems subject to recirculating ball erosion of plastic end cap return paths can easily prevent erosion with lubrication:
True
False

105. Metal end caps and good lubrication are a very good strategy for preventing recirculating ball erosion:
True
False

106. A retainer can help to reduce noise and increase life of recirculating rolling element linear bearings:
True
False

107. Error budgets use a kinematic model of a precision machine to predict how a small unwanted motion in one part of a machine affects the accuracy of the machine:
True
False

108. Creating an error budget for a precision machine is not required if solid modeling and finite element analysis is used:
True
False