FUN*da*MENTALS of Design

Topic 9

Structural Connections & Interfaces

Structural Connections & Interfaces

Take a close look at a bridge or a building as it is being built and compare what you see to the structure of a large crane, automobile, or machine tool. What similarities and differences do you observe? Can you close your eyes and visualize how loads transfer through the system? At every connection or interface, power, loads, or data are transferred, and it is the job of the design engineer to determine the best way to accomplish the connector or interface for minimum cost. Remember, cost is not just the initial (fixed) cost, but includes the cost of ownership (variable cost).

All mechanical things have a structure, and the structure is often made up of parts. *Structural connections* are intended to essentially keep the parts permanently attached to each other. *Structural interfaces* are intended to allow parts to be easily attached and detached. In both cases, the design of structural connections and interfaces requires the design engineer to think in terms of springs and degrees of freedom. Identifying the structural loop and the compliance of elements along it is a critical design skill. The design process for connections and interfaces is similar to that of a machine. The process ***RepeatsRepeatsRepeatsRepeats*****.....**

There are many different ways of forming connections & interfaces, and each has its own set of heuristic rules that enable a designer to layout a joint quickly and conservatively. The mechanics of different joints are also well understood, so the detailed design of the joint can then be done deterministically

Consider a three legged chair, and its interface with the ground. For a three legged chair, leg length and compliance are nominally not critical. Three legs will always contact the ground. However, the chair is more prone to tipping because the load must be applied within the bounds of a triangle. On the other hand, consider a five legged chair where each leg has modest compliance so when a person sits on it, all the legs deform a little bit and so all legs make contact. The chair is more expensive to design and manufacture, but its performance is far greater.

So go forth, connect, interface, and remember the FUNdaMENTAL principles of design! Whenever you think you have a good design, invert it, and think of something completely opposite, and compare it to what you have. Stay connected to the real world and interface with it frequently. Never be complacent, always be curious!

Topic 9
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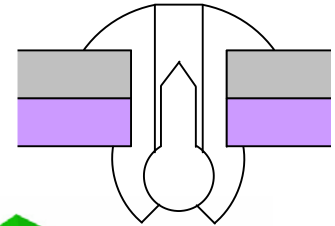
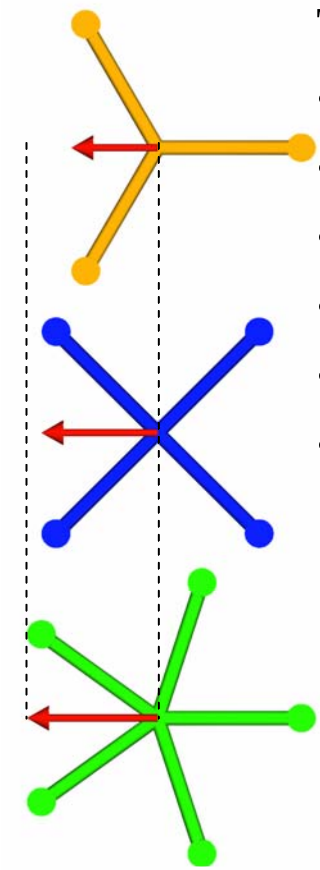
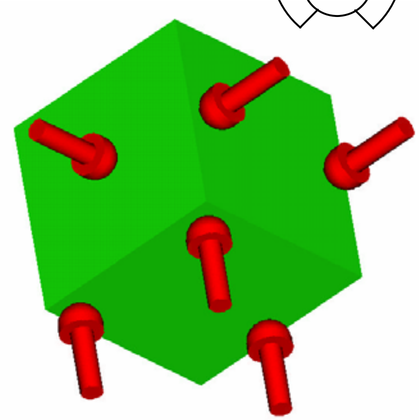
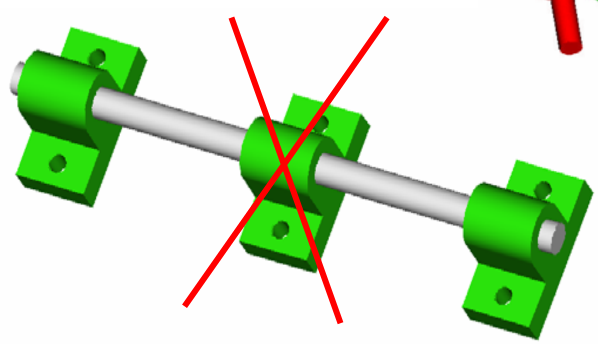


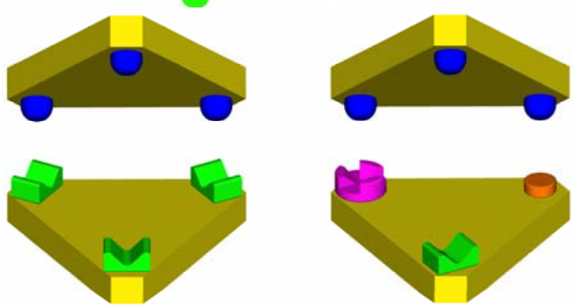
Topics

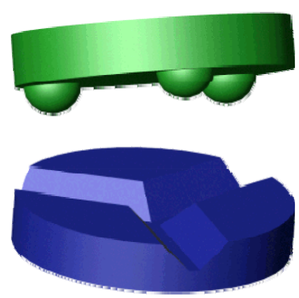
- Connections
- Structural Joints
- Structural Interfaces
- Hertz Contact
- Kinematic Couplings
- Elastic Averaging

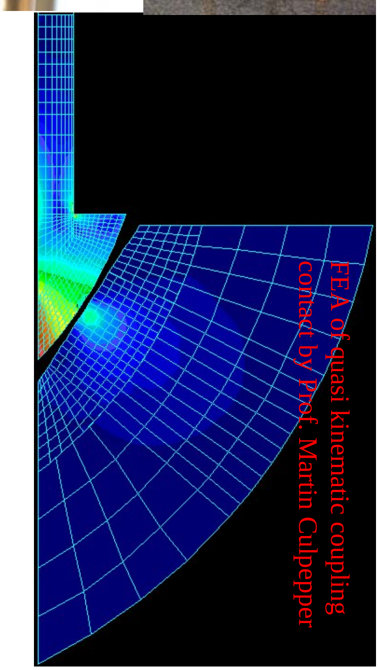












Connections & Interfaces: *Visualization*

The design of a connection or interface follows the same process as the design of a structure, and indeed, the ability to visualize the system, as also discussed on page 8-6, is a critical skill. Using the elements of FRDPARRC, a strategy can first be developed, and then detailed concepts can be created. The ability to close your eyes and open your mind to see how your parts can physically come together to form a connection or an interface is an acquired skill. A good way to develop it is to take things apart and study them. When approaching an old device, such as a printer, that you are going to take apart for practise:

- Looking at the overall device, can you discern what *functional requirements* it fulfills?
- What *design parameters* (parts or features can you see) allow the device to meet its functional requirements?
- Try to visualize the device operating. Can you identify the primary *modules*?
- Try to visualize opening the machine, and removing all the removable parts.
- Physically open the device and remove all the removable parts. Are there any additional functional requirements that you can identify? Were your visualizations accurate? Repeat the above steps until you can accurately play the movie in your head.
- Put the parts back, and close up the device.
- The next step is to consider what *analysis* might the engineers have to have done in order to optimally design each connection and interface to maximize performance and minimize cost.
- Next consider where else you might have seen similar *connections* or *interfaces*.
- Imagine what sorts of problems might occur with the connections or interfaces. What *risks* are there in their use that may affect the ability of the device to achieve its functional requirements? Try to visualize each of the risks occurring.
- Next, try to think of what sorts of *countermeasures* the designers perhaps considered when assessing the risks. Try to visualize each of the countermeasures.
- The next step will be to start taking apart the device into all its component parts. At each step in the disassembly process, repeat the above steps.

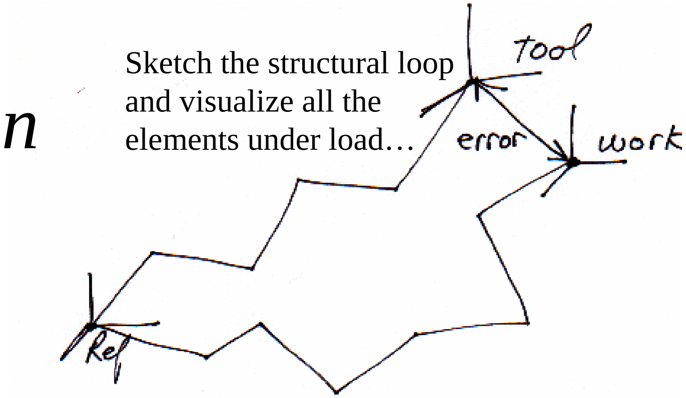
These thought exercises can help you to develop the ability to visualize connections and interfaces. Being able to trace the path that loads take through a device allows you to identify the structural loop. Once the structural loop is identified, you can visualize how loads are transferred through a structure. You can then use principles such as reciprocity to invert the *structural loop* to compare performance. Since this is a virtual exercise, visualization skills are critical.

In addition to take-apart exercises, you can also have fun with seemingly simpler devices such as toys. A very fun device is a *Hoberman Sphere* which uses ingenious mechanism to enable the sphere to fold up into a small shape typically one-quarter the size of the expanded sphere. The number of joints is quite large, but they repeat. Can you visualize how all the parts could be assembled by automated machinery?

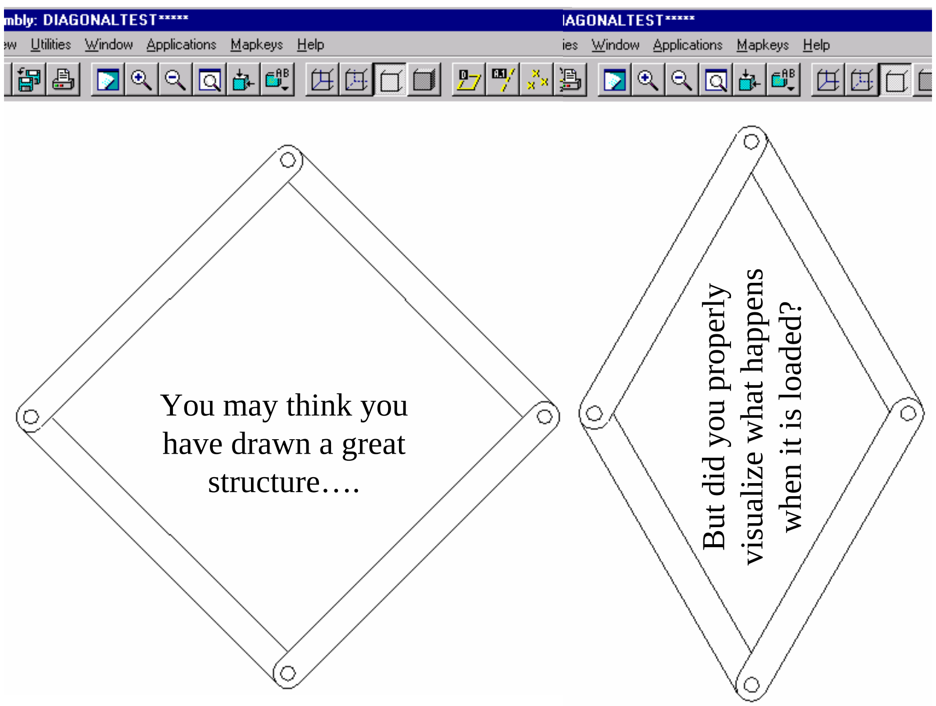
Assembly is another important part of visualization, for in addition to being able to visualize the working motions of a device, the engineer must make sure that the device can be manufactured. Hence it is important to be able to visualize the motions that manufacturing equipment, such as robots, might have to make in order to build your device. Some mechanisms are so small and/or intricate or are made in such small numbers that they require hand assembly; and the engineer needs to ensure that the device can be assembled. This visualization of design for assembly often leads to the inclusion of assemble guidance features to prevent incorrect orientation of assembled parts, and space for tools, such as wrench head clearances.

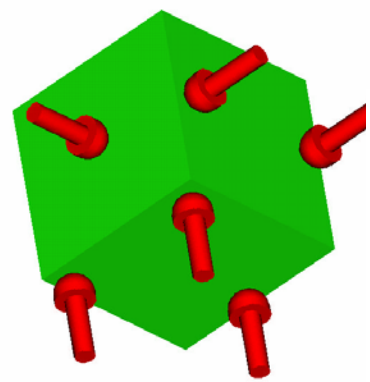
You can develop your visualization skills by taking things apart while following the steps described above. You can also develop your visualization skills for connections and interfaces by imagining what something will look like before you do it. Next time you open a latch, try to imagine the mechanism. Next time you bolt a joint together, imagine the threads screwing in and the forces from the bolts flattening a part so that it makes intimate contact with the rest of the structure. Now close your eyes and imagine taking your machine apart and visualize the flow of forces through it. Will its functional requirements be met?

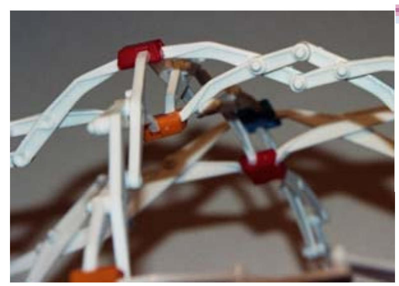
Connections & Interfaces: *Visualization*



- As a visualization tool for a joint of which are unsure:
  - Make a cardboard model of the joint
    - If the model is stable, there is a good chance that the real parts will also be stable!
- What happens to the performance of your structure if you assume the joints are just pinned?







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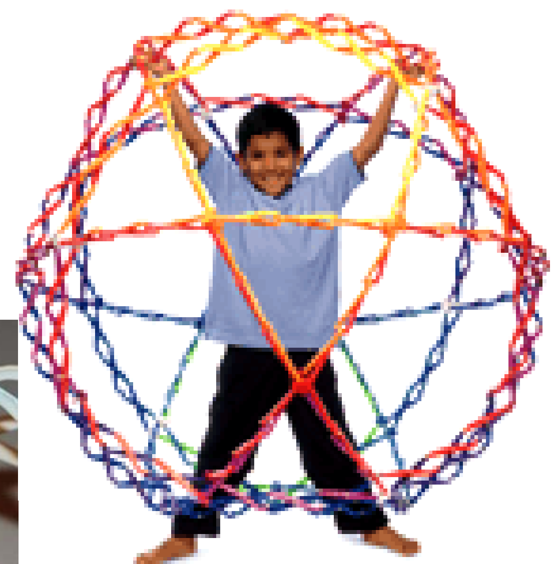


Connections & Interfaces: *Accuracy*

One of the differences between a solid model of a machine and the real machine, is the fact that the solid model is assembled using exact mathematical constraints; thus exactly sized components will mate perfectly with other exactly sized components. In actuality, however, manufacturing tolerances can lead to the condition shown, where the holes in one part may not line up with the holes in a mating part. One part sometimes needs to be rotated to make the holes line up, but this can lead to mounting misalignment as the part twists when tightening torque is applied to the bolts.

Look at the green brackets mounted on the red plate. All but one hole lines up; however, to rotate the green plate so that all the holes line up for bolts to pass through, leaves the green part rather twisted. If the bracket was just for structural reinforcement, this might not be an issue. However, if the green bracket holds some other element, such as a wheel or a gear, the resulting misalignment can be very detrimental to performance. How features are dimensioned can have a huge impact on performance.[1]

A *tolerance analysis* can be performed to make sure that given the range of allowable tolerance on location and size of features, that the parts fit together. For example, for the green bracket shown, it can be assumed that the hole is accurately sized, but its placement may be off in any direction by *X* mm. Does this means that the corresponding hole in the red part should have a radius that is also *X* mm larger? What if the hole in the red plate is also incorrectly positioned by *X* mm? What are the chances that the green hole is off to the left and the red hole is off to the right? In the best worst case, one might average the errors so the red hole radius equals the green hole radius plus *X* mm. In the absolute worst case, the red hole radius must equal the green hole radius plus *2X* mm. In reality, the expected value is typically halfway between the two; hence the diameter of the red hole should be made to be *3X* mm larger than the green hole.

Fortunately, as will be discussed, parts held together with bolts do not need the bolts to be contacting the edges of the holes. It is the clamping action of the bolt combined with the friction in the joint between the parts that enables shear forces to be transmitted across a joint.

What is typical hole placement error *N* for hand-drilled holes? If parts are carefully laid out and scribed, and a center punch used to mark the center of the hole, *N* might typically be 1/2 mm. If a machining center is used, then *N* can be less that 0.1 mm, and 0.05 mm is doable. This simple estimation is often overlooked by novice designers as they create and build their machines for robot design contests, and is thus one of the biggest time sinks.

Error budgets, an advanced topic, model the effect of allowable error amounts applied to each element and interface in the assembly, to ensure the machine meets its required accuracy goals. In particular, an error budget looks at how errors in one part, such as an alignment error, ripple through the device to cause a larger cumulative error between the tool and the workpiece. A practical example for robot design contests is lazy tongs or scissor linkages (see page 4-23). For example, you can create a lazy tongs mechanism, and it works great on the solid modeler, but then when you build it, something different happens. Fully extended, its reach matches that predicted by the spreadsheet *linkage_lazy_tongs.xls*: BUT when retracting, notice that some links are tight, while the end links are still spaced, and CANNOT be closed by the actuator. This is due to the clearance in the joints.

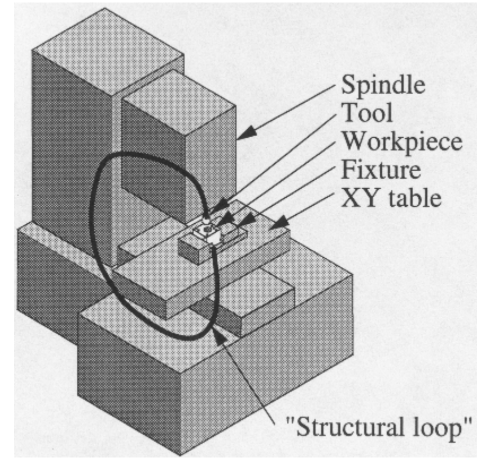
Tolerance discussions often focus on limits and not on how statistical variations can create a range of performance parameters. It is possible to statistically model the affects of parameter variations on the performance of a machine, but it requires a good mathematical model of how the machine operates in the first place. As CAD systems evolve, they may one day allow an engineer to ideally create a device, and then assign possible variations to all the features, in order to model how the system might perform.[2]

Think ahead about how your machines will act BEFORE its design is finalized or built. What parts absolutely must be "perfectly" aligned? Make sure to include generous tolerances so the parts can be fitted together properly. There is a LOT more to engineering than just stress analysis!

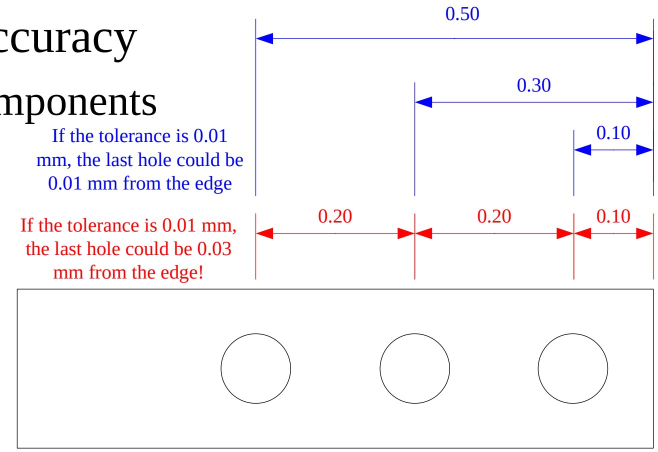
1. See, for example, F. E. Giesecke et al., Technical Drawing, 7th ed., Macmillan Co., New York, 1980, and Dimensioning and Tolerancing Y14.5M-1982, available from the American National Standards Institute, 1430 Broadway, New York, NY 10018.

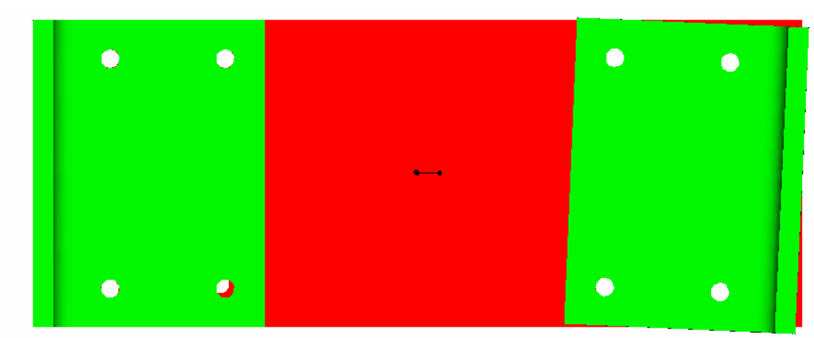
2. See Frey D.D, Otto K.N, and Pflager W., "Swept envelopes of cutting tools in integrated machine and workpiece error budgeting", 1997 Annals of the CIRP, vol. 46, no. 1, pp. 475-480. For a very in-depth discussion, see Frey, Daniel, "Using Product Tolerances to Drive Manufacturing System Design", Ph.D. thesis, Dept. of Mech. Eng., MIT, June 1997.

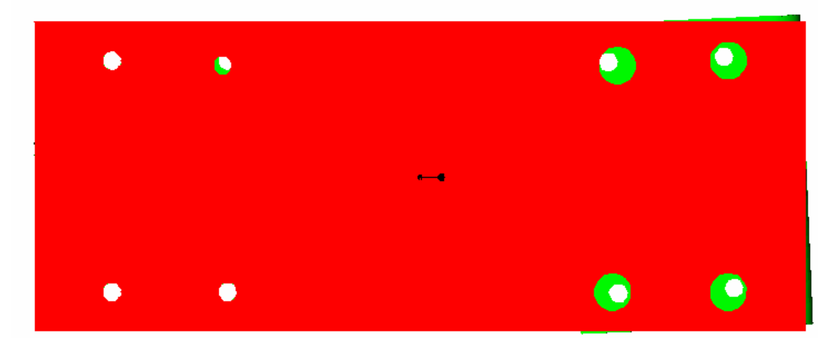
Connections & Interfaces: *Accuracy*



- Interfaces must enable parts to fit together with the desired accuracy
  - You cannot create two sets of exactly matching holes in two components
    - You can oversize the holes
    - The clearance between the bolts and holes means that the components do not have a unique assembly position
- "Error budgets" keep track of interferences & misalignments
  - These methods often assume "worst case tolerance"
  - For complex assemblies, advanced statistical methods are required
  - Deterministic designs are created using *financial*, *time*, and ***error*** budgets







Structural Joints[1]

There are a wide variety of structural joints that are commonly used, and there are many different books that are dedicated to the study of joints. Discrete joints include bolted, riveted, and pinned joints. Continuous joints include welded and adhesive joints. The intricacies of the mechanics of joints are critical to their design when seeking to attain long life and high performance. In this text, only an introduction can be given, but enough of the subtleties are hopefully given to entice designers to seek out the details.

As with the structure itself, a key skill is to be able to visually separate elements and to draw free body diagrams in order to determine the loads imposed on a connection be they welds, adhesives, bolts, rivets, or press-fit members. Is the joint to be designed for strength, stiffness, or both? If the joint is to be designed for strength, there is generally less uncertainty because issues of surface contact and flange deformation are not as critical. If the joint is to be designed for stiffness, a more conservative approach is needed such as making sure all the bolts' stress cones overlap.

One of the primary issues in the design of a structural joint is the attainment of a truly rigid connection: a connection that can support a moment load. Saint-Venant's principle once again is helpful in the initial layout of the joint, where the critical dimension in a beam is the thickness. If three times the member thickness is allocated to the joint, in general, a rigid connection can be made, but beware, this is just a guideline and on thick members, 3x might not be needed. To this dimension must be added the product of the number and diameter of the bolts or rivets that are used. An exception to this guideline is welded joints, where a good weld that fully penetrates a section can yield the effect of the section behaving as a continuous part of the structure to which it is welded. Adhesive joints, on the other hand, work best in shear, and may require more area than bolted joints if they are subject to moment (peeling) loads.

1. A good trade magazine to read is *Assembly Engineering*, Hitchcock Publishing Co., 25W550 Geneva Road, Wheaton, IL 60188. Also see *Machine Design's annual Fastening, Joining, and Assembly* reference issue. Other references include A. Blake, Design of Mechanical Joints, Marcel Dekker, New York, 1985, and the section on joint design in M. Kutz (ed.), Mechanical Engineer's Handbook, John Wiley & Sons, New York, 1986. Also see R. Connolly and R.H. Thornley, "The Significance of Joints on the Overall Deflection of Machine Tool Structures," 6th Int. Mach. Tool Des. and Res. Conf., Sept. 1965, pp. 139-156.

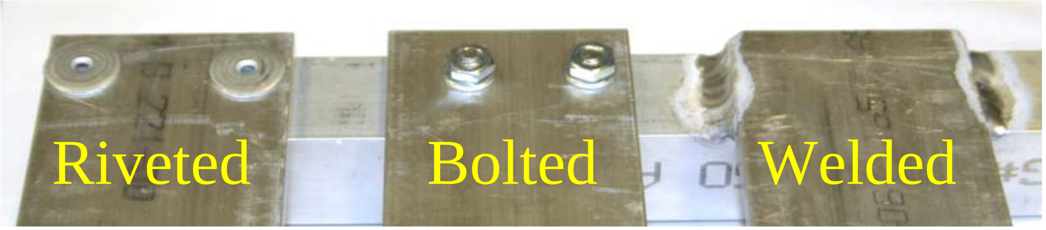
Many times the decision as to which type of structural joint to use is based on the economics of what production methods are readily available. In the case of robot design competitions, a bolted or riveted joint is often the easiest to implement, because the loads are often not that high or the cycles are not that large. However, the space a bolted or riveted joint requires to achieve a rigid connection is typically greater than that required by a welded joint, and hence designers of machines for robot design competitions can benefit from learning to weld.

The components of joint cost include how much space in the design does the joint require, how long it takes to make the joint, and can the joint be taken apart if a mistake is made. Space and simplicity would seem to favor welded or adhesive joints as often as possible; however, the use of dissimilar materials, modular assemblies, and the ability to make changes all point to the use of bolted joints. Bolted joints require more parts and operations, which can lead to the decision to use riveted joints, which can be drilled out and released without too much trouble.

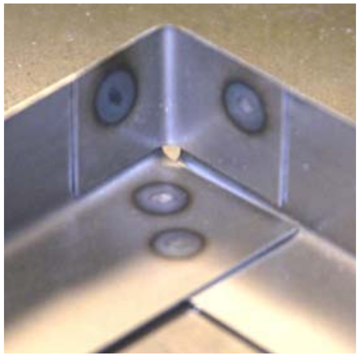
Another consideration is manufacturing resources. When designing a robot for a design contest, anyone can drill a hole for a bolt; but it takes skill to create precision holes for a press-fit. It takes even greater skill to create a good weld. Often a shop for students does not have the resources available to train every student to weld, and the shop personnel do not have time to weld everyone's machine. Good forward thinking and planning are a must.

Parts must also be carefully prepared for marking, manufacturing, and assembly. Parts to be welded must be clean because dirt, oil, paint... can generate gases that create porosity and weakness in the weld. Adhesive joints must be extremely scrupulously cleaned. Bolted joints must also be cleaned, especially the threads so as to reduce friction and enable the torque applied to more efficiently generate clamping forces. The surfaces of steel joints may also require a wipe with an oil cloth to keep them from later rusting together.

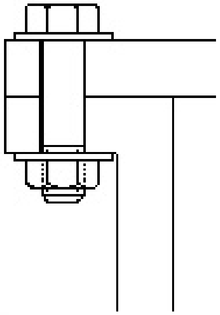
Look at your machine and draw the structure loops, i.e., load transfer paths, and circle the structural joints. List the functional requirements for each joint. How much space have you allocated for each joint and is it enough given Saint-Venant's principle? Are you inclined to weld, glue, bolt, or rivet the joint? Do you really have enough information to decide at this point, or is cost, time in particular, the primary driver?

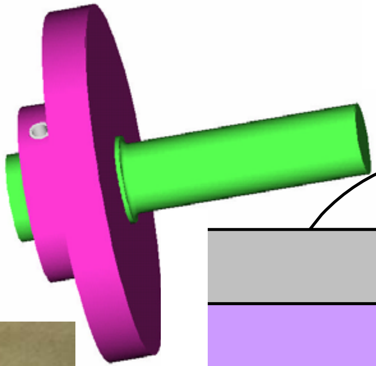


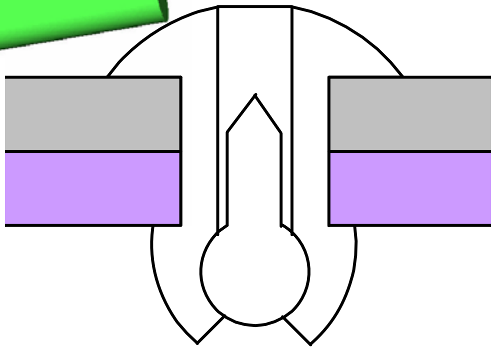
Structural Joints



- *Structural joints* (non moving) transfer loads between members, and are a necessary part of almost all structures
  - They can take up space and add cost
  - They can provide damping and design flexibility
- There are many different types of joints including:
  - Welded
  - Adhesive
  - Bolted
  - Pinned & Riveted
  - Interference-fit (also see page 5-28)







Structural Joints: *Welded*[1]

A welded joint can be as strong as the base metal if the appropriate process and surface preparation are used. Certain metals, like cast iron, are difficult to weld, while other metals, such as mild steel, are easy to weld. Although aluminum is easy to machine and handle, it is covered with a thin native oxide layer which is an electric insulator; and makes aluminum difficult to weld. A good weld looks like one part smoothly transitions into another. A bad weld is porous and looks crunchy. Most welds on machines are made using an electric arc. DC or AC current can be used, but DC is most common.

- *Stick welding* uses a consumable electrode sheathed in a material that releases a protective gas cloud around the pool of molten metal to protect it from oxidation. A reasonable gap between components can be filled, but this increases the chance of distortion.
- *Metal Inert Gas (MIG)* welding uses wire from a spool fed through a gun to provide filler material. Argon gas flows around the wire as it leaves the gun and shields the weld pool from oxygen. MIG welding produces very nice welds in steel, and can also be used to weld aluminum. However, if there is a heavy oxide layer, then a large amount of wire will be melted trying to break through the oxide, and a molten mess occurs.
- *Tungsten Inert Gas (TIG)* welding uses a tungsten electrode, which is not consumed by the heat during the welding process. A shielding gas, such as argon, is used to protect the electrode and the weld pool. Filler metal can be added from a rod that is fed into the joint as needed by the welder. Because filler material does not have to be added, the welder can keep the arc in one location long enough to break through a thick oxide layer in aluminum in order to start a weld. TIG welds are amongst the best welds.
- *Spot welding* is a very useful process whereby two elements to be welded are placed in contact between two electrodes. The parts are squeezed between electrode tips, and then an impulse of current is sent through the electrodes. The current is localized and causes the metal to melt and be quickly cooled by the surrounding metal. Getting the proper current setting can take a few tries, but ideally in a large shop running lots of the same types of materials, spot welding stations will be set up for different thicknesses and types of materials. Thin sheet steel and welding rod are easy to spot weld, and from them can be made car bodies and trusses respectively. Spot welding is perhaps the fastest way to create a joint.

1. A classic is O. Blodgett, Design of Weldments, James F. Lincoln Arc Welding Foundation, P.O. Box 3035, Cleveland, OH 44104, 1963.

All welds depend on achieving good penetration into the components being welded, because a weld just laying on the surface is more likely to crack. If too much heat is applied, the *heat affected zone* can be large which can cause excessive thermal distortion or embrittlement. To reduce thermal effects, if a structure is designed for stiffness, where loads are low compared to what the sections could hold, a *skip weld* is often used: instead of a continuous weld along a long joint, a section of weld can be followed by an unwelded section, and then another welded section etc. 50% skip welds are common in machine tool structures. To help achieve good penetration, the edges of parts to be welded can be beveled with a sander or grinder. Fixturing is also important to the welding process to not only hold the parts together in position at the start of the weld, but to also resist the forces of the contracting cooling metal until enough weld can be formed to completely constrain the joint.

The shear stress in a weld must be less than one-half the weld material's yield strength. Note that when welding a heat treated alloy, the temper will be lost unless the entire part is re-heat treated:

$$\frac{F_{shear}}{A_{weld}} SF_{\text{safety factor}} \le \frac{\sigma_{\text{yield strength}}}{2}$$

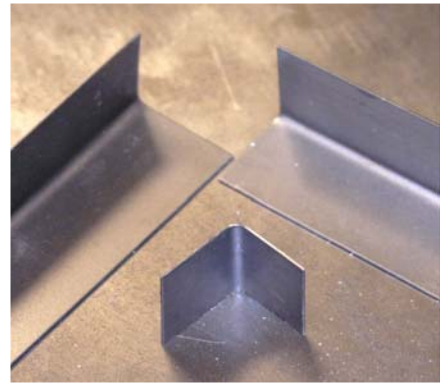
Welded joints in bending can be analyzed by assuming one of the welds on one side of the beam is in compression, and acts like a fulcrum, and the other welds is in tension. If the applied moment is M and the weld areas are $A_{weld\ top\ area}$ and $A_{weld\ bottom\ area}$, which are separated by a distance h, then the stress in the welds must be less than the yield strength:

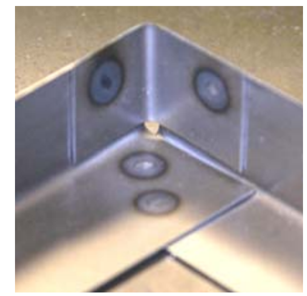
$$\frac{M_{\text{applied moment}}}{A_{\text{weld top area}} h} SF_{\text{safety factor}} \le \sigma_{\text{yield strength}}$$

What joints in your machine are candidates for welding, and how must you prepare and fixture them? How might thermal distortion affect the accuracy of your structure and the performance of your machine? Ask shop personnel to review your welding plans.

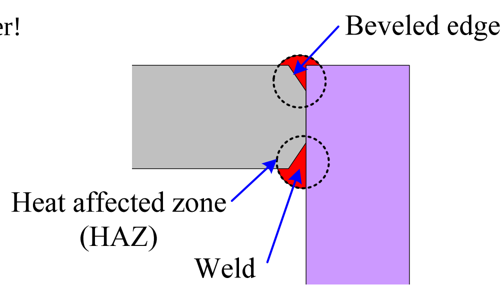
Structural Joints: *Welded*

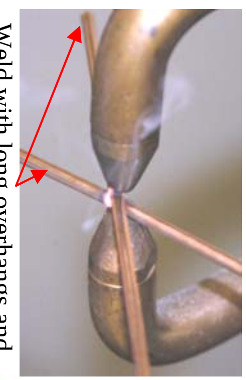
- A good weld is as strong as the base metal
  - Heat treated alloys require re-heat treatment
  - Surface preparation is VERY IMPORTANT
    - Cleanliness
    - On thicker parts, bevel edges to be welded
- Shop personnel will help you with your welding needs
  - Consult with them during the **concept** stage about options
  - Spot welding is used for sheet metal and thin rods
  - Arc welding is typically used for heavier sections
  - TIG (*Tungsten Inert Gas*) is used for welding aluminum, or for very precise welds on steels and special alloys



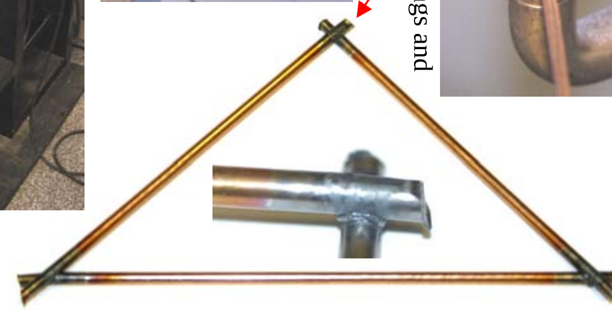


Steve Haberek, master welder!





Weld with long overhangs and then trim



Welded Joint Case Study: *Welded Sprocket-Coupling*

As with many robot design contests, FIRST[1] team design and build their robots using a kit of materials. Sometimes it is hard to use best engineering practice when time is limited and it is known the machine only has to last for a few weeks. However, this can lead to failure at the most inopportune moment.

A common challenge is using the actuators provided in the space available. In the current example, an electric door opener's gearmotor was needed to raise the mast on the robot. The motor was to drive a sprocket which would provide force to a chain that would lift the mast. As the spreadsheet shows, this design was capable of generating very large forces in the chain. however, how could a sprocket be attached to the gearmotor shaft to handle the torque and radial load? If the radial load had any significant offset from the motor shaft support bearings, the resulting bending moment would likely destroy the gearmotor.

The first Functional Requirement of transmitting the torque could be approached with several design parameters. If a keyway was cut into the small output shaft, it would be significantly weakened. An interference fit would make installing and changing the sprocket difficult. A squeeze-type collar seemed like the best option. the spreadsheet shows the calculations which verify that a squeeze-type collar would work. A squeeze-type collar uses one side as a fixed anvil, and the other side radially displaces. The next challenge was to attach the sprocket to the collar.

The second Functional Requirement is to counterbore the sprocket so the teeth are located adjacent/over the gearmotor shaft support bearing (also see page 7-20). This means that the sprocket needs to be attached to just one-half of the collar (the anvil side). What is the best way to do this? Given the small size of the components, it did not appear that a bolted connection would be appropriate. Could a weld hold? There was no formula in a handbook to find the stress σ in the weld for this problem. What to do? It turns out that simple calculus and strength of materials provide the tools needed to analyze this problem.

1. www.usfirst.org

The worst case is when the apex of the arc-shaped weld is in tension from the bending moment caused by the radial force. The diameter on which the weld is made is $D = 2R$, the radius r_{bead} represents constant weld thickness for a weld with good penetration, and the differential segment of a weld element is

$$dA = r_{bead} R\, d\theta$$

The distance between a differential weld segment and the worst-case assumed pivot axis (XX) through the center of the clamping collar is:

$$\mathrm{r} = R\sin\theta$$

The moment is thus found from:

$$dM = \sigma r\, dA$$

$$\mathrm{M}=2\int \mathrm{dM} = 2\int_0^{\frac{\pi}{2}} \sigma r_{bead} R^2 \sin\theta\, d\theta = 2\sigma r_{bead} R^2 = \frac{\sigma r_{bead} D^2}{2}$$

The moment through the X'X' axis is:

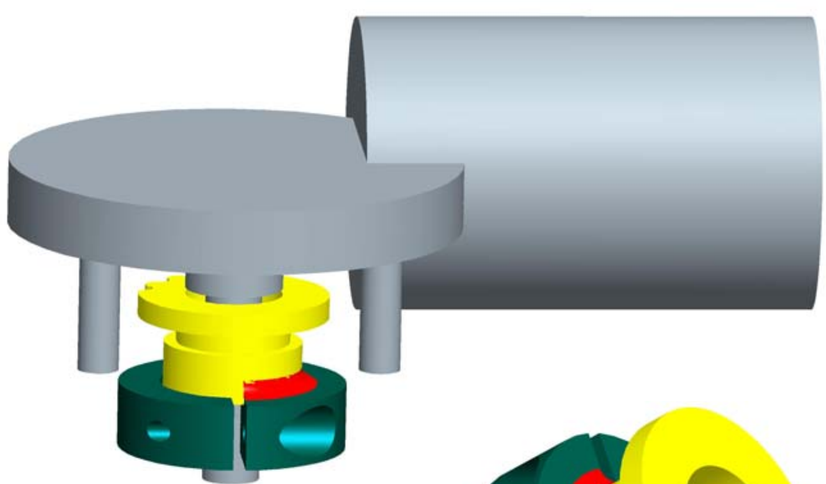
$$\mathrm{r} = R(1-\sin\theta)$$

$$\mathrm{M}=2\int \mathrm{dM} = 2\int_0^{\frac{\pi}{2}} \sigma r_{bead} R^2 (1-\sin\theta)\, d\theta = (\pi-2)\sigma r_{bead} R^2 = \frac{(\pi-2)\sigma r_{bead} D^2}{4}$$

The analysis was used to create the spreadsheet *Z_axis_sprocket_design* which allowed the designers to play "what-if" scenarios with design parameters. In the end, the design worked great, and the FIRST team made it to Nationals.

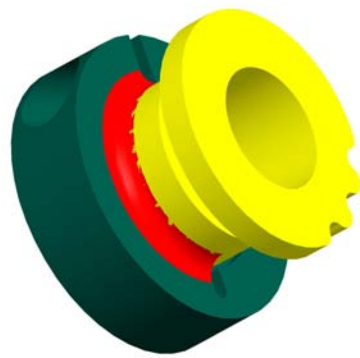
This type of engineering example is a very common experience for good engineers. Many machines can seemingly be built by just assembling components from a catalog, but in reality, they still require significant customization. Your machine's most critical module most likely requires significant engineering analysis, and it should also be documented so others can follow your thought process.

Welded Joint Case Study:
Welded Sprocket & Coupling



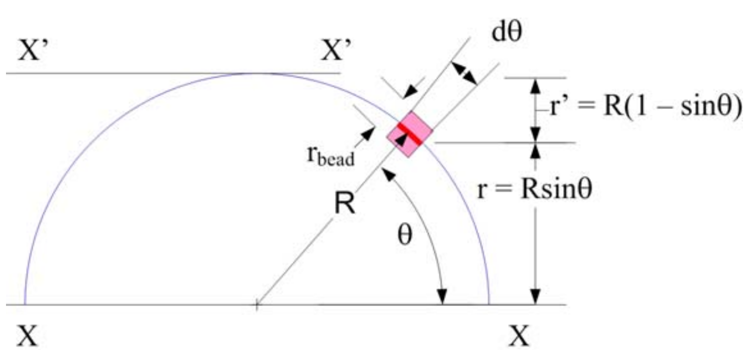
Bishop Brady High School FIRST robotics team
2005 robot (Prof. Slocum was a coach)

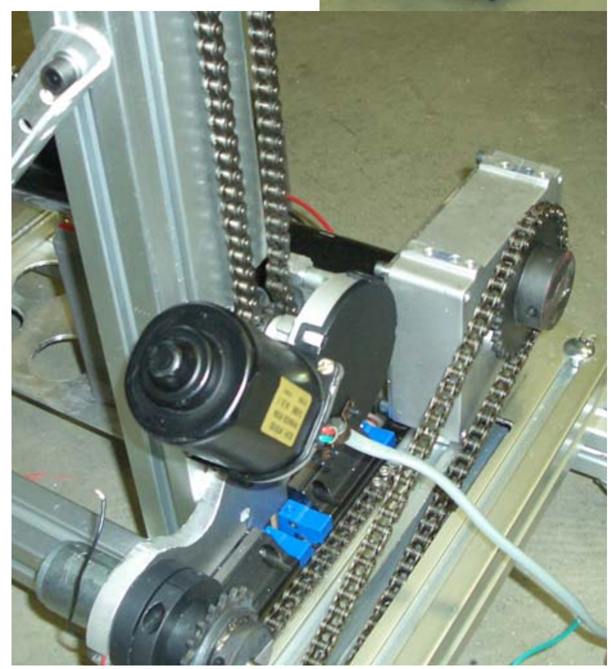
Z_axis_sprocket_design.xls		
To estimate the stress in the weld of the Z-axis sprocket		
Alex Slocum, January 23, 2005		
Motor stall torque (N-m, in-lb)	**50**	**442**
Sprocket pitch diameter (in)	**1.375**	
Overhang distance from weld (in)	**0.625**	
Maximum radial force (lbs)	**643**	
Moment on weld (in-lb)	**402**	
Diameter of weld, D (in)	**1.065**	
Weld bead radius, rbead (in)	**0.125**	
Weld strength, sigma (psi)	**30000**	
Moment capacity of weld, M (in-lb)	**1214**	
Resultant safety factor	**3.0**	



Grade 8 bolt to clamp and hold torque	
Bolt diameter	**0.25**
Thread root cross section area	**0.0276**
Max stress (psi)	**150000**
clamping force	**4142**
Force applied to Two sides	
Force amplified X2 by fulcrum	
Total effective clamping force	**16567**
Coefficient of friction	**0.2**
Shaft diameter	**0.435**
Max torque (in-lb)	**721**
Acceptable	**YES**

Motor Shaft	
Diameter (in)	0.435
Strength (psi)	**109,481**
I/c	**0.00808**
Length of moment arm (in)	1
Load (lb)	**643**
Moment (in-lb)	**643**
Stress (psi)	**79,622**
Safety Factor	**1.4**
Assume max torque, & 2x safety factor	
I/r	**0.0162**
max shear stress	**54740**
max yield strength	**109,481**





Structural Joints: *Adhesive*[1]

Modern adhesives have revolutionized the design and manufacture of countless products due to their simplicity and versatility. The use of adhesives is very desirable because there is no need to drill holes, use fasteners (small parts) and their use does not cause thermal distortion as can welding. However, their strength depends on being applied over a comparatively large extremely clean area because adhesives are polymers which have an order of magnitude lower strength than metal fasteners or welds. On the other hand, since adhesives are applied in thin layers, they take up essentially zero space and thus are invaluable for the manufacture of high strength laminates. In addition, surface preparation is critically important when using adhesives. Surfaces must be clean and free of any oils including those from your fingers.

There are many types of adhesives and they are generally grouped according to the type of curing mechanism. Common mechanisms include solvent evaporation and polymerization. White glues, such as Elmer's, bond by solvent evaporation where the solvent is typically water. When the water evaporates, the polyvinylacetate latex forms a flexible bond between the surfaces onto which it was spread. This means that the surfaces must typically be hydrophillic or porous so the glue can penetrate and adhere. Polymerizing adhesives are often used for structural applications, and these include "super glue", epoxy, and ultra violet light cured adhesives.

"Super glue" deserves its name -- a single drop can join your thumb to your index finger faster than you can say "Whoops!"[2] 1 cm^2 bond can hold a 1000 N tensile load. Super glue, or Cyanoacrylate ($C_5H_5NO_2$), is an acrylic resin that cures almost instantly when exposed to hydroxyl ions in water, which causes the molecules to entangle and form a polymer. Most surfaces have some moisture on them, and cyanoacrylate's low viscosity causes it to quickly spread onto a surface. Thus when two surfaces are forced together, the water molecules on the surfaces cause the glue to cure and bond to the surfaces, adhering the two surfaces together. However, this means that super glue is not useful on porous surfaces. It is extremely useful for bonding rubber, metal, ceramic...For example, super glue is shown being used to bond the ends of a timing belt together with a single strap joint.

Epoxies are two-part adhesives which, when mixed, polymerize to form a strong adhesive that can have a cure time from a few minutes to many hours. Generally, the longer the cure time, and the higher the temperature at which curing is recommended, the stronger and more temperature stable the bond. Epoxies can be moderately viscous (e.g., honey) putty-like, or even in tape form, which is used for creating laminates. In fact, epoxy is an ideal adhesive for bonding metal to wood to form a laminate. The spreadsheet *laminate.xls* can be used to estimate the stress in the adhesive layer to ensure that it does not delaminate. Epoxy can also be placed on a fastener after it has been tightened, to prevent it from loosening. Clear epoxy becomes translucent when fractured, and this can help to show when a fastener has loosened.

Another use for epoxies is as filler materials in machine assemblies. Shown is an OMAX Abrasive Waterjet machining center™ and a schematic cross section through an axis. Epoxy is used as a set-in-place spacer to make a perfect fit between the outer precision surfaces of linear bearing blocks and the rough inner cast surface of the axis carriage. This allows the machine parts to be precisely set-up in precision fixtures. Epoxy is then injected between components to fill gaps and hold all the components precisely in place. Bolts then preload all the joints and add strength. Mold release placed on the parts enables them to be taken apart if needed. However, beware of shrinkage, which will typically be 0.2% in the direction in which the joint is constrained, but it may be 5% or more in unconstrained directions.

Friendly Plastic™ is a polymer with properties similar to nylon, except that when placed in a boiling water, it softens to the consistency of chewing gum. Due to its low thermal conductivity, when removed from the hot water, it can be molded with your hands like clay until it cools and hardens. It can also be used to replicate shapes, such as to form bearings around shafts.

Review the joints in your machine and assess where adhesives can be used. Can a simple laminate replace a more complex truss? Where might the need for precision machining be reduced by the use of replication? ***Make sure that surfaces to be joined are extremely clean!***

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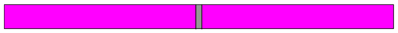
1. See, for example, A. Blake, Design of Mechanical Joints, Marcel Dekker, New York, 1985, and M. Kutz (ed.), Mechanical Engineers' Handbook, John Wiley & Sons, New York, 1986.
2. Who first applied reciprocity to this problem and realized that this would be a great way to close a wound without stitches? By changing the type of alcohol in cyanoacrylate, from ethyl or methyl alcohol to butyl or octyl, super glue becomes less toxic to tissue. Need to get your fingers unstuck? Acetone (finger nail polish remover) will dissolve cyanoacrylate.

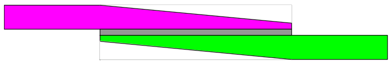
Structural Joints: *Adhesive*

Butt Joint: OK



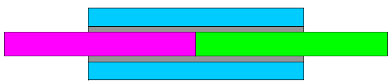
Lap Joint: Good

Tapered Lap Joint: Very Good

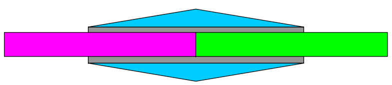


Stepped Lap Joint: Very Good

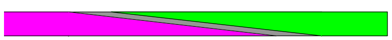
Double Strap Joint: Very Good



Tapered Double Strap Joint: Excellent



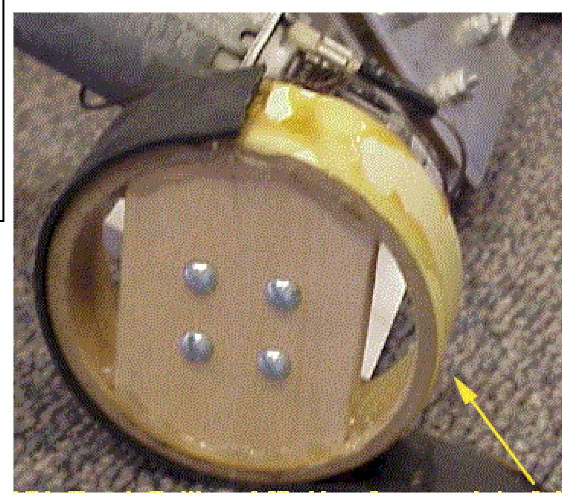
Scarf Joint: Excellent



Single strap joint used to bond toothed belt with Super glue

- Adhesives are often used to bond large surface areas
  - Epoxy is often used for making laminates
  - Adhesive joints are usually not meant to be moment connections
  - Thread locking agents are used to keep screw threads from coming undone
  - ***CLEANLINESS IS OF UTMOST IMPORTANCE***
  - Check out binding recommendations: *http://www.thistothat.com/*
- Strengths vary greatly with the type of adhesive, but the lap shear strength is typically are on the order of 15 MPa at 80 ^{o}F
  - K. Lewis, "Bonds That Take a Beating", *Machine Design*, Aug. 8, 2002 pp 69-72

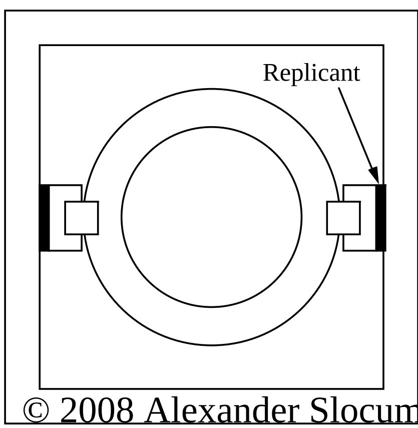
"Double Bubble" two-part epoxy. Make sure to squeeze out all the material from BOTH packets



Improper surface preparation (rubber should be clean and rough), and the rubber should have been scarf joined

Aluminum epoxied to both sides of plywood which acts as a core

Aluminum epoxied to one side of plywood



Structural Joints: *Bolted*

Bolted joints are very common because they are relatively easy to design, and they can be easily taken apart, which is often a necessity when building a robot for a robot design contest! There are many misconceptions about bolted joints, which can result in severe problems. Fortunately, some simple realizations enable a designer to quickly design a bolted joint:

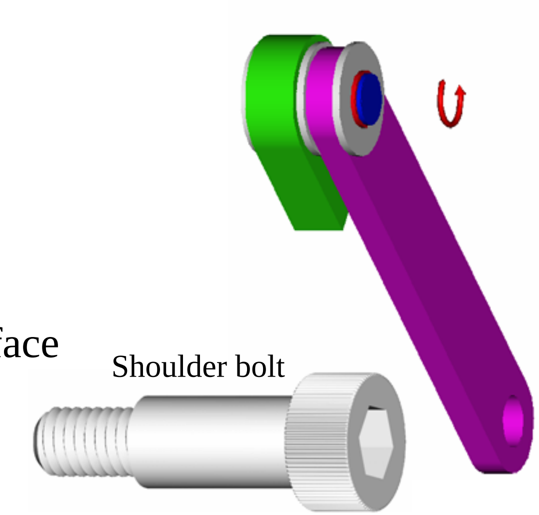
- A bolt is really just a means to clamp a joint together, and the bolt itself does NOT withstand any shear loads.
- A bolted joint withstands moment loads by clamping the surfaces together, where the edge of the part acts as a fulcrum, and the bolt acts as a force to resist the moment created by an external force or moment.
- A bolt is just a form of leadscrew, and hence the analysis presented in Chapter 6 applies equally well (e.g., *leadscrew_design.xls*). Since bolts are sliding contact thread devices, most of the energy spent tightening them is dissipated by friction. How much tension in the bolt is really created? How tight is right?[1] In general, about 50% of the energy goes to friction under the bolt head, 40% goes to friction in the threads, and only about 10% goes to creating tension in the threads. Fortunately, rotation of the bolt head relative to the parts being bolted together is a good measure of the tension in the bolt. In critical applications more elaborate methods may be used to ensure proper tightness (e.g., load-measuring washers).
- Since a bolt is a leadscrew, and the primary functional requirement is to generate a clamping force, the bolt threads should be lubricated. Grease works well, and the threads will still not be back drivable. In high vibration applications, use a thread-locking lubricant (e.g., Loctite 410™).
- A *strain cone* (or *stress cone*) under the bolt head that projects downward at 30 to 45 degrees from the vertical. When sufficient bolts are used in a joint such that the strain cones overlap, the joint behaves as if it were a continuous piece of material and there was no joint at all. To prevent the stress field from affecting components, apply Saint-Venant's principle.
- The strain cone represents a deformation of the material under the bolt head, and if the cones of action are not overlapping, then the bolted joint introduces a waviness into the parts. These deformations can cause bearings to fail if they also cause deformations in the bearings or the surfaces to which they are mounted.
- The tensile/compressive stiffness of the materials in contact at a joint acts in parallel with the stiffness of the bolts. With proper preload, the % variation in tensile load on a bolt will be a small percentage of the bolt preload force. This is the key to reducing fatigue.
- Typically, the thickness of the material under a bolt head should be from one to three bolt diameters, and bolts' center distances should be spaced from three to five bolt diameters apart.
- When a bolt is threaded into a section whose thickness equals the bolt diameter, the tensile strength of the bolt will equal the shear strength of the threads. It does little good to have more thread engagement.
- Parts bolted together can both be drilled with clearance holes for the bolt and some adjustment. A nut is used to tighten the bolt; OR one of the parts can have a threaded hole and one can have a clearance hole. Both parts should NEVER have threaded holes because the threads will not be in phase.
- Self-tapping screws can reduce the number of parts and manufacturing operations required for a joint and thus reduce cost.
- Surfaces to be bolted together should be clean and dry, but if they are subject to corrosion, then a corrosion inhibitor can be applied before bolting.
- Bolting a component to an imperfect surface, without allowing for forced geometric congruence, can warp a structure so critical elements are no longer aligned! Bolted joints *can* cause warping of components.

In addition to the details of the bolted joint itself, the designer must consider the effects of the bolted joint on the system. Many seemingly great ideas fail because the mechanisms are over or under constrained after they are assembled into a system. A module may work great on the bench, but then fail after being bolted into the system. This can be prevented with the use of kinematic mounts, better tolerances, or controlled compliance (elastic averaging) of the mounting points.

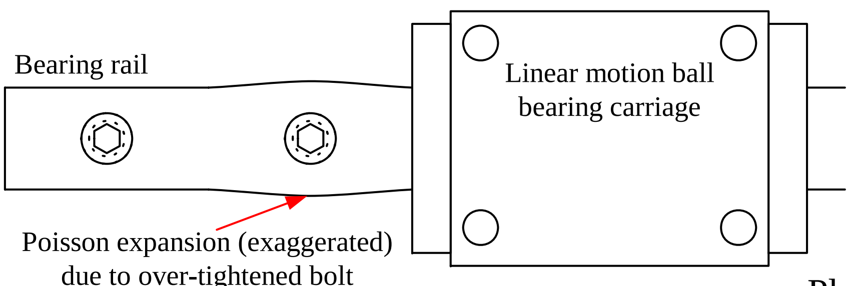
Review the joints you plan to bolt and reassess which joints are in shear, tension, or bending. How will combined loads affect them? Imagine the bolted parts are made of rubber, and visualize the bolts squeezing them. How will the resulting deformations adversely affect other parts? Make sure that your bolted joint designs conform to the above guidelines.

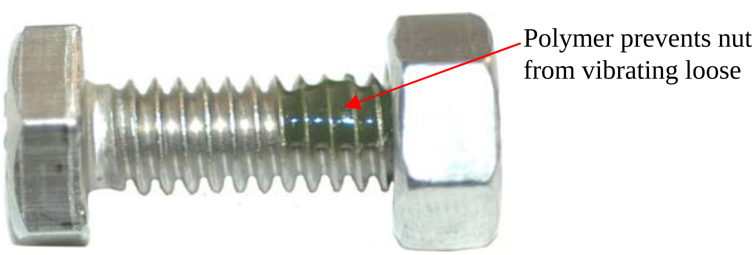
1. D. Miller "How Tight is Right?", Machine Design, August 23, 2001, pp 74-78. In addition to previous references on the design of joints, see for example, J. H. Bickford, An Introduction to the Design and Behavior of Bolted Joints, Marcel Dekker, New York, 1981, as well as A. Blake's book.

Structural Joints: *Bolted*



- Bolts and screws ONLY clamp one element to another!
  - Friction and the clamping force are what hold the joint together
  - Washers are used to keep hex-nut edges from chewing up the surface
- Bolts and screws DO NOT themselves take shear loads
  - Unless you use a shoulder bolt
- A shoulder bolt can act as a shaft or element of a linkage (pin):
  - When a bolt is to be used to support a bearing, or act as an axle (pin) in a linkage:
    - One end of the bolt must be firmly anchored so it is preloaded and rigid
    - The cantilevered end ideally has a precision ground shoulder that acts as an axle





Plain nut

Lock-nut

JointCompliance.xls	
Spreadsheet to estimate bevel gear tooth strength	
Production gears must be designed using AGMA standards	
Last modified 9/8/2003 by Alex Slocum	
Enter numbers in **BOLD,** results in RED	
Be consistant with units! (in, lb or N, m or N, mm)	
Angle of cone of influence, theta (degrees)	**45**
Diamter of bolt head, Done	**20**
Diameter of affected zone, Dtwo	32
Thickness of material, t	**6**
Diameter of bore for bolt, Dbore	**12**
Modulus of elasticity, E	**200000**
Compliance, C	3.24941E-07
Stiffness, k	3.08E+06
From FEA	
Load (4x load for quadrant)	**1000**
displacement	**4.55E-04**
resulting stiffness, k	2.20E+06
Ktheory/Kfea	1.40
With shear stiffness term (assumes theta = 45 degrees)	
Poisson ratio, n	**0.29**
Shear stiffness	4216149
Total stiffness (1/(1/compression + 1/shear))	1.78E+06
Ktheory/Kfea	0.81

Bolted Joints: *Mechanics*[1]

Bolts hold joints together by clamping. For a joint subject to a force F a distance L away from the center of stiffness of a bolt pattern, the force is attempting to twist the joint as well as translate it; hence the bolt preload force $F_{preload}$ must have a component that resists the moment FL and the shear F. Assuming each of N bolts is a distance r_i from the bolt pattern's enter of stiffness, and each bolt has the same preload force and there is a coefficient of friction μ between the bolted members, the minimum preload force per bolt is:

$$F_{\text{preload}} = \frac{F}{N\mu}\left(1 + \frac{L}{\sum_{i=1}^{N} r_i}\right)$$

Preloading stretches the bolts and compresses the joint. When a tensile load is applied to the joint, the joint stays in compression. For a pressure vessel, this means that the joint will not leak. For a structural connection, this will increase the stiffness and fatigue life of the joint. The figures show how the region under a bolt head acts like a spring, so when preloaded they act to seemingly support tensile loads. The system is modeled with a simple one-dimensional *Free Body Diagram*. The bolts collectively can be modeled as a spring k_B, and the flange can be modeled as a spring k_F.

Phase 1: Preload force F_p is applied to the bolt by tightening the nut. The bolt pulls on the plate with F_p, and the plate pushes on the flange with F_p. The initial compression of the joint and stretch of the bolts is:

$$\delta_{Bpreload} = \frac{F_P}{k_B} \qquad \delta_{Fpreload} = \frac{F_P}{k_F}$$

Phase 2: A load F_L is applied to the plate. The plate is held back by the collective bolt force F_B and the flange force F_L. The two unknowns cannot be solved for with the one available equilibrium equation ΣF = 0. Thus the problem is *statically indeterminate*, and *constitutive equations* relating deflections and forces and *geometric boundary conditions* must be considered.

The *unknowns* of the problem are thus F_B, F_F, δ_B and δ_F.

The *equilibrium equation* for the plate is $F_L - F_F + F_P - F_P - F_B = 0$

The *constitutive* equations (force/deflection) for the system are:

$$\delta_B = \frac{F_B}{k_B} \qquad \delta_F = \frac{F_F}{k_F}$$

The *geometric boundary condition* for the system is once the preload force is applied, an additional applied force F_L cause flange and bolt forces F_F and F_B which cause equal deflections ($\delta_B = \delta_F$) of the flange and bolt "springs". These four equations and four unknowns can be solved:

$$F_B = \frac{F_L k_B}{k_F + k_B} \qquad F_F = \frac{F_L k_F}{k_F + k_B} \qquad \delta_B = \delta_F = \frac{F_L}{k_F + k_B}$$

If the force F_L is too large, the initial compression of the flange by the bolts will be overcome. The flange stiffness effectively drops to zero and the joint opens up and starts to leak. The bolt force will then suddenly increase and the bolts could break. The goal is to have the force F_L cause the preload force F_P to be just overcome so the joint will leak, and hence release pressure, without breaking the bolts:

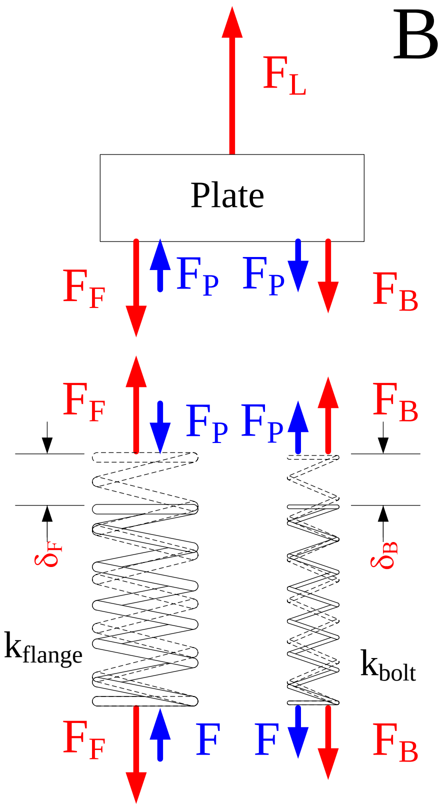
Leak: $$F_F = F_P = \frac{F_{Leak} k_F}{k_F + k_B} \Rightarrow F_{Leak} = \frac{F_P\left(k_F + k_B\right)}{k_F}$$

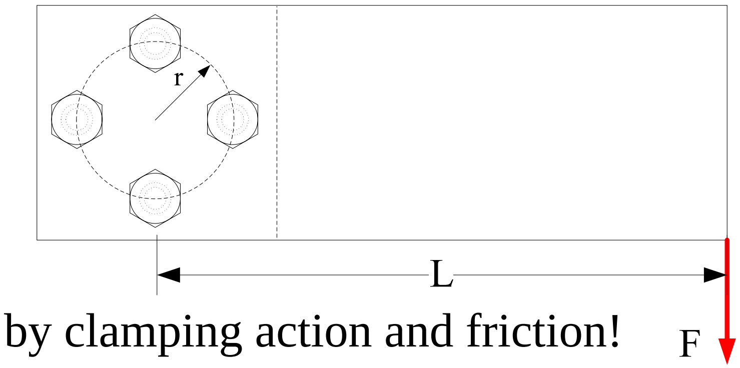
Bolts do not break: $$F_{bolt\max} > \sum_{\text{forces on bolts}} F = \frac{F_{Leak} k_B}{k_F + k_B} + F_P \Rightarrow F_{bolt\max} > F_{Leak}$$

Can you see the "magic" of preloading a bolted joint? It is really an amazing effect! Note the sequence used to analyze this problem. It can be used for many different statically indeterminate problems!

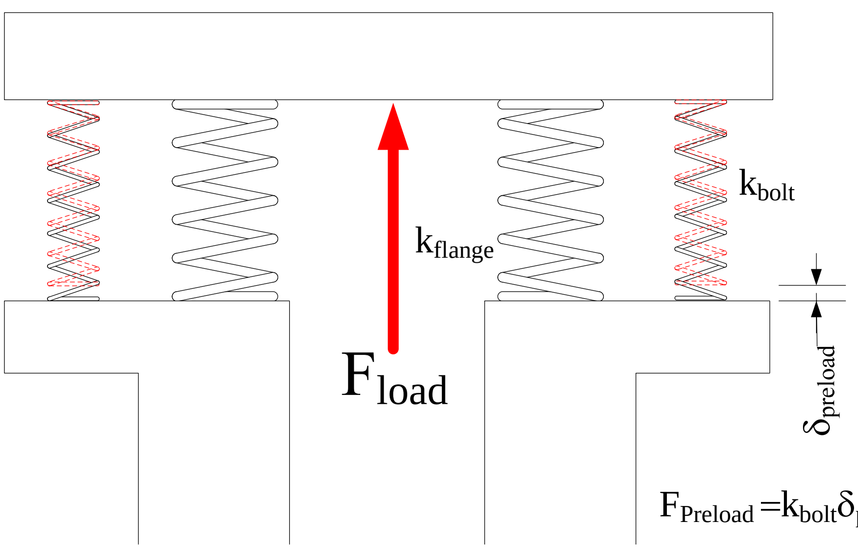
1. The mechanics of applying torque and generating bolt force, including the effect of the thrust region under the bolt head, are discussed in detail on pages 6-3 to 6-5 and applied in the spreadsheet *leadscrew_design.xls*.

Bolted Joints: *Mechanics*

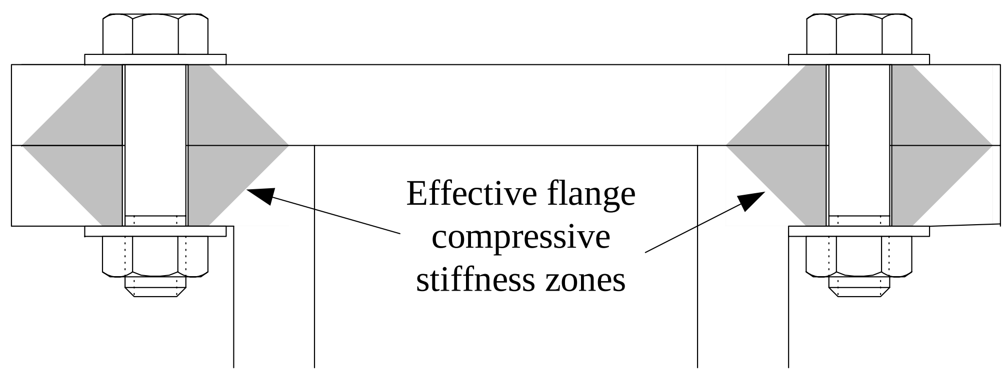




- Bolted joints resist shear ONLY by clamping action and friction!
  - Thread lubrication enables consistent conversion of tightening torque into bolt force
  - NEVER rely on a bolt to withstand a shear load, UNLESS the bolt is a shoulder bolt!
- Shear and moment capacity: Find the center of the bolt pattern, and compute the moments about it
- Bolts act in parallel with the stiffness of the joint
  - By tightening bolts to create a preload higher than the applied load, the effects of alternating stresses created by a load are reduced



$F_{Preload} = k_{bolt}\delta_{preload}$



Bolted Joints: *Stiffness*

The previous analysis depends on being able to estimate the flange stiffness. The stiffness of the flange includes the compressive stiffness of the material under the bolt head in series with the stiffness of the physical interface, which increases with pressure, and the stiffness of the threaded material. In general, the compressive stiffness of the flange material is much less than the compressive stiffness of the preloaded physical interface[1]. In order to determine the stiffness of this cone-like section under the bolt head, use the compliance *C = 1/k*, and define the differential compliance in terms of the outer diameter *d* of the cone of material in compression at a distance *y* below the bolt head, the effective cone angle α, the flange thickness h_{flange}. The compliance term for the clearance hole diameter, d_{bore}, for the bolt:[2]

$$d(y) = d_{\text{bolt head}} + 2y\cos\alpha_{\text{cone angle}}$$

$$C_{\text{flange comp}} = \frac{4}{\pi E}\int_0^{h_{\text{flange}}} \frac{dy}{\left(d_{\text{bolt head}} + 2y\cos\alpha_{\text{cone angle}}\right)^2 - d_{bore}^2}$$

$$\Rightarrow \frac{4h_{\text{flange}}}{\pi E_{\text{flange}}}\left(\frac{1}{d_{\text{bolt head}}\left(d_{\text{bolt head}} + 2h_{flange}\cos\alpha_{\text{cone angle}}\right) - d_{bore}^2}\right)$$

This makes sense, because if α were to be zero we are left with the compliance of a cylinder with a hole in it. But what about shear deformations in this region? Using energy methods (see page 8-21), the shear compliance of the flange region is estimated to be[3]

$$C_{\text{flange shear}} = \frac{(1+\nu)\ln 2}{\pi h_{flange} E_{flange}}$$

The total compliance is the sum of the compliance due to compression and shear. The stiffness is $1/C_{total}$:

$$k_{\text{flange}} = \frac{\pi E_{flange}}{4h_{flange}\left(\frac{1}{d_{\text{bolt head}}\left(d_{\text{bolt head}} + 2h_{flange}\cos\alpha_{\text{cone angle}}\right) - d_{bore}^2}\right) + \frac{(1+\nu)\ln 2}{h_{flange}}}$$

The stiffnesses of the bolt shaft in tension, and the head and nut (if a nut is used) in shear, all act in series, so their stiffnesses combine to give the total stiffness of the bolt:

$$k_{\text{bolt}} = \frac{1}{\frac{1}{k_{\text{boltshaft}}} + \frac{1}{k_{\text{bolthead}}} + \frac{1}{k_{\text{nut}}}} = \frac{\pi E_{\text{bolt}}}{\frac{4L_{\text{bolt}}}{d_{\text{bolt}}^2} + \left(1+\nu_{\text{bolt}}\right)\ln 2\left(\frac{1}{h_{\text{nut}}} + \frac{1}{h_{\text{bolthead}}}\right)}$$

As the flange thickness increases, the length of the bolt to pass through the flange thickness also increases, so the bolt stiffness decreases in a linear fashion. On the other hand, the diameter of the strain cone increases, which offsets much of the height increase, and the flange stiffness decreases far more slowly than that of the bolt. The analysis agrees reasonably well with Finite Element Analysis when a strain cone angle of 30 degrees from the vertical is used. The ramifications are that the bolt stiffness is typically only 20% of the flange stiffness, and bolt preload force is typically 70% of the maximum allowable bolt force to realize F_{break}/F_{leak} = 1.2. Of course there is a spreadsheet *Bolts_preload.xls* with which you can play "what if" scenarios! The details of bolt design can be studied using *leadscrew_design.xls*.

What appropriate analysis is required for the bolted joints in your robot? Do you have to worry about fatigue life of bolted joints, or is the ability to resist shear loads the primary issue? Experiment with *Bolts_preload.xls*

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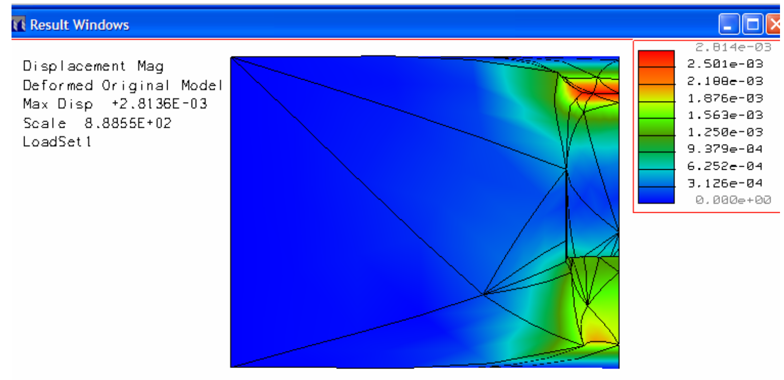
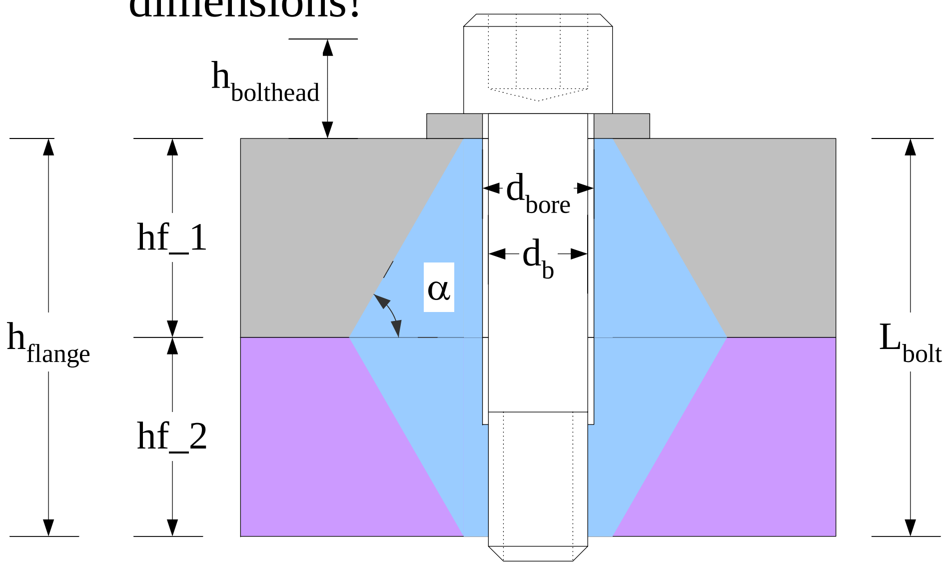
1. See M. Yoshimura, "Computer Aided Design Improvement of Machine Tool Structure Incorporating Joint Dynamics Data," Ann. CIRP, Vol. 28, 1979, pp. 241-246. The test specimens were made of 0.55% carbon steel and the surfaces were ground and coated with a light machine oil.
2. Direct integration produces an inverse hyperbolic tangent function, thus initially ignore the d_{bore}^2 term, and later add it as a "negative stiffness" term in series with the flange in compression and shear.
3. See A. Slocum, Precision Machine Design, SME, Dearborn, MI, pp 371-378. The same method is used to determine the shear stiffness of the bolt head and nut

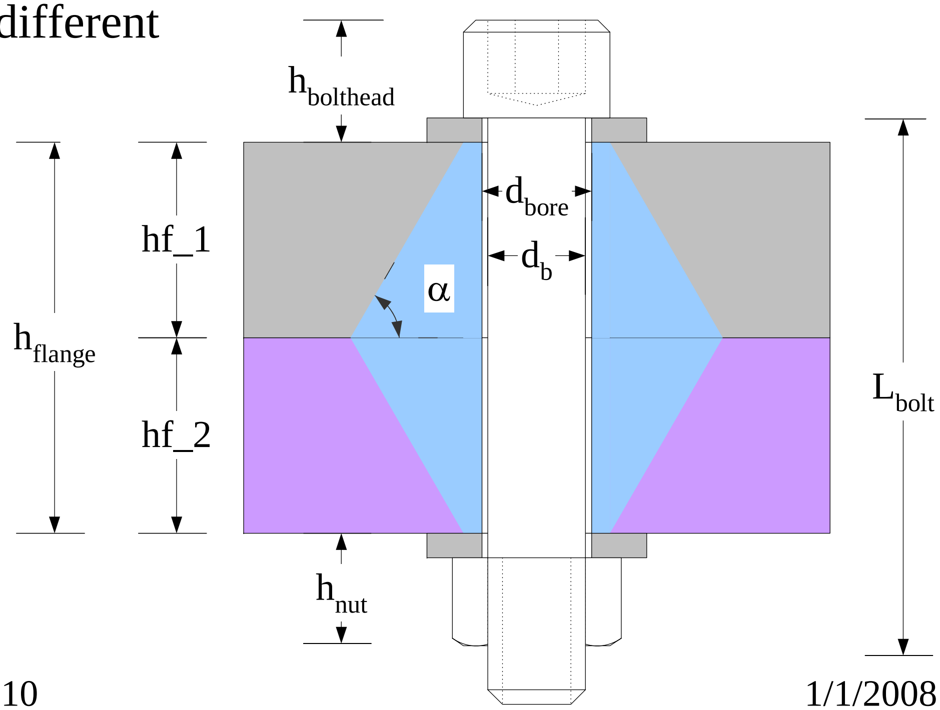
Bolted Joints: *Stiffness*

- As bolts are tightened (preloaded), their stiffness acts in series with the flange stiffness
- As external loads are applied to the joint, bolts' stiffness acts in parallel with flange stiffness
- Preloading bolts allows large loads to be applied to a joint while minimally affecting the bolt stress
- A joint can be designed so it "leaks" before a bolt breaks
  - Make the stress cones overlap!
- *Bolt_preload.xls* lets you experiment with different dimensions!



FEA results compared to analysis (60° stress cones)	
Applied load (N)	**4000**
deflection under bolt head (mm)	**0.002810**
deflection from threaded region (mm)	**0.002100**
Total deflection (mm)	**0.004910**
Stiffness (N/mm)	**814664**
FEA/Analytical	**0.86**





Bolted Joints: *Bracket Case Study*

From mounting a winch on a ship to a simple bracket to hold up shelves loaded with books, a common simple but critical design problem is the design of a bracket to hold a load. You should have the ability as a design engineer to take a seemingly simple problem, identify the issues to be addressed and then investigate them in detail from the most obvious to the not-so-obvious yet critical because overlooking them could cause failure when loads are high and over design is costly. The purpose of this particular case study is not to present all the information in step by step detail, but rather to present enough information to allow you to work through the details and compare your answers with those in the spreadsheet *Bolted_Bracket_Design.xls* (but do not look at it until you are done!).

Consider the problem of designing a bracket to support shelves that are to be heavily loaded. If you buy brackets from the hardware store that look very strong and may even have a rating that says they will support the loads you intend, will the screws you use to screw the bracket to the wall in your room hold? The starting point for the problem is the figures. Think of how you would mount brackets to a wall to support a shelf. If any information is missing, add it. Start with the FUNdaMENTALS: draw the structural loop of the bracket mounted to a wall. For each element in the structural loop draw Free Body Diagrams, label forces and dimensions, list knowns and unknowns and equilibrium equations. What are the risks associated with each element? The screws are in tension from the load of the shelf on the bracket but are there any other loads on them?

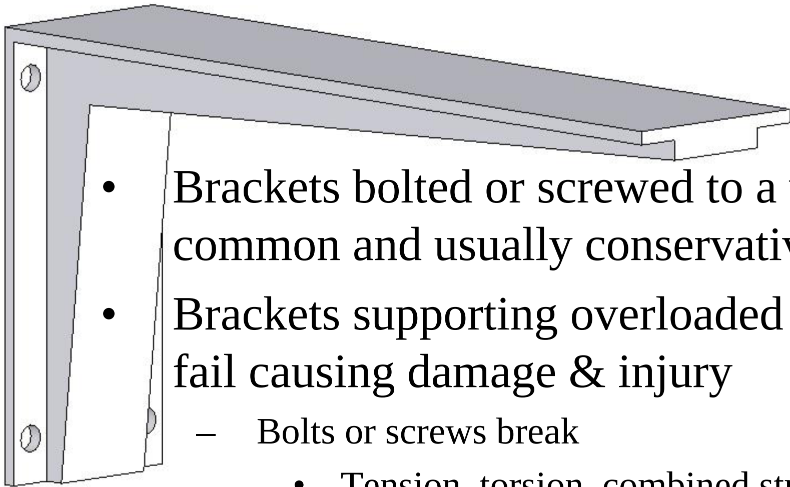
Write down your thought process as if you were writing a book detailing the design process and analysis of all the elements. In the end you should take the equations you have created to predict forces and stresses and create some sort of computer code (spreadsheet, MATHCAD, MATLAB…) so you can enter data and get an answer as to will the brackets hold.

First off, you should see that when the bolts holding the bracket fail, the bracket will pivot about the lower edge. Your analysis should probably first find the forces in the bolts based on this assumption. Your first challenge will likely be that you have more unknowns than knowns (the problem is statically indeterminate). There are two options: 1) as an engineer recognize that the lower bolts are there for preload to create friction force with the wall to resist shear, so ignore them. 2) because you will be writing a spreadsheet, you might as well put in all the details, so as with any statically indeterminate system, replace the bolts with springs, assume the bracket is much stiffer, and now find the forces in the springs.

Next determine the stresses in the bolts from their initial tightening. There will be a torsional stress and a tensile stress and they occur simultaneously so you can use the Von Mises criteria to get an equivalent stress. What is the maximum stress for a structural screw? Check the internet and see what you find, and record the reference. After the screw is installed and the shelf is loaded, the preload force and the force from the shelf loading add. Which total stress state is more severe, tightening or shelf loading? If you have ever used a drill to tighten a screw you will usually drive the screw head into the wood and stop long before you break the screw. However, if you are using screws to attach a stiff metal plate to a piece of wood, you will be surprised how easy it is to twist off the heads of the screws! What design thoughts does this trigger?

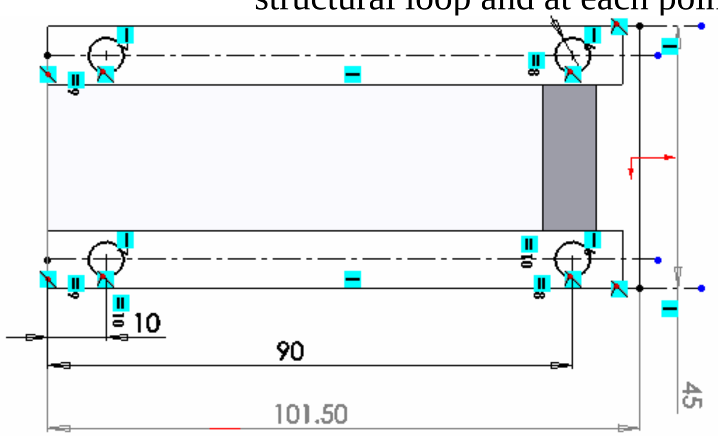
If the screws do not seem like they will break, what is the next element in the structural loop? What about drywall if the bracket is screwed to a drywall covered wall? If you have ever worked with drywall you probably recall how crumbly it can be. What are the contact pressures? The preload pressures will be localized around the bolt heads because of local sheet metal deformation, but the shelf loads will be distributed along the stiff spine of the bracket, so assume a triangular pressure distribution from zero at the top to maximum at the bottom. What is the differential force element at any section? Integrate from top to bottom to get the moment, set it equal to the moment from the shelf load and solve for the maximum pressure!

If you get stuck, you can peak at the formulas in the spreadsheet *Bolted_Bracket_Design.xls* . Complete your spreadsheet, compare them, and see where they differ. There may be an error in either one and errors should be found and rectified. If you find an error in someone else's work, contact them and tell them you think you found an error and why!



Bolted Joints: *Bracket Case Study*

- Brackets bolted or screwed to a wall are very common and usually conservatively designed
- Brackets supporting overloaded shelves can fail causing damage & injury
  - Bolts or screws break
    - Tension, torsion, combined stress?
  - Screw threads pull out from the wall
    - Wood, masonry, drywall…
  - Too much screw thread force
    - Crushes sheet metal brackets, wood, drywall
- Use the figures here as a starting point and as an exercise
  - Draw the Free Body Diagram
  - Systematically follow the chain of elements in the structural loop and at each point calculate the stress



Bolted_Bracket_Design.xls		
NOTE: If you are using this spreadsheet for designing shelf brackets, do not use drywall screws which are brittle. Use "deck screws" or screws intended for structural assembly		
By Alex Slocum, 1/2/3007		
Enters numbers in **BOLD,** Results in **RED, All units in N, mm**		
Distance from wall to load, Ll (mm)	**200**	
Load, W (N)	**200**	
Moment, M (N-mm)	**40000**	
width of bracket, wb (mm)	**45**	
length of bracket along wall, LB (mm)	**110**	
Preload		
Bolt torque, tb (N-mm, lbf-ft)	**2000**	**1.47**
Bolt lead, lead (mm)	**3**	
screw thread efficiency	**0.2**	
Preload force per bolt, FP (N)	**838**	
coefficient between bracket and wall, mu	**0.3**	
Max load from preload force, coefficient of friction, Fmax (N)	**1005**	
Bolts		
Bolt pitch diameter, dbp (mm)	**4.0**	
Bolt root diameter, droot (mm)	**3.0**	
Bolt engagement length in wall, Le (mm)	**25**	
Number of bolts upper row, NBU	**2**	
Number of bolts lower row, NBL	**2**	
Distance upper row of bolts from fulcrum, LBU (mm)	**90**	
Distance lower row of bolts from fulcrum, LBL (mm)	**10**	
Maximum stress, sigmax (N/mm^2, psi)	**800**	**115984**
Bolt forces: Assume rigid bracket, fulcrum point at bottom		
(bolt stiffness)*(deflection angle), Kba	**4.9**	
Force on each upper bolt, FBU (N)	**220**	
Force on each lower bolt, FBL (N)	**24**	
Bolt stresses		
Bolt tensile stress from preload, sigbp (N/mm^2, psi)	**119**	**17183**
Max bolt tensile stress from load sigbl (N/mm^2, psi)	**31**	**4502**
Shear stress from torque, tt (N/mm^2, psi)	**377**	**54695**
Von Mises stress during screwing, sigvms (N/mm^2, psi)	**664**	**96279**
Von Mises stress during loading, sigvml (N/mm^2, psi)	**150**	**21685**
Resulting safety factor	**5.3**	
Wall		
Structure into which bolt is threaded		
Load shear stress along thread engagement, taus (N/mm^2, psi))	**2.7**	**387**
Preload shear stress along thread engagement, taus (N/mm^2, psi))	**0.7**	**101**
Total shear stress	**3.4**	**488**
Wall covering (e.g., drywall)		
Assume triangular over length from fulcrum point to upper bolts		
Moment contact pressure, pc (N/mm^2, psi)	**0.7**	**95**
Preload contact pressure (5 bolt diameters) (N/mm^2, psi)	**2.7**	**387**
Total contact pressure on surface covering	**3.3**	**482**
Bracket stresses: to be developed by the user for the bracket they have		

Bolted Joints: *Finite Element Analysis*

A solid model is a great tool for visualizing a design. Finite Element Analysis is a great tool for adding constraints and loads to a model and then using FEA to predict stresses and deflections. Illustrated is one of the biggest problems with using a solid model as modeled for producing part drawings and then sending the model direct to the FEA program: The connections between parts will be assumed to be continuous wherever parts are in contact. Most parts are typically bolted together, and the "real" constraint is best conservatively modeled only by the effective area of contact under the bolt head as discussed on page 9-10. To create this effective contact area in a solid model of the part, the flange should be modeled slightly thinner than intended, and then protrusions added under the bolts to obtain the desired actual thickness. A solid that overlays the protrusions and makes the part appear as would be manufactured is added, but suppressed for FEA study. It is unsuppressed for creating the part drawings.

The figure shows a column that is to be bolted at its base to another structure with the solid overlay suppressed. Three bolt options are shown: a minimum of thee bolts, a typical six bolt pattern used for structural attachment, and a twelve bolt pattern that would typically be used if a tight seal were to also be maintained (a gasket would also have to be added).

The results are summarized in the table below. Note that it is important in the FEA results to specify deflection in the horizontal (Z) direction as that what is calculated by the theory. The "displacement magnitude " results include the vertical motion also, and hence represent the vector deflection of a point at the end of the column which is larger. The results show that the twelve bolt pattern indeed models the column as if it were uniformly attached at the base, and the six bolt pattern is a pretty good model, but the three bolt model deflects significantly more; hence if an engineer used a three bolt pattern in manufacture but used FEA to model the system without putting in the virtual pads shown in the figures, the stiffness would have been over estimated.

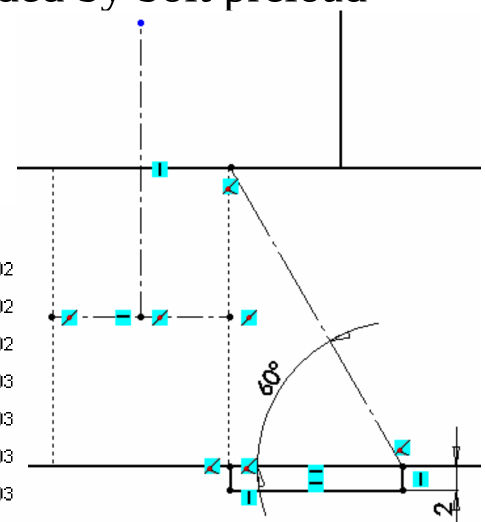
The theoretical and FEA results for a simple cantilever beam are very close, differing primarily because the FEA also includes shear deformations. The flanged part is more complex, but essentially is the same as a straight cylinder with respect to a closed-form model.

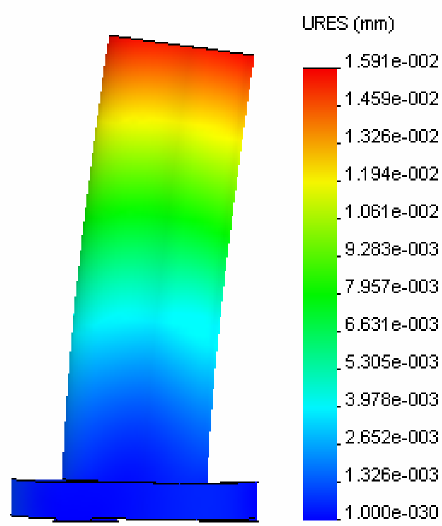
Radial force, F (N)	1000		
Length of column (top to top of flange, top), L, L1 (mm)	327	300	
Outer diameter, OD (mm)	100		
Inner diameter, ID (mm)	50		
Modulus of elasticity, E (N/mm^2)	200028		
I (mm^4)	4601942		
Cantilevered beam (theory)			
	Theory	FEA radial	FEA vector
	w/o flange		
300 mm tall structure (no flange)	0.00978	0.0107	0.0110
327 mm tall structure (includes 27 mm flange)	0.01266		
		with flange	
327 mm tall structure (includes 27 mm flange)			
3 bolt		0.01585	0.01628
6 bolt		0.01363	0.01394
12 bolt		0.01303	0.01334
solid flange		0.01231	0.01259

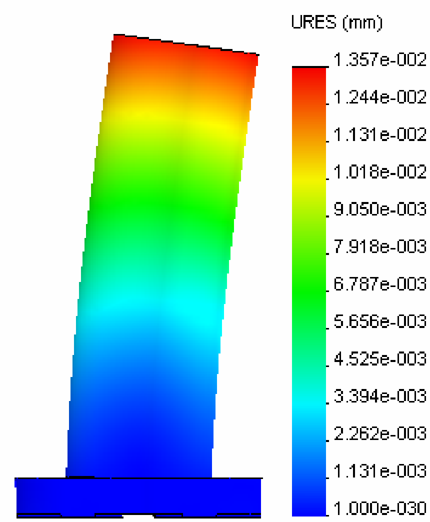
Conducting these sorts of studies helps an engineer develop design intuition, so if you are ever unsure about just how to create an FEA model, try several different scenarios and compare the outputs!

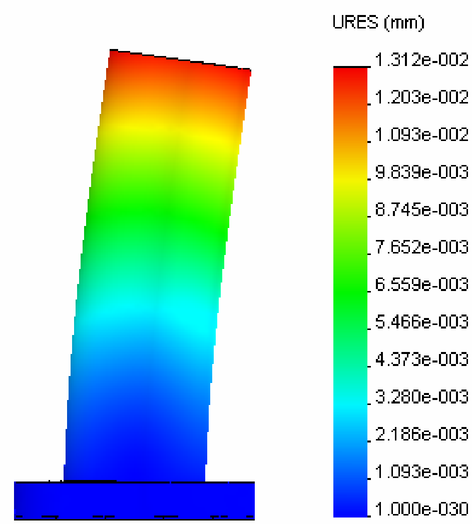
Bolted Joints: *Finite Element Analysis*

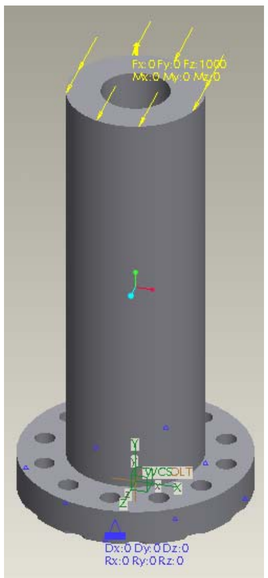
- Rarely will you do FEA of the threads in a bolt, but it is not uncommon to do FEA on a flange to determine local stresses, or the stiffness of a bolted joint
- Remember Saint-Venant's Principle: The contact area that is preloaded by bolt preload force only extends a few bolt diameters
  - Conservatively, assume a 60 degree cone under the bolt head
  - FEA elements should only make contact in this region

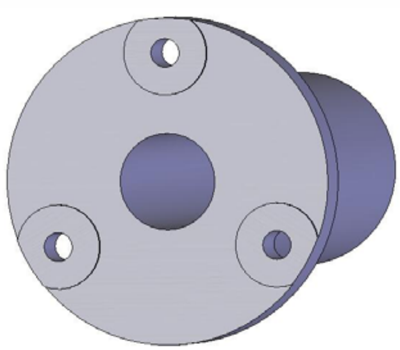


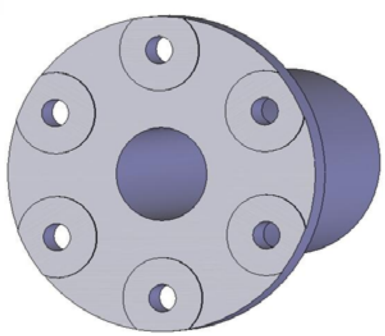


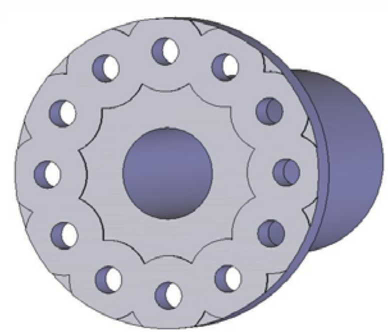












Structural Joints: *Pinned & Riveted*

Recall that bolted joints maintain relative lateral position between two parts by relying on friction and clamping force. The bolt itself has clearance between it and the hole in which it is placed. *Pinned* or *riveted* joints, on the other hand, completely fill the holes into which the pins or rivets are placed, and thus maintain alignment by the shear strength of the pin or rivet. Press-fits are in this same category (recall that interference-fit joints were discussed in detail on page 5-25. See also *Joint_interference_fit.xls*)

There are many different types of pins, including solid (dowel), spring, and spiral pins. All allow a joint to be taken apart after it has been pinned together; hence pins are typically used to locate and bolts are used to hold a joint together against tensile and moment loads. As the pins go from solid to having more radial compliance to accommodate undersized or misaligned holes, their ability to resist shear forces in the joint also decreases. Note that regardless of the type of pin used, if a hammer is used to pound the pin in, it is likely that the impact force will flare the head of the pin and then it will not go all the way in. Use an arbor press whenever possible! The three primary types of pins are:

A *dowel pin* is pressed into a hole, and thus requires a very precisely made hole made by first drilling and then reaming. The insertion force can still be very high and typically requires an arbor press.

A *spiral pin* is essentially a cylinder made from rolled sheet, and when pressed into a hole smaller than it, the radial pressure causes the rolled sheet to contract. The elastic energy stored in the deformed pin material maintains its tight fit in the hole. This type of pin is somewhat forgiving of undersized or misaligned holes.

A *spring pin* is a cylinder with an axial slit, and when pressed into a hole smaller than the pin, the radial pressure causes the axial slit to close. The elastic energy stored in the deformed pin material maintains its tight fit in the hole. This type of pin is most forgiving of undersized or misaligned holes.

Pinning two parts together can also be useful for maintaining alignment during manufacturing or assembly. In order to maximize the alignment of holes in two parts, clamp both parts into a vise and then drill holes through both parts simultaneously. Then with the parts pinned together, other features, such as a milled common reference edge and bores for bearings or shafts, can be machined. Boring holes in two parts simultaneously so as to ensure alignment is referred to as *line boring*.

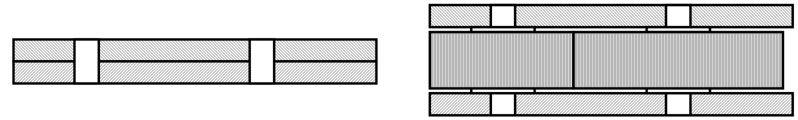
A riveted joint is similar to a pinned joint in that it can transmit shear loads, but it can also support tensile and moment loads on a joint. Rivets are most often used to attach sheet-metal parts. A riveted joint has the benefits of both bolted and pinned joints, but it is a permanent connection, unless the rivet is drilled out. There are many different types of expanding rivets, and *pop rivets* are very common. A pop rivet has a hollow cylinder into which a stem with a spherical end is placed. As the stem is drawn up, the spherical end deforms the cylinder causing its end to flare and its body to expand until the stem fractures below the surface of the rivet head.

A riveted joint uses cylinders which are expanded into holes in which they are first placed. The expansion is caused by compressing the cylinder which also acts to clamps the parts together. The expanding nature of the rivet allows many holes to be drilled in parts to be fastened together, and the rivets can deform to accommodate some misalignment. Unless you are using an NC milling machine, to be conservative, align and clamp the parts, and then drill all of the holes through both parts at once (line bore them). In large structures, such as bridges and buildings, red-hot rivets are pounded in place. In transportation vehicles such as planes, trains, and automobiles, holes are made by NC machine or they are drilled in-place. Since riveting is a very fast and economical process, once you are sure a joint is ready to be permanent, rivet it!

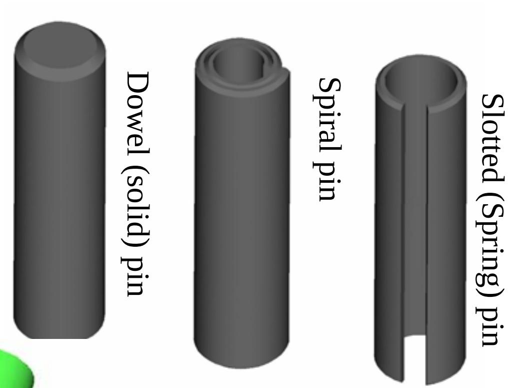
For pinned and riveted joints, the shear strength can be estimated to be equal to the product of the total cross sectional area of each of the fastening elements and the allowable shear stress of the fastener. Usually the joint itself around the fastener will deform significantly, so the allowable shear stress in the fastener is less than the allowable shear stress of the fastener metal.

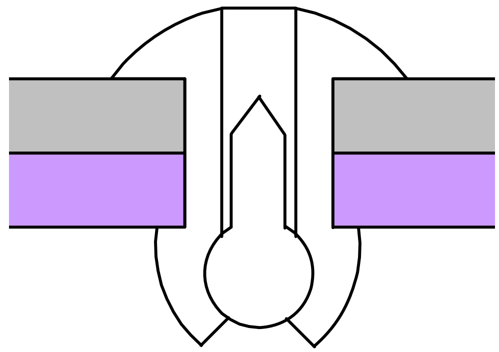
Review the functional requirements of the attachments between your components. Which require careful alignment, and perhaps could benefit from line boring? Which joints must have the ability to be taken apart and could benefit from being pinned together? Which joints are permanent and could be riveted together during assembly?

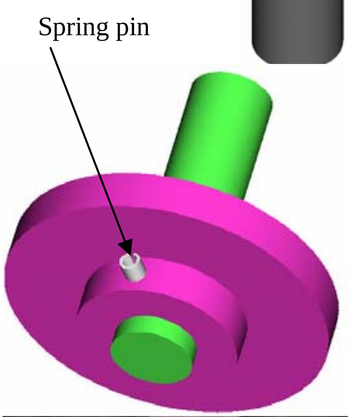
Structural Joints: *Pinned & Riveted*

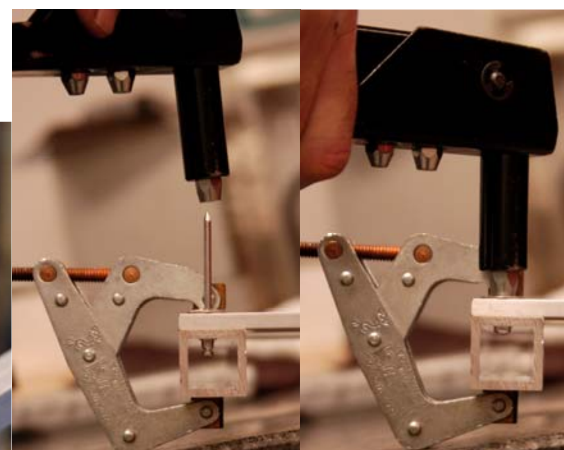


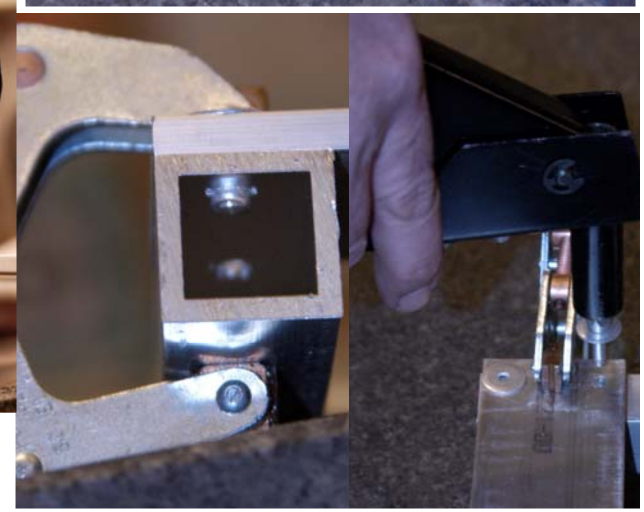
- Pinned joints use pins pressed into holes to transmit forces (or torque) ***(see page 5-25)***
- Pinning parts together can help during alignment during manufacturing or assembly
  - Line-bore holes for shafts and bearings by pinning plates together and drilling all the holes at once!
- A riveted joint uses expanded members to transmit shear forces and resist peeling forces
  - The expanding nature of the rivet allows many holes to be drilled in parts to be fastened together











Structural Interfaces

A structural interface is considered to be a repeatable mechanical connection capable of withstanding structural loads, and it can be routinely taken apart. This is in contrast to structural joints which are not intended to be routinely taken apart. A structural interface must therefore provide constraints to control all the intended degrees of freedom. According to the principle of *Exact Constraint Design* (ECD) as discussed on page 3-26: The number of points of constraint should be equal to the number of degrees of freedom to be constrained. This is the minimum, although some interfaces may utilize more constraints in order to achieve higher load capacity and high repeatability and accuracy using the principle of *Elastic Averaging* as discussed on page 3-27.

Most commonly, structural interfaces are designed using keyways or pinned connections. Because they would typically be over constrained if an attempt were made to create an exact fit, tolerances are set so there is always some room between components. This loose fit ensures that parts can be assembled, but the accuracy and repeatability that can be obtained is limited by the toleranced gaps. The alternative is to use spring pins (page 9-11) or numerous elastic elements that accommodate misalignment and tolerances by elastic deformation. However, designing a system that is over constrained takes exceptional care to ensure that deformations do not occur that may overload sensitive components such as bearings. Thus if possible, a good strategy is to try and create an exactly constrained design.

ECD often concentrates loads at single points which can lead to a smaller region of stability (e.g., 3 legged chair); however, ECD does not always mean that systems have to be designed like three-legged chairs. Ponder the following: Can you support a plate at multiple points yet not get the “four legged chair with one short leg” syndrome? How do windshield wiper blades distribute the point force applied by the arm uniformly across the blade? The answer to both questions is to use a *wiffle tree* as shown on page 3-26.

Exact constraint design can sometimes be visualized by imagining how support points need to be applied to uniquely define the position of a cube using a 3-2-1 fixturing philosophy. Each of the support points can in fact be the center of stiffness of an array of points on a *wiffle tree* arm; however, eventually at the connection to ground, 3-2-1 points are established:

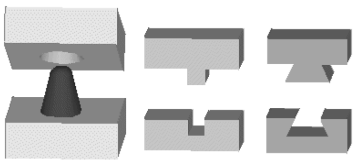
- One side is placed on three support points
- A second side is pushed up against two support points
- The first side slides across the three support points
- A third side is pushed against one support point
- The first and second side slides across their support points

With the above, ideally, exact constraint is theoretically achieved, and for all practical purposes, it is when the loads are very light; however, when heavier loads are applied, which may be due to the weight of the object itself, point loads cause local deformations that act like additional orthogonal constraints. These point contact deformations are called *Hertz contact* deformations and are discussed in detail in the next section. Since the Hertz contact deformations and friction fully constrain the cube when it is first placed, as the cube is pushed against the other constraints, the Hertz deformations and friction reduce the repeatability of the system. Hence the 3-2-1 fixturing method practically has a repeatability on the order of 3-5 microns.

What countermeasure can be used to address the risk of Hertz contact deformations essentially exactly constraining the system when the first three contact points are established? The answer is to use the fundamental principles of *stability* (see page 3-18), or rather that of an unstable system, and *self-help* (see page 3-17). If initial contact points can be established such that the weight of the system creates a moment that requires more contact points to be established in order to create force equilibrium, then “exact” constraint can be achieved that is far less sensitive to Hertz contact deformations. This type of coupling is called a *kinematic coupling.* The primary Hertz contact issue becomes: what is the maximum point load that can be applied at any support point without causing permanent deformation?

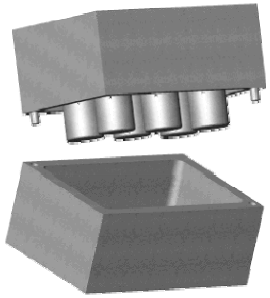
Like any principle, ECD is a guideline, a catalyst for synthesis, but never an absolute! For your robot, how many wheels do you really need? If a shaft were to expand, would it overload the bearings? Do a constraint sensitivity analysis on your machine and base your design decisions on sound philosophical, analytical, or experimental results.

Structural Interfaces



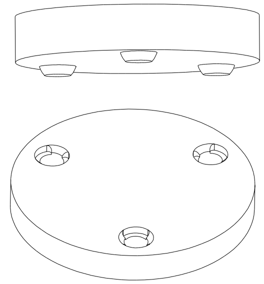
Keys

Over Constrained



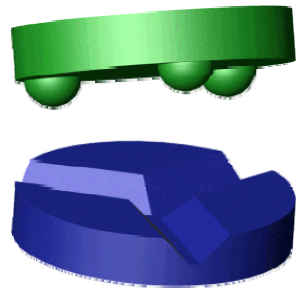
Pinned Joints

Often over Constrained



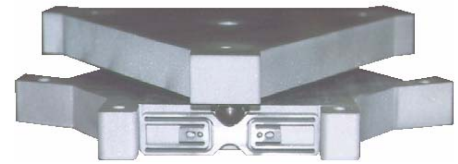
Quasi-Kinematic Couplings

Near Kinematic Constraint



Kinematic Couplings

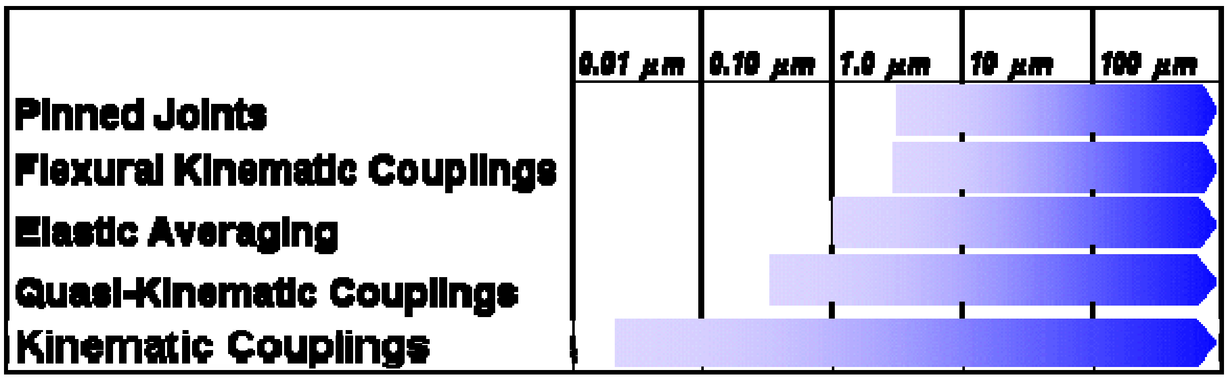
Exact Constraint

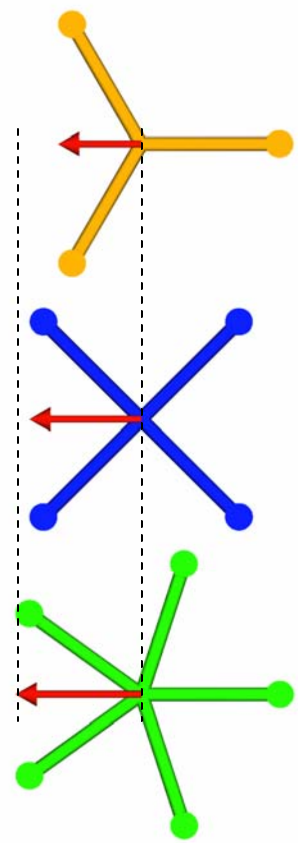


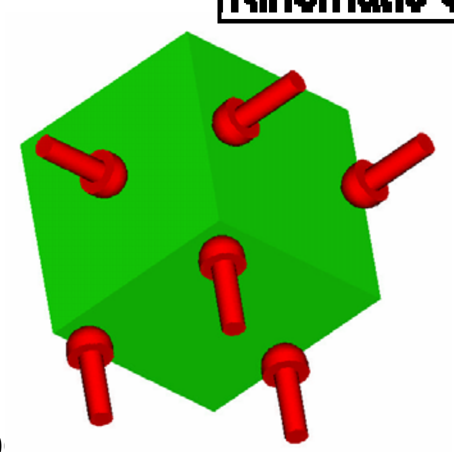
Flexural Kin. Couplings

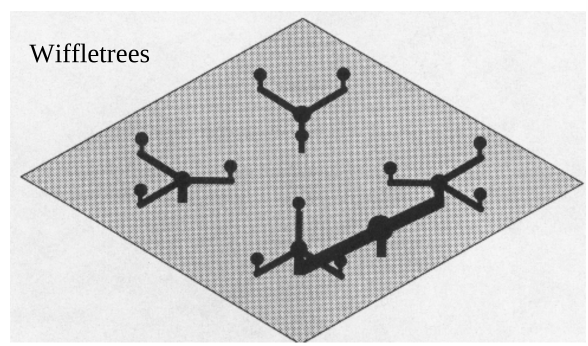
Exact Constraint

Repeatability









Hertz Contact

With the discussion of exact constraint design, and the use of points of contact to define the position of an object, care must be taken to not create stresses at the contact points that could cause local yielding of the materials in contact.

To *feel* the importance of *Hertz contact*, take your pen and put the point to your temple and push with increasing force until it *hurts*. You are now experiencing the maximum Hertz stress of your temple. Next, turn the pen 90 degrees (reciprocity!) and push on your temple with the side of the pen (the barrel) and note you can push a lot harder before you feel discomfort. It is the difference in the radii of curvature that makes the difference in the stress at the contact zone. Being able to predict this stress is critical for the success of many a design. Perhaps just as important, understanding the nature of the stress and the relations between the design parameters (i.e., curvatures of the bodies at their contact points) enables designers to think creatively and synthesize new designs. All this was made possible by a brilliant mathematician named Heinrich Hertz (1857-1894).

Heinrich Hertz, like many mathematicians, saw no real difference between different types of problems, only different boundary conditions to be applied to differential equations. As a result, he made major contributions to the fields of mechanical and electrical engineering. In the latter discipline, the symbol for frequency "Hertz" comes from his name. In the former discipline, "Hertz contact" symbolizes the high stresses that arise between bodies in point or line contact. They are of such great importance because the contact stress analysis methods pioneered by Hertz allowed the industrial revolution to roll forward. Although his life was short, his impact will long be remembered as time rolls on.[1]

As machinery rapidly evolved in the 1800's, the need for gears and transmissions became more critical; however, there were no equations that could allow designers to predict performance. As a result, designs evolved slowly based on experience (failure!). In addition, railroads were becoming more widespread, and the sizes of locomotives were increasing to allow them to haul more freight for less cost. It was a positive feedback system: better machines led to the creation of better and cheaper components, which led to better machines. However, there was a high cost associated with creating new designs, because trial and error was the rule, and experience played a key part. The need for experience meant that new designs were created primarily only by a few who were extensively skilled in the art.

Imagine a large gear that is too heavily loaded. This could cause the teeth to fail prematurely and lead to an expensive repair. Now imagine a railroad locomotive that is heavily loaded so it has the tractive effort to pull a longer train. If the contact stresses between its wheels and the rails are too high, the rails could prematurely fail, resulting in the need for replacement of hundreds of miles of tracks and massive disruption of the transportation system. Such was the motivation to create a formula (Hertz Contact Theory) for determining how much load a train could carry.

Because Hertz stresses can locally be so high, they often act as initiation sites for spalling, crack growth and other failure mechanisms. An understanding of Hertz contact stresses is thus of fundamental importance in machine design, and intense volumes have been written about Hertz-induced failures.[2] The resulting equations for Hertz contact are developed from analysis methods that require the evaluation of elliptical integrals, which are not simple to evaluate. The results are often shown as plots, from which designers must interpolate the values. As will be shown on the next page, approximate polynomials can be used, which when incorporated into a spreadsheet, such as *Hertz_Point_Contact.xls* and *Hertz_Line_Contact.xls*, make it easy to evaluate Hertz contact stresses. Hence there is no reason any practising designer should not be able to make at least a first-order evaluation of contact stresses in a design.

Identify regions of your design where there are point or line contacts between elements. Estimate the loads and use the spreadsheets to determine if the point contact stresses exceed the yield limit of the materials. What are the risks of high contact stresses, and what are some viable countermeasures, other than simply decreasing the forces?

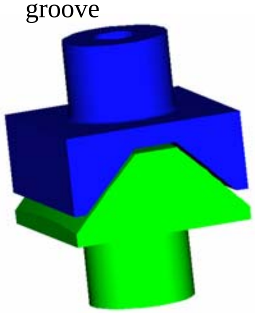
1. See K.L. Johnson, "One Hundred Years of Hertz Contact", Proc. Instn. Mech. Engrs. Vol. 196, pp 363-378

2. See for example, T.E. Tallian, Failure Atlas for Hertz Contact Machine Elements, ASME Press.

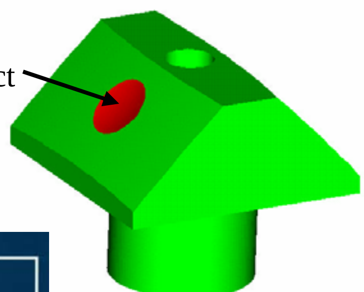
Hertz Contact

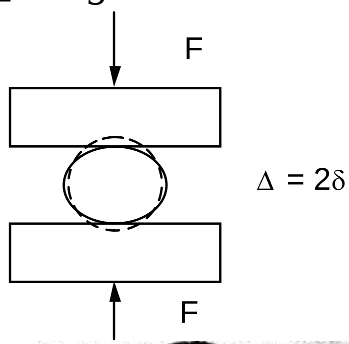
- A most important aspect of interface design are the stresses at the contact points
- In the 1800's, railroad ***wheels*** were damaging tracks, and rolling element bearing designs were very limited
  - Heinrich Hertz, the mathematician famous for his work in the frequency domain, created the first analytical solution for determining the stress between two bodies in point contact
  - *Hertz_Contact_point.xls, Hertz_Contact_line.xls, Kinematic_Coupling_3Groove_Design.xls*

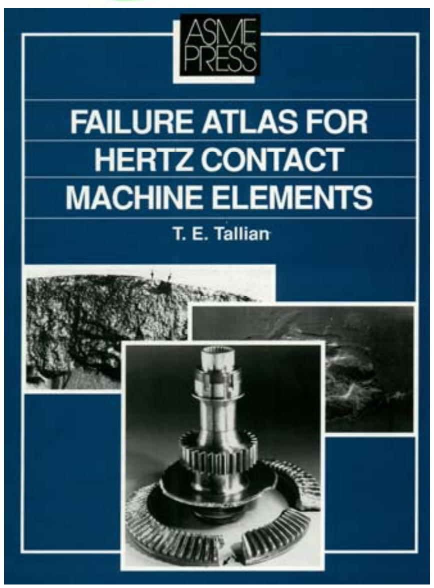
Canoe ball and vee groove



Hertz contact zone







Crowned cone

Heinrich Hertz 1857-1894

HertzContact.xls	
To determine Hertz contact stress between bodies	
By Alex Slocum, Last modified 1/17/2004 by Alex Slocum	
Last modified 12/28/03 by Alex Slocum	
Enters numbers in **BOLD,** Results in **RED**	
Be consistent with units!!	
Ronemaj	**1.00E+06**
Ronemin	**1.00E+06**
Rtwomaj	**0.500**
Rtwomin	**0.500**
Applied load F	**4,358**
Phi (degrees)	**0**
Ultimate tensile stress	**3.45E+08**
Elastic modulus Eone	**1.93E+11**
Elastic modulus Etwo	**1.93E+11**
Poisson's ratio vone	**0.29**
Poisson's ratio vtwo	**0.29**
Equivelent modulus Ee	**1.05E+11**
Equivelent radius Re	**0.2500**
ellipse c	**2.50E-03**
ellipse d	**2.50E-03**
Contact pressure, q	**3.33E+08**
Stress ratio (must be less than 1)	**0.97**
Deflection at the one contact interface	
Deflection (μunits)	**12.4**
Stiffness (load/μunits)	**350.8**
for circular contact a = c, a	**2.50E-03**
Depth at maximum shear stress/a	**0.634**
Max shear stress/ultimate tensile	**0.324**

Hertz Contact: *Point Contact*[1]

The cornerstone of Hertz contact theory is the *gap bending hypothesis* which states that the effect of geometry on the system in the contact region is a function of the algebraic sum of the curvatures of the two surfaces in contact. Thus the contact between two surfaces can be approximated by an equivalent contact problem between a sphere of radius R_e and a plane, for which the solution is known[2]. Note that convex surfaces' (e.g., a ball) radii are positive, concave surfaces' (e.g., a groove) radii are negative, and flat plane radii are infinite. The next step is to determine the equivalent modulus of elasticity of the system based on the elastic modulii and Poisson ratios of the two materials in contact.

As the bodies in contact approach being spherical, the approximate (gap bending hypothesis) and "exact" solutions developed by Hertz converge. When the bodies are far from being spherical (e.g., the case of a friction drive roller in contact with a round bar), the gap bending hypothesis yields conservative results: The gap bending hypothesis gives estimates for the stress and deflection that are much higher (by up to 30%) than they really are.

The equations shown allow a designer to determine the size of the contact region between the bodies, the stress, and the deflection. The size of the contact zone is very important: Regions in contact should be 3-5 times the size of the contact zone in order to prevent the regions' edges from breaking down. In other words, if the contact zone is 1 mm in diameter, no edge should be closer than 3mm from the center of the contact point. This is yet another practical application of Saint-Venant's principle: BE CAREFUL TO NOT ALLOW THE CONTACT ELLIPSE TO BE CLOSE TO THE EDGE OF A SURFACE!

1. There are many excellent references for more detailed study of Hertz contact stresses, including the limits of the theory. See for example, F. Seely and J. Smith, Advanced Mechanics of Materials, John Wiley & Sons, New York, 1952; and K. L. Johnson, Contact Mechanics, Cambridge University Press, 1985.
2. When the problem approaches that of a sphere indenting an elastic cavity (R1/R2 < 1.1), the Hertz theory assumptions start to break down. In this region, Hertz theory overestimates the deflection; thus for precision machine designers, Hertz theory provides a conservative estimate of point contact deflections. See, for example, L. E. Goodman and L. M. Keer, "The Contact Stress Problem for an Elastic Sphere Indenting an Elastic Cavity," Int. J. Solids Structures, Vol. 1, 1965, pp. 407-15.

The dominant form of failure is excessive contact pressure. It is important to note that the highest stresses are shear stresses that occur below the surface of the contact region. This is why, for example, soft aluminum with a thin hard anodized (aluminum oxide) surface layer still makes a poor bearing. As a designer, it is important to be able to identify trends in performance. For point contact between two bodies:

Contact pressure is proportional to:

- **Force to the 1/3rd power**
- **Radius to the –2/3rd power**
- **Modulus to the 2/3rd power**

Deflection[3] is proportional to:

- **Force to the 2/3rd power**
- **Radius to the –1/3rd power**
- **Modulus to the –2/3rd power**

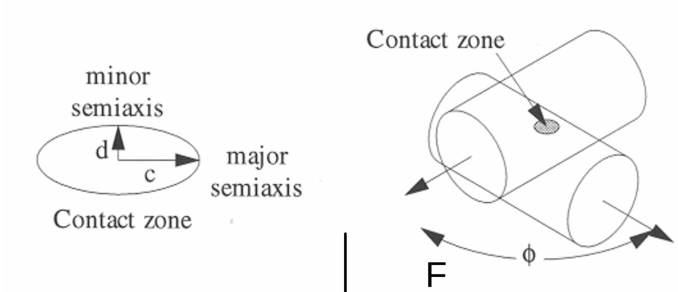
Contact ellipse diameter is proportional to:

- **Force to the 1/3rd power**
- **Radius to the 1/3rd power**
- **Modulus to the –1/3rd power**

Often enough, a designer will create a system with a point contact for location or to establish a rolling interface to reduce friction. Familiarize yourself with *Hertz_Contact_Point.xls* and be prepared for contact!

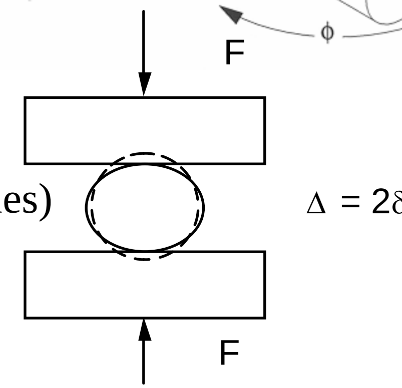
3. Note that the magnitudes of deflections of two bodies in contact are often in the submicron range; thus surface finish characteristics can play an important role.

Hertz Contact: *Point Contact*



- Equivalent radius R_e and modulus E_e (ν is Poisson ratio):

$$R_e = \frac{1}{\frac{1}{R_{1\,\text{major}}} + \frac{1}{R_{1\,\text{minor}}} + \frac{1}{R_{2\,\text{major}}} + \frac{1}{R_{2\,\text{minor}}}} \qquad E_e = \frac{1}{\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}}$$



- $Cos\theta$ (ϕ is the angle between the planes of principal curvature of the two bodies)

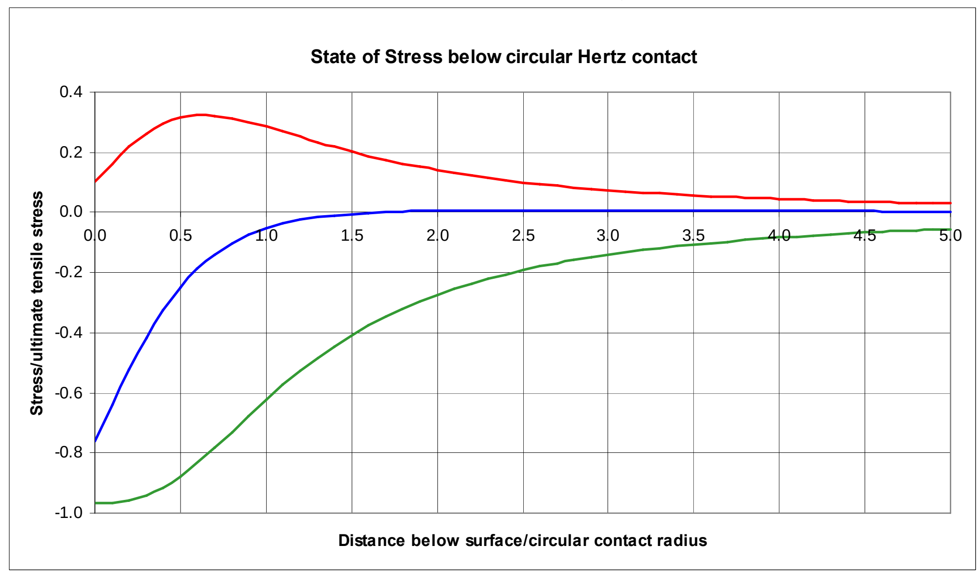
$$\cos\theta = R_e\sqrt{\left(\frac{1}{R_{1\,\text{major}}} - \frac{1}{R_{1\,\text{minor}}}\right)^2 + \left(\frac{1}{R_{2\,\text{major}}} - \frac{1}{R_{2\,\text{minor}}}\right)^2 + 2\left(\frac{1}{R_{1\,\text{major}}} - \frac{1}{R_{1\,\text{minor}}}\right)\left(\frac{1}{R_{2\,\text{major}}} - \frac{1}{R_{2\,\text{minor}}}\right)\cos 2\phi}$$

- Elliptic Integrals

$$\alpha = 1.939e^{-5.26\theta} + 1.78e^{-1.09\theta} + \frac{0.723}{\theta} + 0.221$$

$$\beta = 35.228e^{-0.98\theta} - 32.424e^{-1.0475\theta} + 1.486\theta - 2.634$$

$$\lambda = -0.214e^{-4.95\theta} - 0.179\theta^2 + 0.555\theta + 0.319$$



- Major and minor contact area elliptical semi-axes

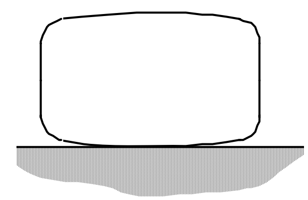
$$c = \alpha\left(\frac{3F R_e}{2E_e}\right)^{1/3} \qquad d = \beta\left(\frac{3F R_e}{2E_e}\right)^{1/3}$$

- Contact pressure & Deflection

$$q = \frac{3F}{2\pi cd} \le 1.5\sigma_{\text{for metals, ultimate tensile strength}} \qquad \delta = \lambda\left(\frac{2F^2}{3R_e E_e^2}\right)^{1/3}$$

- State of stress for circular contact of radius a as a function of depth z below the surface

$$\sigma_z(z) = q\left(-1 + \frac{z^3}{\left(a^2+z^2\right)^{1.5}}\right) \qquad \sigma_r(z) = \sigma_\theta(z) = \frac{q}{2}\left(-(1+2\nu) + \frac{2(1+\nu)z}{\sqrt{a^2+z^2}} - \frac{z^3}{\left(a^2+z^2\right)^{1.5}}\right) \qquad \tau(z) = \frac{\sigma_\theta(z) - \sigma_z(z)}{2}$$



Hertz Contact: *Line Contact*

Consider a cylinder on a flat plane as shown. There appears to be a flat region where the cylinder and plane meet. What is the dimension δ required to obtain the observed width B?

$$\frac{B}{2R} = \sin\theta \approx \theta \qquad \delta = R\left(1-\cos\theta\right) \approx \frac{R\theta^2}{2} \qquad \Rightarrow \qquad \delta = \frac{B^2}{8R}$$

Could this δ be due to surface roughness factors? If B = 1.5 mm, R = 37.5 mm, and then δ = 7.5 μm, which seems small, but if smooth metal surfaces are used, δ is likely due to Hertz contact. Indeed, contact between two cylinders with parallel axes is a special case of Hertz contact stress theory[1] but it has been studied in great detail[2], especially given the fact that a cylinder in contact with a surface is far more rigid than a sphere in contact with the same surface[3]. Given two elastic cylinders of diameter d_1 and d_2 in contact over a length L, the rectangular contact area is of width 2b:

$$b = \sqrt{\frac{2F d_1 d_2}{\pi L E_e\left(d_1 + d_2\right)}}$$

For a cylinder of diameter d_1 compressed between two flat rigid surfaces (E_2 is infinite), the displacement of the center of the cylinder relative to one of the surfaces is:

$$\delta_{cyl} = \frac{2F}{\pi L E_e}\left[\ln\left(\frac{2d_1}{b}\right) - \frac{1}{2}\right]$$

For an elastic cylinder on an elastic flat plate, this equation diverges, which is a peculiar result of the plane stress theory used. To determine the displacement of the center of a cylinder with respect to a point at distance d_o below the surface, a two-part solution is used. The first part is the displacement due to the deformation of the cylinder as given above. The second part is for the deformation of the elastic flat plate as a rigid cylinder is pressed into it, whereby Saint-Venant, a reasonable assumption is $d_o = 3d_1$:

$$\delta_{plate} = \frac{2F}{\pi L E_e}\left[\ln\left(\frac{2d_0}{b}\right) - \frac{\nu}{2\left(1-\nu\right)}\right]$$

The deflection of the cylinder center with respect to a point three cylinder diameters below the surface is $\delta_{cyl} + \delta_{plate}$. The contact pressure is:

$$q = \frac{2F}{\pi b L}$$

The maximum shear stress is 0.3q which occurs at a distance of 0.786b below the surface. This is why many failures seem to result in a delamination of material. The state of stress below the contact surface is given by:

$$\sigma_x = -2q\nu\left[\sqrt{1+\frac{z^2}{b^2}} - \frac{z}{b}\right] \qquad \sigma_y = -q\left[\left(2-\frac{b^2}{b^2+z^2}\right)\sqrt{1+\frac{z^2}{b^2}} - \frac{2z}{b}\right]$$

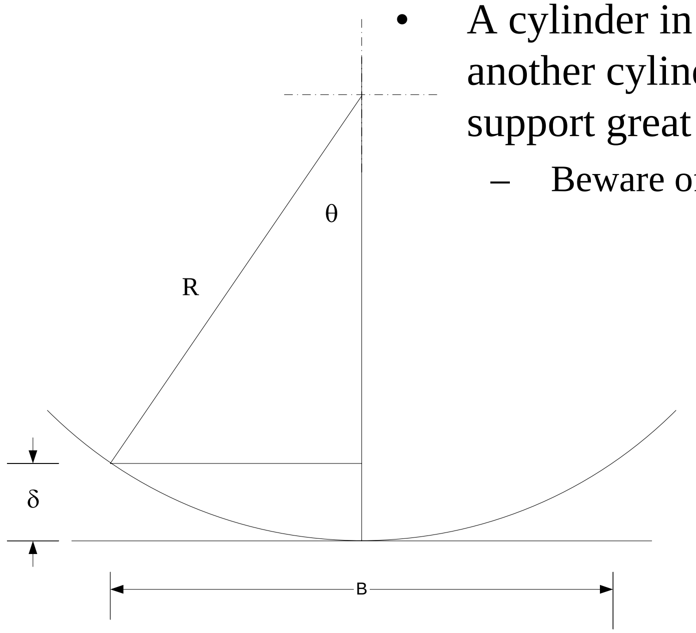
$$\sigma_z = -qb\sqrt{\frac{1}{b^2+z^2}} \qquad \tau_{yx} = \frac{\sigma_y - \sigma_x}{2} \qquad \tau_{zx} = \frac{\sigma_z - \sigma_x}{2} \qquad \tau_{zy} = \frac{\sigma_z - \sigma_y}{2}$$

The spreadsheet *Hertz_Line_Contact.xls can be used to easily evaluate these equations.* In practise, it is very difficult to achieve line contact between surfaces with more than one cylinder at a time because the line contact is so rigid and deformations so small compared to surface flatness and misalignment errors. To prevent edge, loading barrel-shaped rollers are often used in applications such as roller bearings.

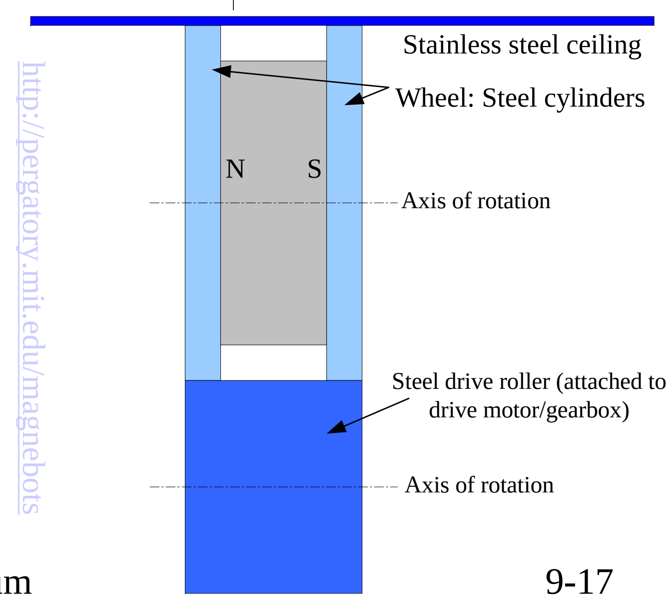
It is wise to create a design where an object that was to make line contact instead has a very large major radius and thus makes point contact even when there is misalignment. Check your design and imagine how contacts change with misalignment.

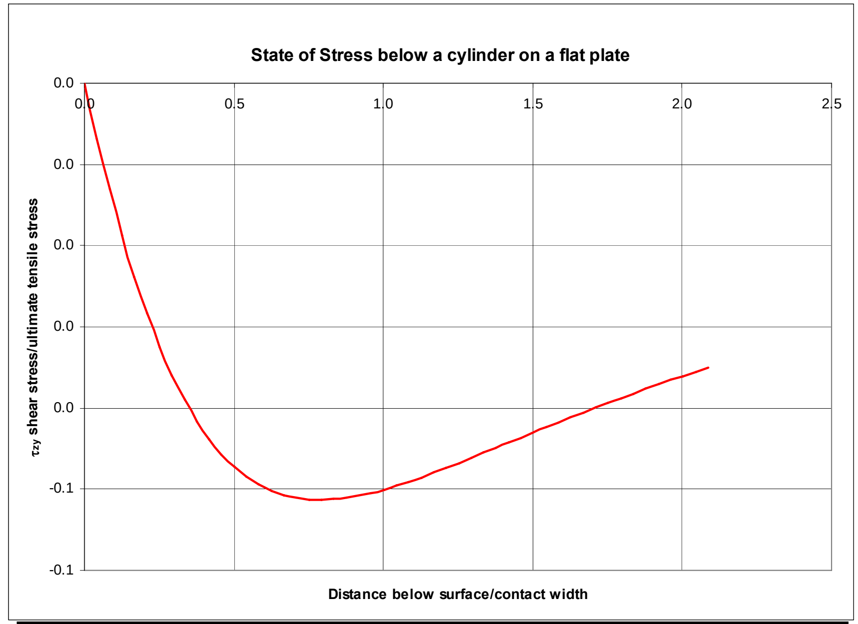
1. See A. Slocum, Precision Machine Design, Society of Manufacturing Engineers, Dearborn, MI, 1996, pp 236-237
2. See J. Lubkin "Contact Problems," in Handbook of Engineering Mechanics, W. Flugge (ed.), McGraw-Hill Book Co., New York, 1962; and T. Harris, Rolling Bearing Analysis, John Wiley & Sons, Inc., New York, 1991.
3. This makes rolling cylinders extremely important machine elements; however, the stress concentrations associated with the edges of the cylinders requires them to be generously rounded.

Hertz Contact: *Line Contact*



- A cylinder in contact with another cylinder or a plate can support great forces
  - Beware of edge loading!





Hertz_contact_line.xls	
To determine Hertz contact stress between two cylinders	
By Alex Slocum, last modified 2/10/2004 by Alex Slocum	
Enters numbers in **BOLD**, Results in **RED**	
Smaller cylinder 1 diameter, d_1 (mm)	**10**
Larger cylinder 2 (or flat plane) diameter, d_2 (mm)	**100**
Length, L (mm)	**10**
Applied load, F (N)	**8,184**
Elastic modulus Eone (N/mm^2)	**2.00E+05**
Elastic modulus Etwo (N/mm^2)	**2.00E+05**
Poisson's ratio vone	**0.29**
Poisson's ratio vtwo	**0.29**
Ultimate tensile stress, sigult (N/mm^2)	**1500**
Depth below contact surface for evaluating deflection, do	**300**
Rectangular contact zone width, 2b (mm)	**0.42**
Contact pressure, qcyl (N/mm^2)	**2502**
Deflection motion of d_1 center, defl_1 (mm)	**0.0104**
Deflection motion of d_2 center, defl_2 (mm)	**0.013159**
Total relative displacement of the cylinder's centers, dcyls (mm)	**0.0236**
Stress factor: Must be less than 1	
Maximum shear stress/(ultimate tensile/2)	**1.00**
Manufacturing issues	
Surface roughness, Ra (mm)	**0.005**
Potential induced contact width, Bra (mm)	**0.4**

Kinematic Couplings

Kinematic couplings are exact constraint design couplings because they use six contact points to locate one component with respect to another. They have long been known to provide an economical and dependable method for attaining high repeatability in fixtures[1].

A *3-2-1 coupling* is a form of kinematic coupling, where the three points can be established between 3 point contacts and a surface, then two point contacts against another surface, and then one point against a third surface. Typically the surfaces are planes and are perpendicular to each other. However, in this sequence, Hertz contact deformations act as physical detents and repeatability is typically limited to about 10 µm. When a ball rests in a concave tetrahedron, another ball rests in V-groove, and third ball rests on a flat plate, and the balls or hemispheres, are attached to one body, and the tetrahedron, groove, and flat are located on the other body, the coupling is referred to as a *Kelvin Coupling* (or *Kelvin Clamp*) after Lord Kelvin who favored this design. The primary advantage of this non-symmetric design is that its instant center of rotation is always located at the center of the tetrahedron contact points. However, it suffer from the 3-2-1 Hertz contact problem if same-diameter balls are used. James Clerk Maxwell, on the other hand, preferrd the symmetry and ease of manufacture of three-groove couplings.

Kinematic couplings are deterministic because they only make contact at a number of points equal to the number of degrees of freedom that are to be restrained. This makes performance predictable and helps to reduce design and manufacturing costs[2]. On the other hand, contact stresses are often very high and no lubrication layer remains between the elements that are in point contact. For high-cycle applications it is advantageous to have the contact surfaces made from corrosion-resistant materials (e.g., stainless steels, carbides, or ceramic materials). When non-stainless steel components are used, one must be wary of fretting at the contact interfaces, so steel couplings should only be used for low-cycle applications.

1. J. C. Maxwell, "General Considerations Concerning Scientific Apparatus," in The Scientific Papers of J. C. Maxwell, Vol. II, W.D. Niven (ed), Cambridge University Press, London, 1890, pp. 507-508.
2. See A. Slocum, "Kinematic Couplings for Precision Fixturing - Part 1: Formulation of Design Parameters," Precis. Eng., Vol. 10, No. 2, 1988, pp. 85-91; and A. Slocum "Design of Three-Groove Kinematic Couplings," Precision Eng., Vol. 14, No. 2, April 1992, pp 67-76.

Tests on a heavily loaded (80% of allowable contact stress) steel ball/steel groove system have shown that sub-micron repeatability can be attained;. However, with every cycle of use, the repeatability worsened until an overall repeatability on the order of ten microns was reached after several hundred cycles[3]. At this point, fret marks were observed at the contact points. Tests on a heavily loaded (80% of allowable contact stress) silicon nitride/steel groove system have shown that 50 nm repeatability could be attained over a range of a few dozen cycles, and that with continued use the overall repeatability asymptotically approached the surface finish of the grooves. An examination of the contact points showed burnishing effects, but once the coupling had worn in, submicron and better repeatability was obtained. The tests also showed that with the use of polished corrosion-resistant (preferably ceramic) surfaces, a heavily loaded kinematic coupling can easily achieve submicron repeatability with little or no wear-in required. Regretfully, too many designers still consider kinematic couplings to be useful only for instrument or metrology applications.

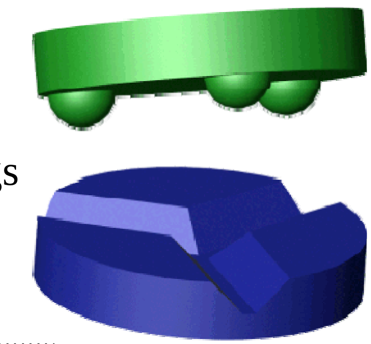
How can kinematic couplings be useful in a robot design competition? Many robot design contests have machines start at one end of a contest playing field, but the contestants do not know on which side of the field they will be on until just before the contest. If their machines include offensive modules, they may have to rapidly reconfigure their machines. Three-groove kinematic couplings are very easy to build, and for such applications, the balls and grooves can be made from plastic, and the repeatability can easily be on the order of fractions of a millimeter (depending on the repeatability of the preload). To enable a module to be rotated 180 degrees, six grooves, spaced 60 degrees apart on a circle, can be made. A magnet can then be used to create a highly repeatable preload on the coupling.

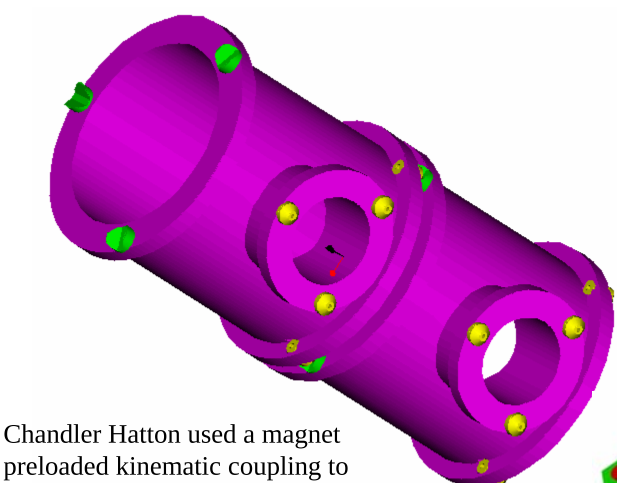
Do you have modules that need to be placed according to which side of the contest playing field your machine starts? How are you planning to rapidly re-orienting your modules when you may only have a few minutes between rounds? How can you ensure that your method will be highly repeatable?

3. A. Slocum and A. Donmez, "Kinematic Couplings for Precision Fixturing - Part 2: Experimental Determination of Repeatability and Stiffness," Precis. Eng., Vol. 10, No. 3, 1988, pp. 115-122

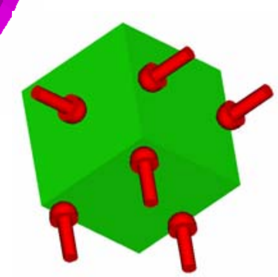
Kinematic Couplings

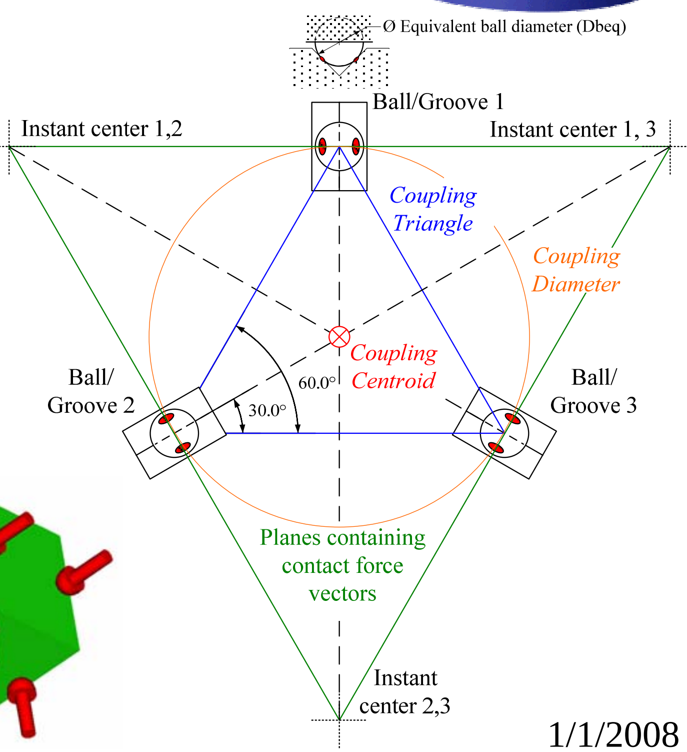
- When a component is constrained by a number of points equal to the number of degrees of freedom, it is said to be *exactly constrained*
  - Kinematics is the study of motion, assuming bodies are rigid, so when a design is “kinematic” it means it is exactly constrained, and geometric equations can be written to describe its motion
- Kinematic Couplings are couplings that exactly constrain components
  - They are not stable unless ALL six contact points are engaged
  - There are no intermediate stability configurations like those in 3-2-1 couplings
  - They can provide repeatability on the order of parts’ surface finish
  - ¼ micron repeatability is common
- Managing the Hertz contact stresses!





Chandler Hatton used a magnet preloaded kinematic coupling to enable her machine’s module to be easily flipped depending which side of the table on which she had to setup





Kinematic Couplings: *2D*

A common problem faced in many applications is how to repeatedly fixture (locate) a 2D object. For example, you have made small silicon devices, and you need to stack up multiple layers before you bond them together, yet you need alignment to within a few microns. The die saw that cuts the chips (die) from the wafer leaves inexact edges. What do you do?

The first thing to note are the sensitive directions and precision features that must be aligned. This allows you to establish reference features. In the case of silicon Micro Electro Mechanical Systems (MEMS), the features are created by a pattern that is formed by photolithography and etching of the silicon. Thus the process itself can be used to create reference edges.

Given that the devices are very flat, it is reasonable to assume that when you place the die on a flat surface, that one degree of freedom (normal to the surface) is defined by the contact interface, which in turn also defines the pitch and roll of the die. However, the two translational degrees of freedom parallel to the surface and yaw (rotation) about an axis normal to the surface need to be defined.

Consider the problem of silicon wafer fixturing. If someone says that they have a wafer of diameter D with a flatness (bow) Δ, and they want to know the flatness δ of a square chip (die) of size w cut from the wafer, what do you do? To the first order, the process is to determine the radius of curvature ρ of the large wafer, and then essentially reverse this process to find the flatness δ of the chip (die):

$$\rho^2 = \left(\frac{D}{2}\right)^2 + (\rho - \Delta)^2$$

$$\rho = \frac{D^2 + 4\Delta^2}{8\Delta} \approx \frac{D^2}{8\Delta} \quad \text{for } \Delta << D$$

$$\frac{D^2}{8\Delta} = \frac{w^2}{8\delta}$$

$$\delta = \frac{w^2}{D^2}\Delta$$

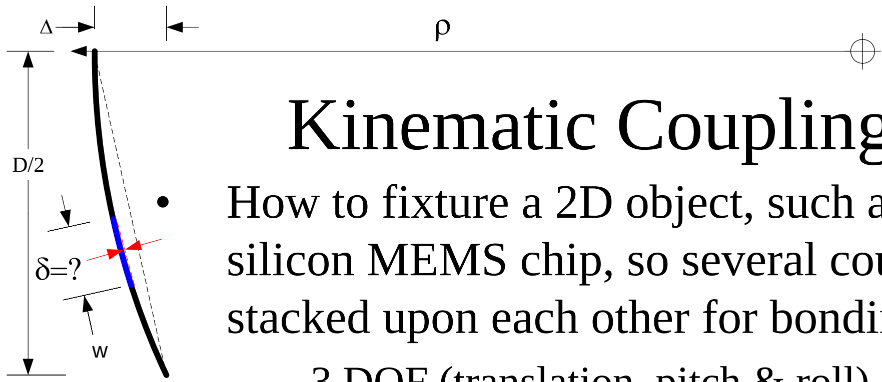
Some typical numbers obtained from *diebow.xls* show that the chip is indeed essentially flat:

D (mm) =	**300**	Δ (μm) =	**50**	ρ (m) =	**225**
w (mm) =	**20**	δ (μm) =	**0.222**		

Because there are three degrees of freedom to define, at least three contact points must be established. But where should they be placed? There are four candidate locations. Dotted lines drawn normal to the contact points and their intersections represent the instant centers of rotation for the body (see page 4-16). If you study the figures carefully and take note of where the forces are applied with respect to the instant centers, you can see that in configuration 1, the preload force is applied between and below the instant centers. Configuration 2 has the preload force applied between but above the instant centers. Configuration 3 has the preload applied below the instant centers, *but* it is in line with one of them, which makes this design therefore marginally stable in one direction. Configuration 4 has the preload force above one of the instant centers and also directly in line with one of them... To feel what is more stable, make the bench level experiment (BLE) and play with it! How would you model it analytically in order to optimize?

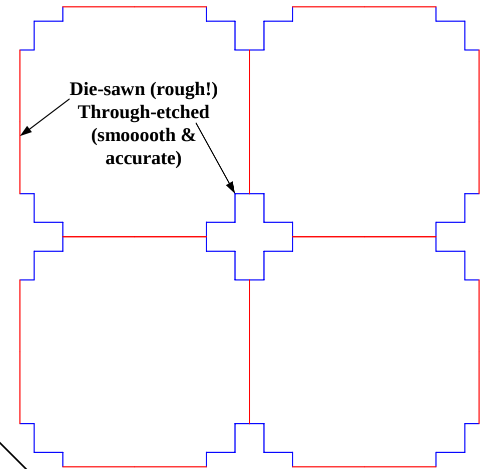
The set of images shown can be easily made into a *Bench Level Experiment* so you can feel the stability of the various configurations. The red arrows indicate unstable directions when forces or torques are applied. The blue arrows indicate stable directions. The green gravity arrow indicates the preload direction. As long as the preload is not overcome, configurations 1, 2, and 3 are OK, with 1 (blue!) being the best. Why? There is also the issue of friction between the die and the fixture, and between die. Vibration can be used to help the die settle. Does this affect your opinion as to which is the most stable configuration?

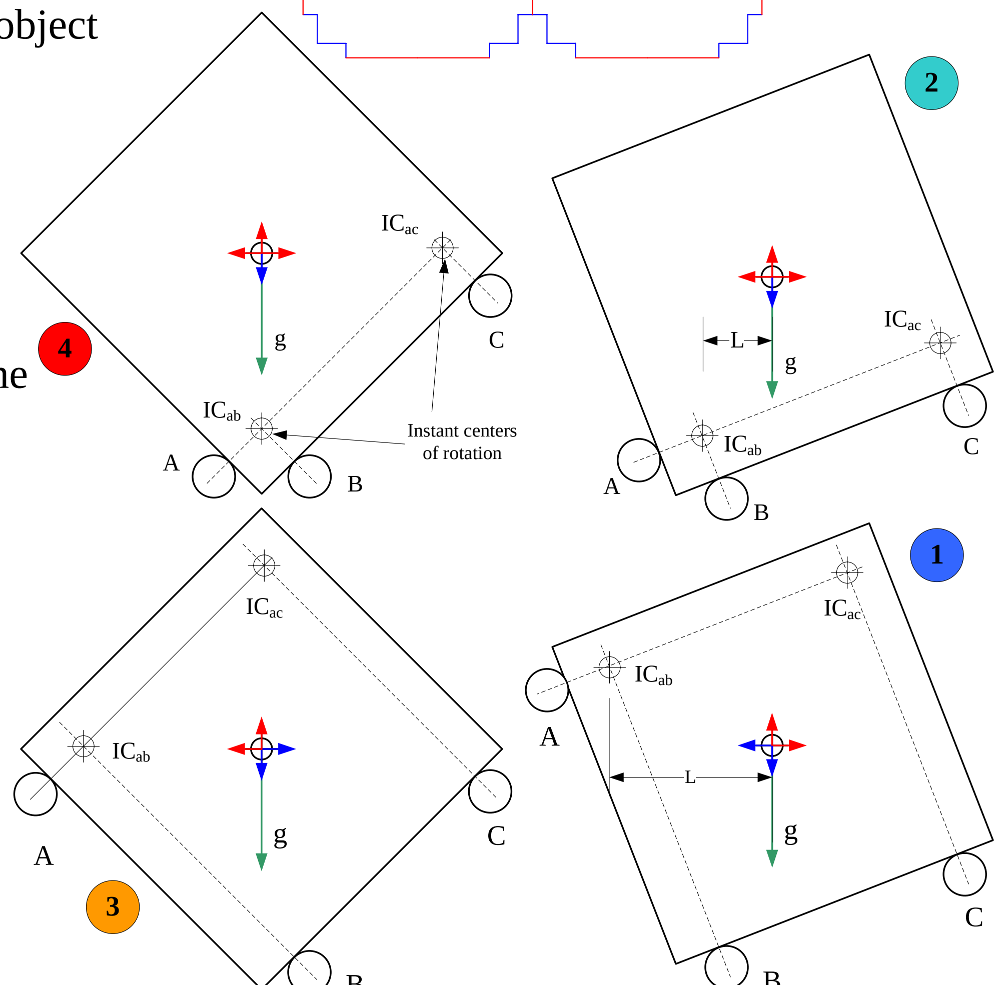
Make a BLE and experiment with it. Can configuration 3 be rotated to provide good stability? Does it not just become then configuration 1? Remember reciprocity (see page 3-14)? If uncomfortability occurs in one position, literally rotate...

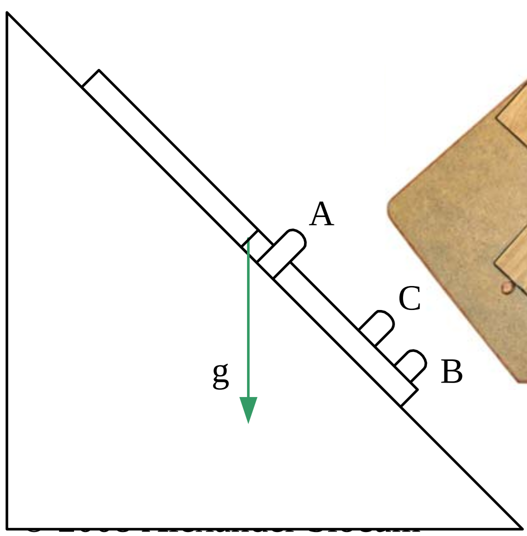


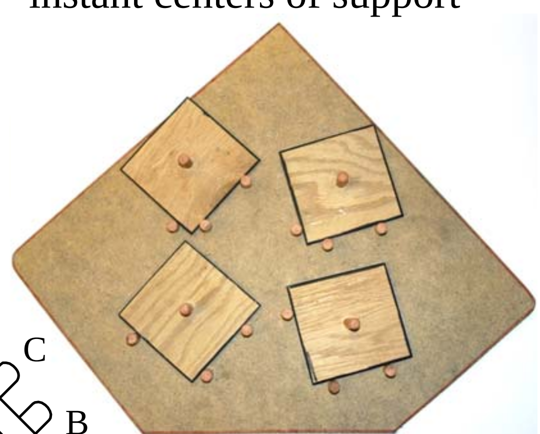
Kinematic Couplings: *2D*

- How to fixture a 2D object, such as a silicon MEMS chip, so several could be stacked upon each other for bonding?
  - 3 DOF (translation, pitch & roll) are defined by the plane on which the object rests
  - 3 DOF (2 translations and yaw) must be established
    - 3 contact points are needed
    - Gravity provides preload
    - Align the gravity vector wrt the instant centers of support









Wooden bench
level experiment

Kinematic Couplings: *3D*

A 3D kinematic coupling is one that has six contact points that define the position and orientation of one component with respect to another. The design equations for 3D kinematic couplings are well established[1], and are incorporated into a spreadsheet *Kinematic_Coupling.xls*. The primary design parameters include the effective ball radii, the diameter of the circle on which the ball centers lie (the coupling diameter), the coupling materials, and the preload force. The performance of the coupling is then affected by variations in the preload force, and the applied loads. The sensitivity to these parameters can be studied using the spreadsheet.

The load capacity is a direct function of the Hertz contact stress, and the contact stress is a very strong function of the shape of the contact interface. Consider the following types of ball-groove contact shape options:[2]

- *25 mm diameter stainless steel half-sphere on 25 mm diameter cylinders*
  F_{max} = 111 N
  Vertical deflection = 3.2 µm
  Contact ellipse major diameter = 0.425 mm , minor diameter = 0.269 mm
- *25 mm diameter stainless steel half-sphere in a Vee*
  F_{max} = 229 N
  Vertical deflection = 4.7 µm
  Contact ellipse major diameter = 0.488 mm , minor diameter = 0.488 mm
- *25 mm contact diameter x 125 mm radius crowned cone in a Vee*
  F_{max} = 1106 N
  Vertical deflection = 11 µm
  Contact ellipse major diameter = 2.695 mm , minor diameter = 0.603 mm
- *250 mm diameter stainless steel half-sphere in a Vee*
  F_{max} = 16160 N
  Vertical deflection = 47 µm
  Contact ellipse major diameter = 4.878 mm , minor diameter = 4.878 mm

1. A. Slocum "Design of Three-Groove Kinematic Couplings," Precision Eng., Vol. 14, No. 2, April 1992, pp 67-76.
2. Based on maximum contact pressure q = 1.3 GPa, and both components having a modulus of elasticity E=193 GPa

Preload is the force applied to the coupling to hold it together, and it is one of the most important parameters that affects repeatability of the coupling. Preload establishes the initial stiffness, given that the Hertzian contact stiffness is nonlinear. Remember, Hertzian deflection is proportional to applied force to the 2/3rds power. To get good stiffness from the coupling, the preload must be high, repeatable, and must NOT deform the rest of the structure. For heavy duty applications, such as fixtures for machine tools, this can be accomplished by preloading through the center of the kinematic elements with bolts that compress springs. For lightly loaded systems, such as instruments, magnets can be used as preload elements.[3]

Materials also play a major role in the performance of the coupling. They not only dictate the maximum stress, and hence the loads that can be applied, they must also have low relative friction between each other, and they must not be subject to corrosion. For super precision high load applications, silicon nitride or silicon carbide are the best materials for the spherical parts of the coupling to minimize the coefficient of friction and to prevent corrosion at the Hertz contact interface.

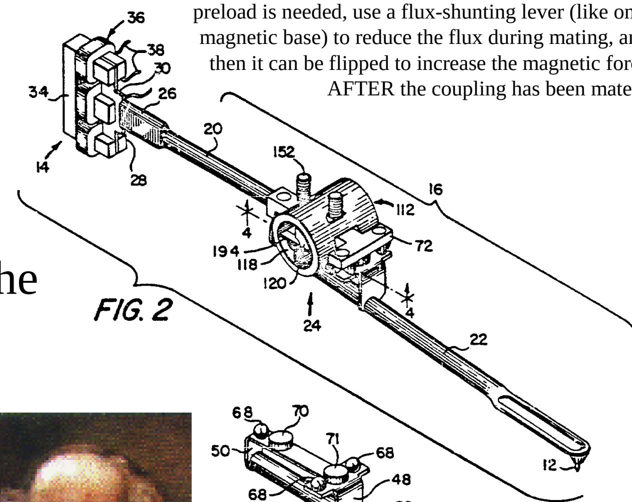
There are many different variations on the kinematic coupling theme[4] and they can be used for more than just routine relative positioning of objects, such as fluid couplings.[5] With the spreadsheet, *Kinematic_Coupling.xls*, the design engineer can easily play "what-if" design games to arrive at a theoretically workable kinematic coupling for virtually any application.

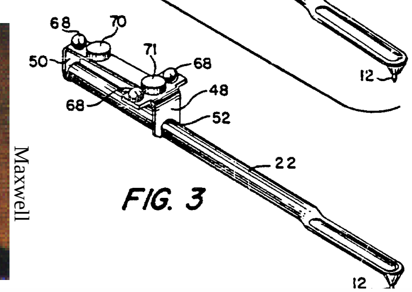
Where might you use a kinematic coupling in your machine? Can use us a magnet to preload it? Never use a component just for the sake of using that component, but if you need fast repeatability, a kinematic coupling is hard to beat!

3. See for example kinematic coupling used to hold probe arm to an electronic indicator (US Patent 4,574,625, assigned to Federal Products Corp.).
4. www.kinematiccouplings.org
5. A. Slocum, "Kinematic Coupling Fluid Couplings and Method", US Patent 5,683,118.

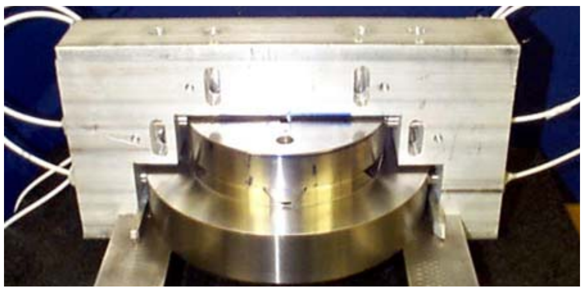
Kinematic Couplings: *3D*

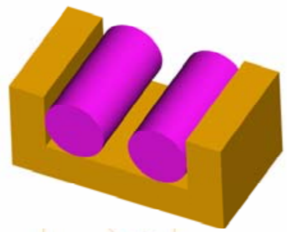
See US patent 4,574,625. NOTE magnet preload needs to be applied gently else the sudden THWAP (impact) of contacts drawn together can cause subsurface failure or surface indentation (*Brinelling*); hence if a LOT of preload is needed, use a flux-shunting lever (like on a magnetic base) to reduce the flux during mating, and then it can be flipped to increase the magnetic force AFTER the coupling has been mated!



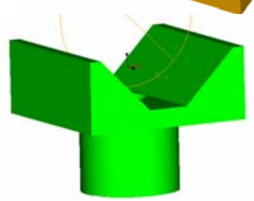


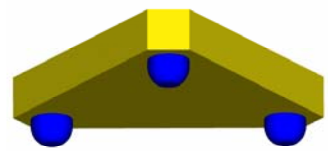
- James Clerk Maxwell (1831-1879) liked the three-grooves
  - Symmetry good for manufacture, dynamic stability
  - Easy to obtain very high load capacity
- William Thomson (later Lord Kelvin) (1824 - 1907) liked the ball-groove-tetrahedron
  - More intuitive, and applicable to non-planar designs

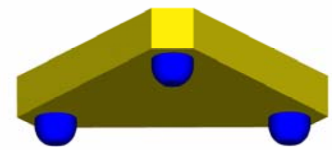


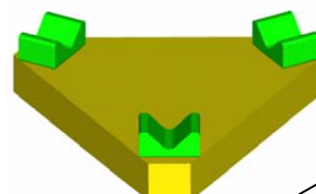


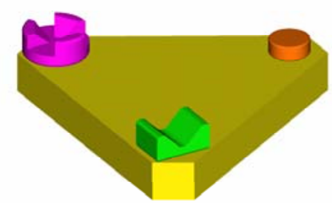
Maxwell

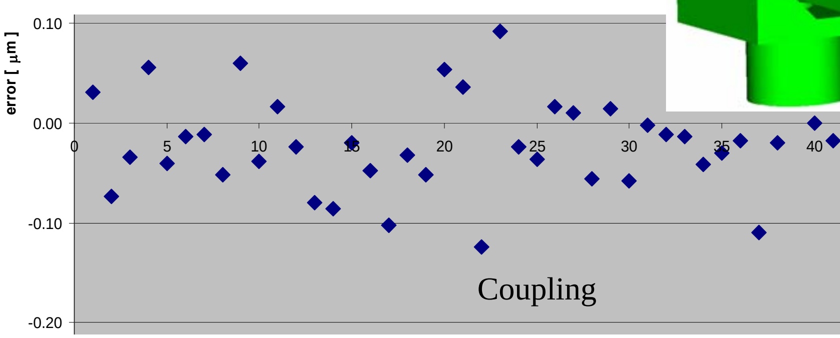


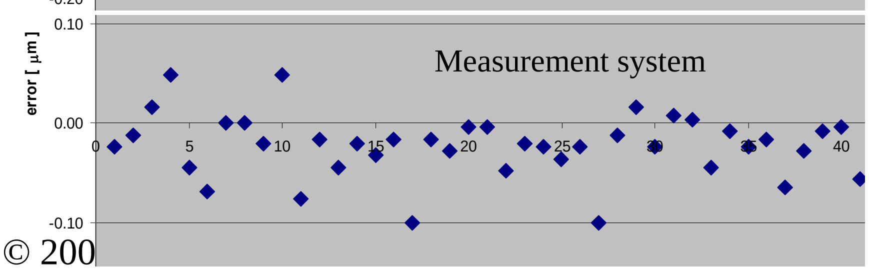


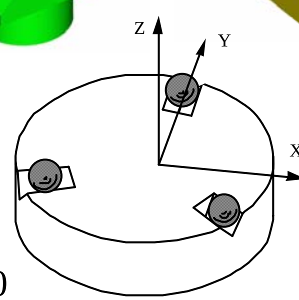


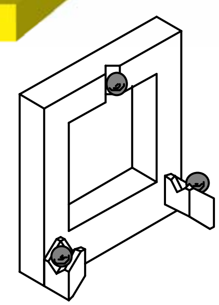












Kelvin

Kinematic Couplings: *Three-Grooves*

Three-groove kinematic couplings are particularly easy to make because the grooves and spherical contact elements, typically hemispheres, can be made all at once and then simply bolted in place. With three grooves, the question naturally arises as to what is the best orientation for the grooves. Mathematically, to guarantee that the coupling will be stable, James Clerk Maxwell stated the following:[1]

When an instrument is intended to stand in a definite position on a fixed base it must have six bearings, so arranged that if one of the bearings were removed the direction in which the corresponding point of the instrument that would be left free to move by the other bearings must be as nearly as possible normal to the tangent plane at the bearing. This condition implies that, of the normals to the tangent planes at the bearings, no two coincide; no three are in one plane, and either meet in a point or are parallel; no four are in one plane, or meet in a point, or are parallel, or, more generally, belong to the same system of generators of an hyperboloid of one sheet. The conditions for five normals and for six are more complicated."

With respect to practical implementation of the theoretical requirement for stability, for precision three-groove kinematic couplings, stability, and good overall stiffness will be obtained if the normals to the plane of the contact force vectors bisect the angles of the triangle formed by lines joining the centers of the hemispheres (e.g., balls) that lie in the grooves.

Furthermore, for balanced stiffness in all directions, the contact force vectors should intersect the plane of coupling action at an angle of 45 degrees. Note that the angle bisectors intersect at a point that is also the center of the circle that can be inscribed in the coupling triangle. This point is referred to as the *coupling centroid* and it is only coincident with the coupling triangle's centroid when the coupling triangle is an equilateral triangle. Fortunately designers of precision kinematic couplings are not faced with the generic grasp-a-potato problem faced by researchers in robotics. Indeed, any three-groove kinematic coupling's stability can be quickly assessed by examining the intersections of the planes that contain the contact force vectors (two per ball/groove interface). For stability, the planes must form a triangle as illustrated. In terms of mechanism design, this means that the instant center for each ball/groove interface must lie outside the coupling triangle.

Two forms of three-groove couplings are illustrated. *Planar couplings* are often found in metrology applications. They can also be used in the manufacture of precision parts. For example, a planar three-groove coupling can be used to hold a grinding fixture on a profile grinder. A matching three-groove plate on a CMM allows the grinding fixture to be transferred to the CMM with the part. The part can be measured and then placed back onto the grinder so the errors can be corrected.

To minimize Abbe errors in some applications, *vertical couplings* can be designed where the preload is obtained with a clamping mechanism or by gravity acting on a mass held by a cantilevered arm. An industrial example would be in a machine where a module must be precisely located with respect to the rest of the machine, yet for maintenance purposes, it has to be easily disconnected and rolled away to gain access to the interior of the machine.

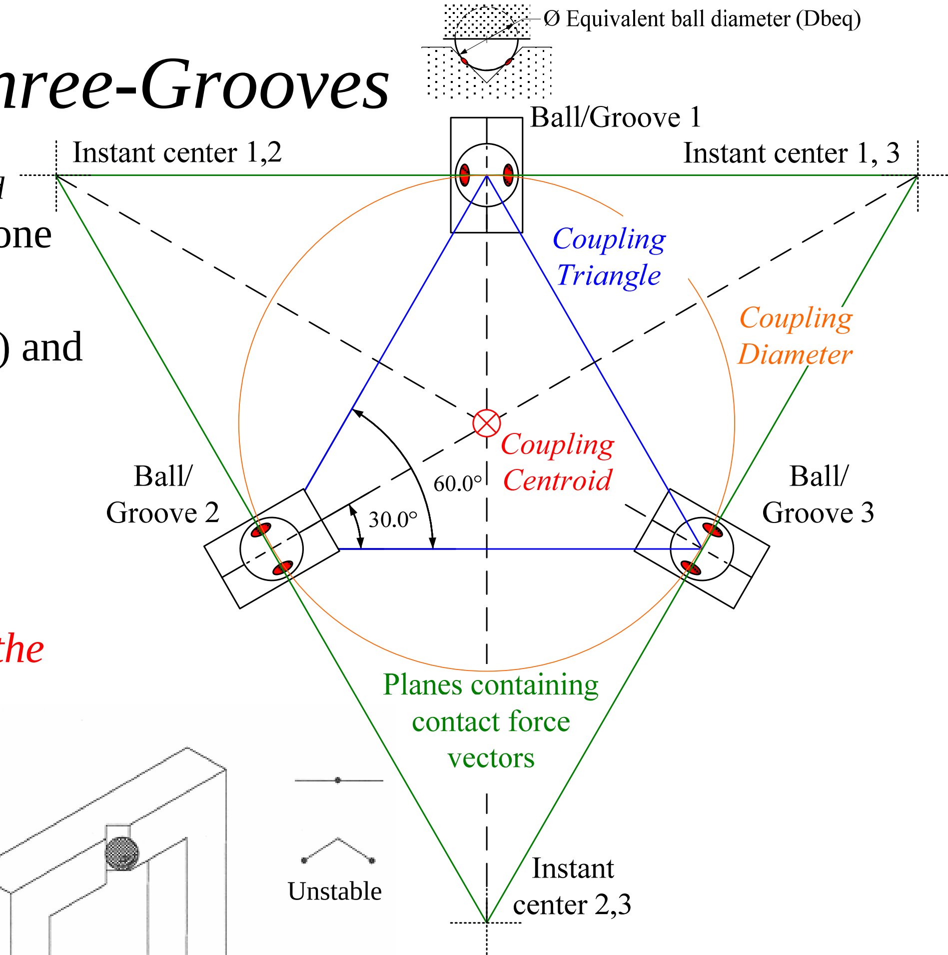
As a design exercise, imagine the following: For a robot contest, three-groove kinematic couplings can be easily made by milling a long V slot in a block of material, drilling and tapping threads in the center of the blocks (through the vees), and then slicing the milled block into three pieces. These vee's can then be easily bolted down to one component. The hemispheres can be made on a lathe by rounding the end of a bolt. The spherical end need only be spherical at the 45 degrees position. The bolt can then be threaded through a hole in the second component, and a nut tightened on the bolt locks it rigidly in place. Now, how will you preload it? Can you somehow mount a magnet in the middle of one component that is attracted to a piece of steel attached to the other component? Can you sketch the picture that should have formed in your mind?

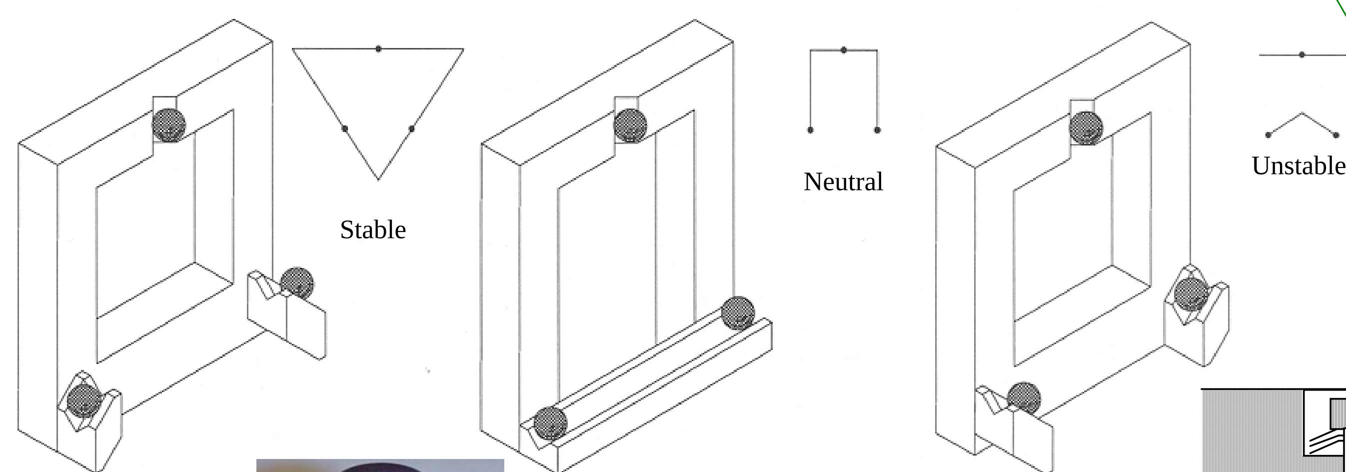
---

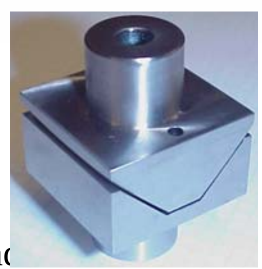
1. In a footnote to this discussion, Maxwell references Sir Robert Ball's pioneering work in *screw theory.* Screw theory asserts that the motion of any system can be represented by a combination of a finite number of screws of varying pitch that are connected in a particular manner. This concept is well illustrated for a plethora of mechanisms by Phillips (J. Phillips, Freedom in Machinery, Vol. I, Cambridge University Press, London, 1982, p 90). Ball's work on screws spanned the latter half of the 19th century and a detailed summary of his work on screw theory was published in 1900 (R. S. Ball, A Treatise on the Theory of Screws, Cambridge University Press, London, 1900.). Ball's treatise describes the theory of screws in elegant, yet easily comprehensible linguistic and mathematical terms. Screw theory is an elegant and powerful tool for analyzing the motion of rigid bodies in contact, but it is not always easy to apply.

Kinematic Couplings: *Three-Grooves*

- For long life, Hertz contact pressure $q < \sigma_{yield}$
  - Contact area center should not be closer than one contact ellipse diameter from groove edge
  - Materials must be non-galling (no AL on AL!) and non-fretting
  - Preload to keep coupling from tipping
- *Ideally align the grooves with the coupling triangle's angle bisectors*
  - *The coupling centroid will NOT always be at the coupling triangle centroid!*

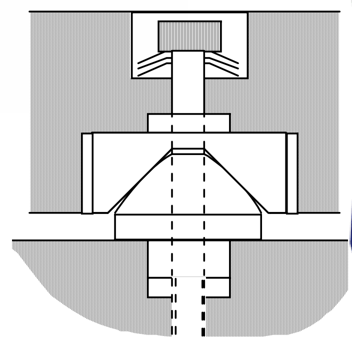


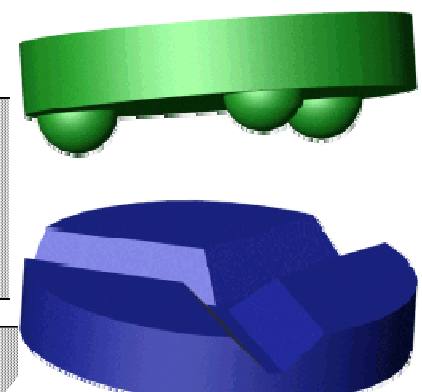




See www.kinematiccouplings.org for spreadsheets, articles, and suppliers

Don't forget the potential of using magnets for light load applications!





Kinematic Couplings: *Three-Groove Design*

Three-groove kinematic couplings are commonly used to create repeatable interfaces for machines and instruments. Modular components are available off-the-shelf[1], but can also be made custom when needed. The spreadsheet *Kinematic_Coupling_3Groove_Design.xls* allows designers to play what-if scenarios. There is no lack of resources for designers who want to design a three-groove kinematic coupling.

The spreadsheet is written to enable a designer to assume a standard common shape for kinematic couplings, where the grooves are spaced 120 degrees apart on a circle which is referred to as the *coupling diameter.* It is also assumed that the " balls" that engage the grooves make contact at a 45 degree angle. If the design engineer needs to make a kinematic coupling where the grooves are not 120 degrees apart, then the direction cosines through the contact points need to be defined. In this case, the designer can modify the spreadsheet accordingly for each groove. Check *www.kinematiccouplings.org* to see if a more generic updated form of the spreadsheet has been posted. The inputs are named, and after the value is a brief description of the design parameter. In more detail:

- ***Dbeq (mm)*** **=** the diameter in millimeters of a ball that could fit in the groove. This is important because the plane through the virtual ball's centers is assumed to be the plane of the coupling. As discussed before, a *canoe ball* or a *crowned cone* could be used to increase the load capacity by entering in the actual radii of curvature at the contact points.
- ***Rbminor (mm)*** **=** the minor radius of contact of the "ball"
- ***Rbmajor (mm)*** **=** the major radius of contact of the "ball"
- ***Rgroove (mm)*** **=** the radius of curvature of the groove at the contact point. For a Vee groove, enter in a huge number, like 109 to indicate a flat plane. If a gothic arch groove is used, it should be no more conformal that about 1.5 times the radius of the ball, else edge loading is likely to occur. Remember, for a conformal groove, the radius of curvature is negative. hence one would enter, for example, Rgroove = -1.5Rbmajor.

1. See www.kinematiccouplings.org which provides many references and design ideas. Components are commercially available, for example from Bal-Tec, Inc., 1550 E. Slauson Avenue, Los Angeles, CA 90011, (800) 322-5832, http://www.precisionballs.com/

- ***Costheta*** **=** the angle between the major diameters of the contact surfaces. this value has always been TRUE
- ***Dcoupling (m)*** **=** the diameter of the *coupling circle* on which the centers of the balls' centers lie. the bigger the coupling diameter, the greater the moment load capacity of the coupling.
- ***Fpreload (N)*** **=** the preload applied over each ball. Because the Z direction is pointing upwards from the coupling plane, the preload force is negative. Even if the preload is centrally applied, enter the preload force over each ball. it is assumed that all balls have the same initial preload.
- ***Xerr (mm)*** **=** the X location at which the deflections of the coupling are to be determined with respect to the coupling coordinate system located at the center of the coupling circle.
- ***Yerr (mm)*** **=** the Y location at which the deflections of the coupling are to be determined.
- ***Zerr (mm)*** **=** the Z location at which the deflections of the coupling are to be determined.
- ***Matlabball*** **=** material label for the ball. Select a standard material, or enter in your own values (to the right).
- ***Matlabgroove*** **=** material label for the ball. Select a standard material, or enter in your own values (to the right).
- ***Min. yield strength*** reminds the designer what stress on which the stress ratio will be based.
- ***Largest contact ellipse major diameter (mm)*** from cells below which report the contact ellipse dimension at each contact point. This is important to ensure that the contact zone does not extend to the edge of the Vee groove. It should be one characteristic dimension away.
- ***Largest contact ellipse minor diameter (mm)*** similar to major diameter.
- ***Largest contact stress ratio*** shows if the subsurface shear stress due to Hertz contact > material's maximum allowable shear ($\sigma_{yield}/2$).
- ***RMS applied force (N)*** shows the root mean square of the contact forces.
- ***RMS stiffness (N/micron)*** shows the root mean square of the δx, δy, δz deflections at Xerr, Yerr, Zerr.
- ***FLx (N)*** **=** the applied X direction force. All all the force components are applied at XL, YL, ZL.
- ***FLy (N)*** **=** the applied Y direction force at XL, YL, ZL.
- ***FLz (N)*** **=** the applied Z direction force at XL, YL, ZL.

Enjoy!

Kinematic Couplings: *Three-Groove Design*

Kinematic_Coupling_3Groove_Design.xls						
To design three groove kinematic couplings						
Written by Alex Slocum. Last modified 10/27/2004 by Alex Slocum						
Metric units only! Enters numbers in **BOLD**, Results in **RED**					**Material properties**	
Standard 120 degree equal size groove coupling? (contact forces are inclined at 45 to the XY plane. For non standard designs, enter geometry after results section)				**TRUE**	*User defined material*	***aluminum***
					Yield stress	
System geometry (XY plane is assumed to contain the ball centers)					plastic	**3.45E+07**
Dbeq (mm) =	**5**	Equivalent diameter ball to contact the groove at the same points			RC 62 Steel	**1.72E+09**
Rbminor (mm) =	**2.5**	"Ball" minor radius			CARBIDE	**2.76E+09**
Rbmajor (mm) =	**2.5**	"Ball" major radius			user defined	**2.76E+08**
Rgroove (mm) =	**1.00E+06**	Groove radius (negative for a trough)			*Elastic modulus*	
Costheta =	**TRUE**	Is ball major radius along groove axis?			plastic	**2.07E+09**
Dcoupling (mm) =	**150**	Coupling diameter			RC 62 Steel	**2.04E+11**
Fpreload (N) =	**-100**	Preload force over each ball			CARBIDE	**3.10E+11**
Xerr (mm) =	**0.0**	X location of error reporting			user defined	**6.80E+10**
Yerr (mm) =	**0.0**	Y location of error reporting			*Poisson ratio*	
Zerr (mm) =	**0.0**	Z location of error reporting			plastic	**0.20**
Auto select material values (enter *other_4* to the right)					RC 62 Steel	**0.29**
Matlabball =	**1**	Enter 1 for plastic, 2 for steel, 3 for carbide, 4 for user defined, 5			CARBIDE	**0.30**
Matlabgroove =	**4**	where each ball and groove is defined individually			user defined	**0.29**
Min. yield strength (Pa, psi)		**3.45E+07**	**5,000**			
Largest contact ellipse major diameter (mm)		**0.831**				
Largest contact ellipse major diameter (mm)		**0.829**				
Largest contact stress ratio		**3.826**	Max Hertz shear stress/Material's max shear stress (tensile yield/2)			
RMS applied force F (N)	**17.32**					
RMS deflection at F (micron)	**2.238**					
RMS stiffness (N/micron)	**7.74**					
Applied force's Z,Y,Z values and coordinates				**Coupling centroid location**		
FLx (N) =	**10.00**	XL (mm) =	**0**	xc (mm)	**0.000**	
FLy (N) =	**10.00**	YL (mm) =	**0**	yc (mm)	**0.000**	
FLz (N) =	**10.00**	ZL (mm) =	**100**	zc (mm)	**0.000**	

Kinematic Couplings: *Compliant Mounts*[1]

Kinematic couplings have an inherent problem in that unless sufficient preload is applied, externally applied loads can cause them to tip. A countermeasure to this risk is to use a high preload, but this can create the need for expensive kinematic contact elements. Furthermore, for applications such as locating forming dies, where the process forces on the dies are huge, or locating mold components, where the surfaces have to seal against each other tightly, conventional kinematic couplings cannot be used.

There are two functional requirements. The first functional requirement is for locating. The resulting design parameter is then a kinematic coupling. The second functional requirement is for having the two part surfaces to touch. The resulting design parameter is to mount the kinematic coupling elements in such a way that they have a degree of freedom normal to the interface surfaces. The first FR has been discussed in detail. The second FR can be achieved through the use various types of linear bearings, which are described in great detail in Chapter 10.

Only one set of kinematic coupling elements needs to be mounted upon movable members. The motion needs to have minimal parasitic error motions (unwanted motions) so as to not lose precision as the mating surfaces come into contact. This means that in addition to not having any backlash (clearance) in the mechanism, the mechanism must have high lateral stiffness in the sensitive directions.[2] Perhaps the simplest way to accomplish these functions with minimal cost is to use a compliant mechanism. The compliant mechanism (flexures) can support rigid elements, such as the vees, or the function of the Vees can be incorporated into the flexural elements themselves.[3]

The blue-background drawings show how a solid model can be used to size the vee-grooves which can be made from bent sheet metal. For heavy duty applications, they would be made from spring steel. The outline of a small circle shows the diameter of a circle that would be tangent to the sides and bottom of the Vee; thus the balls (or hemispheres) used should have a slightly larger diameter. As shown, the small circle has a diameter of 21.49 mm, and if a 23 mm diameter hemisphere is selected, then after the kinematic coupling is seated, the component to which the balls is attached can travel 0.31 mm in the vertical direction before the bottom of the ball touches the bottom of the Vee. More clearance can be obtained by flattening the end of the ball.

The solid model shows an upper plate to be coupled to a lower plate using hemispheres and sheet-metal vees. The sheet metal vees are easily made, and they are attached to the base plate by sheet metal screws or rivets. This would be a typical application for a robot contest, where low-cost vees are sought, and greater vertical stiffness is obtained when the balls then bottom out in the vee-grooves. If the bottoms of the balls were flat, then the vertical load capacity and stiffness would be higher when they contacted the vee bottoms.

How much vertical travel can the sheet metal vees tolerate? Is it reasonable to install them, use the coupling, and just allow them to plastically deform? If they are bent beyond their elastic limit, when the load is released, some recovery will occur due to *elastic springback* (see page 8-5). However, the vertically compliant sheet-metal vees will not have high lateral stiffness. For this, the flexure supported vees, or the die-set supported hemispheres shown need to be used; however, such heavy duty systems would only be needed in industrial applications. Is this system not over constrained? It would appear to have too many points of contact given the original six plus three more after the system is preloaded and deflected into place. How can this be? The elastic averaging effect provided by the compliant vees enables "over-constraint" to provide large load capacity and high repeatability.

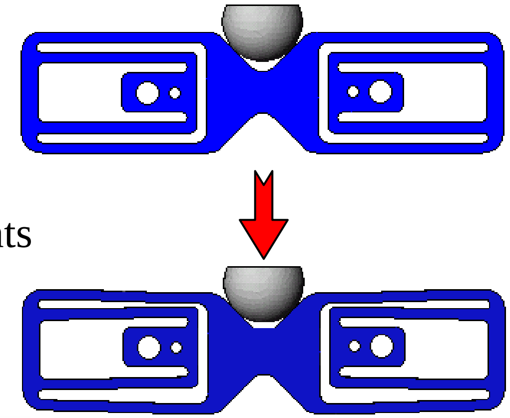
Also shown are drawings for aligning sand-cores used in metal casting. In this case, the compliance is provided by the hemispheres deforming sand vees. They only have to align the sand cores once so steel or wood balls can mate with sand grooves. Once aligned, the balls press into the sand to allow the cores' surfaces to mate.

Does the concept of quick-to-make sheet-metal vees and the ability to preload into a stable surface-to-surface condition make the use of kinematic couplings any more viable for your coupling needs? Manufacturing ingenuity is often the key to design success!

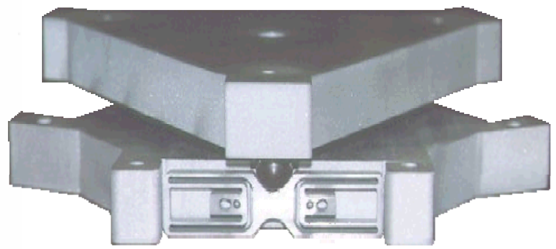
1. The Vees can also have compliance orthogonal to the contact point normal to reduce friction effects. This can greatly increase repeatability. See C. H. Schouten, P. C. J. N. Rosielle and P. H. J. Schellekens, "Design of a kinematic coupling for precision applications" Precision Engineering, Volume 20, Issue 1, January 1997, Pages 46-52
2. If some of these terms do not seem familiar to the reader, the reader should go back and review Topic 3, or check the index.
3. Two great functions in one machine element, this is perfect better components. See A. Slocum, D. Braunstein, L. Muller, "Flexural Kinematic Couplings", #5,678, 944

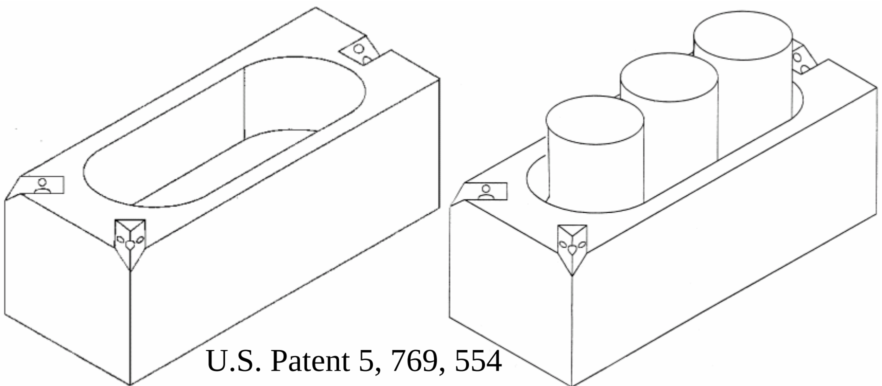
Kinematic Couplings: *Compliant Mounts*

- Allows a component to be kinematically located
  - Application of the preload force deflects the kinematic components until surface-to-surface contact occurs to resist tipping loads
- Many forms from simple sheet metal to flexure-based linkages
- Deformation can be elastic, or permanent
  - Even sand cores can be aligned

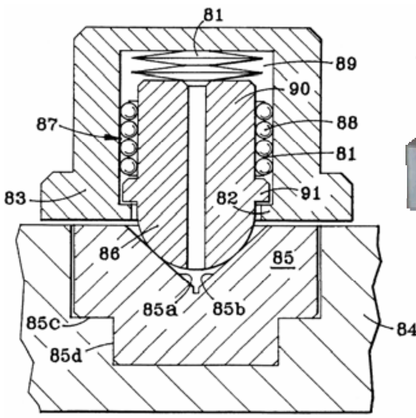


U.S. Patent 5, 678, 944

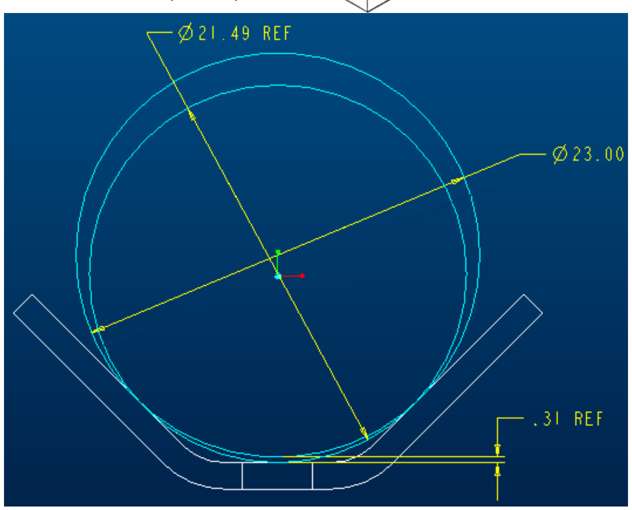


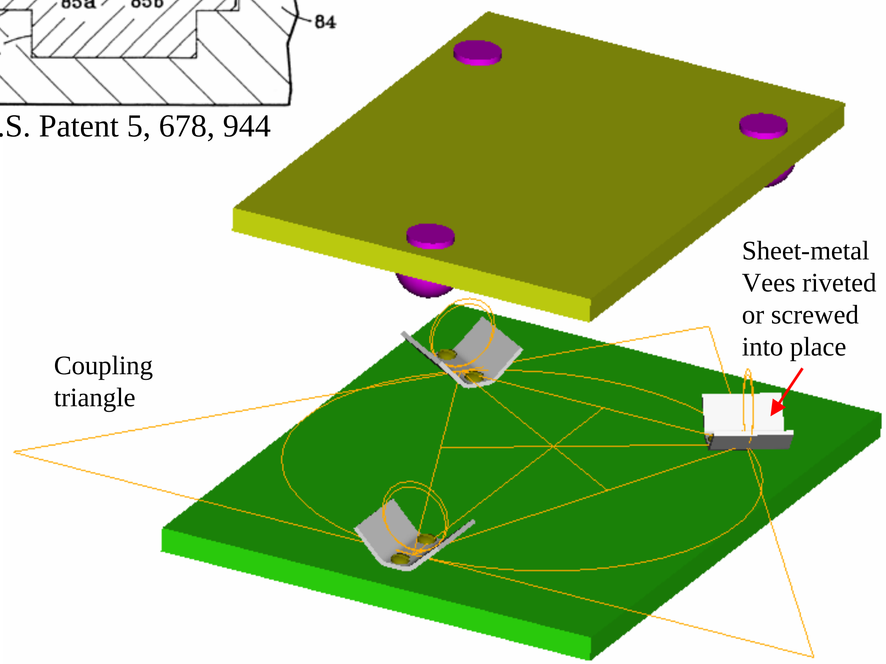


U.S. Patent 5, 769, 554



U.S. Patent 5, 678, 944





Kinematic Couplings: *Three-Tooth*

To prevent thermal expansion, often it is desirable to make precision instrument components from Super Invar, which is an iron alloy with 36% nickel. However, Invar cannot be hardened, and hence it cannot support significant point-contact loads. Three-tooth couplings were invented by Dr. Layton Hale of the Lawrence Livermore National Laboratory to overcome this limitation[1]. The three-tooth coupling forms three theoretical lines of contact between the cylindrical teeth on one component and flat teeth on the other component. Practically, both members can be made identical with cylindrical teeth, which yields the best repeatability. Good performance for low cost can also be obtained if the members are made with flat teeth, which yields the greatest load capacity, least cost, and good repeatability. Each line of contact across two teeth represents a two-degree-of-freedom constraint, thereby giving a total of six constraints. Ideally manufactured with three identical cuts directly into each member, the teeth must be straight along the lines of contact but other tolerances may be relatively loose.

A three-tooth coupling can also be easily molded into plastic to enable very repeatable coupling and orientation of cylindrical members, such as required for lens assemblies. With added snap-fits, a very precise assembly can be obtained.

Compare the load capacity of a single Super Invar tooth-to-tooth line contact verses the load capacity of a hardened 20 mm diameter ball on a hardened flat steel contact. Using the spreadsheet *Hertz_contact_line.xls* a very high and unrealistic load capacity is predicted. This can be seen because the deflection is large compared to the critical dimensions. This indicates that the contact zone has also probably saturated and in effect, the teeth have flattened and face-to-face contact has occurred.

Smaller cylinder 1 diameter, d_1 (mm)	**100**
Larger cylinder 2 (or flat plane) diameter, d_2 (mm)	**100**
Length, L (mm)	**10**
Applied load, F (N)	**62,950**
Elastic modulus Eone (N/mm^2)	**2.00E+05**
Elastic modulus Etwo (N/mm^2)	**2.00E+03**
Poisson's ratio vone	**0.29**
Poisson's ratio vtwo	**0.29**
Ultimate tensile stress, sigult (N/mm^2)	**250**
Depth below contact surface for evaluating deflection, do	**300**
Rectangular contact zone width, 2b (mm)	**19.25**
Contact pressure, qcyl (N/mm^2)	**416**
Deflection motion of d_1 center, defl_1 (mm)	**0.0825**
Deflection motion of d_2 center, defl_2 (mm)	**4.023097**
Total relative displacement of the cylinder's centers, dcyls (mm)	**4.1056**
Stress factor: Must be less than 1	
Maximum shear stress/(ultimate tensile/2)	**1.00**

Contrast the output shown in this spreadsheet to a 20 mm diameter hard steel ball on a flat hard steel contact surface which can support three orders of magnitude less force:

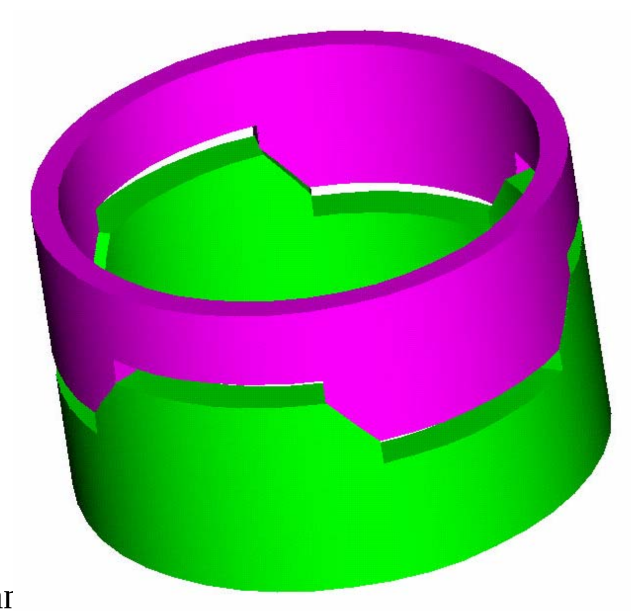
Ronemaj (mm)	**10**
Ronemin (mm)	**10**
Rtwomaj (mm)	**1000000**
Rtwomin (mm)	**1000000**
Applied load F (N)	**34**
Ultimate tensile stress (N/mm^2)	**2500**

Use the spreadsheets to load both the elements with the same load, and then compare the stresses. Does the three-tooth coupling's apparent ease of machinability make it a more reasonable choice for coupling tasks that your machine requires? How will repeated cycles affect its repeatability? Can it be made from plastic or aluminum instead of steel?

1. See http://www.llnl.gov/tid/lof/documents/pdf/235415.pdf for a full copy of Dr. Hale's Ph.D. thesis.

Kinematic Couplings: *Three-Tooth*

- A semi-kinematic effect can be achieved by having three teeth each on two coupling halves mate at six points
  - 3-5 micron repeatability can be obtained with this simple design
- Layton Hale at LLNL put crowns on one set of the teeth to create a nearly true kinematic three tooth coupling:
  - 1 micron repeatability can be obtained with this simple design



United States Patent [19] **Hale**

[11] **Patent Number:** **6,065,898**
[45] **Date of Patent:** ***May 23, 2000**

[54] **THREE TOOTH KINEMATIC COUPLING**

[75] Inventor: **Layton C. Hale**, Livermore, Calif.

[73] Assignee: **The Regents of the University of California**, Oakland, Calif.

[*] Notice: Under 35 U.S.C. 154(b), the term of this patent shall be extended for 634 days.

[21] Appl. No.: **08/511,980**

[22] Filed: **Aug. 7, 1995**

[51] **Int. Cl.**[7] **F16D 1/00**
[52] **U.S. Cl.** **403/364**; 403/190; 403/340; 403/381; 464/157; 192/69.83
[58] **Field of Search** .... 403/190, 291, 403/364, 311, 340, 381; 192/114 T, 69.81, 69.82, 69.83; 464/149, 157

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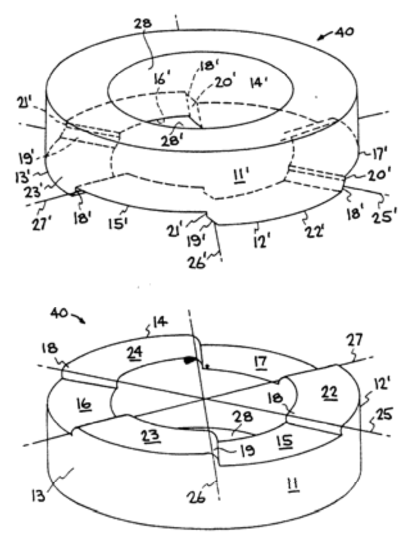
Primary Examiner—Daniel P. Stodola
Assistant Examiner—Bruce A. Lev
Attorney, Agent, or Firm—Alan H. Thompson; L. E. Carnahan

[57] **ABSTRACT**

A three tooth kinematic coupling based on having three theoretical line contacts formed by mating teeth rather than six theoretical point contacts. The geometry requires one coupling half to have curved teeth and the other coupling half to have flat teeth. Each coupling half has a relieved center portion which does not effect the kinematics, but in the limit as the face width approaches zero, three line contacts become six point contacts. As a result of having line contact, a three tooth coupling has greater load capacity and stiffness. The kinematic coupling has application for use in precision fixturing for tools or workpieces, and as a registration device for a work or tool changer or for optics in various products.

15 Claims, 2 Drawing Sheets



Kinematic Couplings: *300mm Wafer Transport*

This section describes an important case-study application example of kinematic couplings. Semiconductors are manufactured using highly automated processing equipment. Typically, a cassette of wafers is placed in the tool and a robot within the tool unloads the silicon wafers and places them in the process stream within the tool. In the past, injection molded plastic cassettes for 150 and 200 mm wafers had an H-shaped protrusion on their bottoms, and the lines of the H-bar coupling nested in corresponding grooves in a tool's loadport. Plastic had to be used for light-weight, low-cost, and chemical inertness reasons.

An incredibly bad thing would happen when a tool's robot zoomed in at high speed to pick up a wafer from a cassette and instead its gripper collided with a silicon wafer; Robot accuracy was of the utmost importance, but with use and time, the casettes would warp. The warping caused them to not sit flat on their H-bars and one could not be sure then about the orientation of the wafer. The first hint of trouble occurred when the switch was made from 150 mm diameter wafers to 200 mm diameter wafers and the pitch (spacing) between the wafers in the cassettes was kept constant. With the anticipated shift to 300 mm wafers, even greater problems were feared. SEMATECH[1] determined that they needed to do something before 300 mm tools were developed.

One of the activities that SEMATECH sponsored was Precision Machine Design Reviews, which were led by Prof. Slocum for equipment manufacturers who wanted an unbiased assessment of their equipment. During such reviews, Slocum made models (*error budgets*) to predict the accuracy of casette unloading/loading robots, and he found that tolerance requirements were increasing in a non-linear manner. It was not just a simple scaling issue, the larger cassettes warped more than the 150 mm casettes. Current practise would be OK for 200 mm wafers, but tool makers were already being asked to think about tools for 300 mm wafers. The error budgets predicted that robots would simply not be able to unload a scaled-up 300 mm cassette because all the tolerance would be used up by the cassette!

An analysis of the cassettes revealed that the *fat-rabbit* (most critical item) was the H-bar, followed by the stability of the plastic itself. The latter could likely be addressed, but as the H-bar increased in size, so did its molding tolerances. The H-bar had to go. Based on his previous experiences with kinematic couplings in precision fixturing, Slocum suggested the use of kinematic couplings, which his spreadsheet *Kinematic_coupling.xls* showed would allow plastic-on-plastic couplings to bear the anticipated loads. The response was initially "this is too different, and manufacturers will never change". Undaunted, Slocum and his then-Ph.D. graduate student Michael Chiu built a sketch model using a 200 mm wafer cassette, and at a meeting of tool manufacturers and users, Slocum passed around the model and gave a presentation on why the industry should be thinking ahead.

A period of discussion followed, and several months later, Ron Billings at SEMATECH was assigned the task of studying the feasibility of creating a kinematic coupling standard for 300 mm cassettes, which were undergoing a major design review. The general idea was soon accepted, and Ron headed up the task force to create the standards necessary to define the kinematic coupling standard. Many months of discussions and negotiations followed, including some suggestions by some interesting individuals at the meetings that "just to be sure, maybe we should use 4 grooves"[2]. 4 grooves will not only not work, they will cause the system to utterly fail and be as bad as the old H-bar. More tests, discussions, and reporting by others on the fact that plastic kinematic couplings really work, and the result was a new standard[3] and the industry moved forward.

Kinematic couplings are a FUNdaMENTAL machine element which have been around for a long time. New applications, however, are discovered all the time by never being afraid to ask questions, to dream, and to open a spreadsheet and play "what-if" scenarios. Let the formulas do the talking, not the naysayers!

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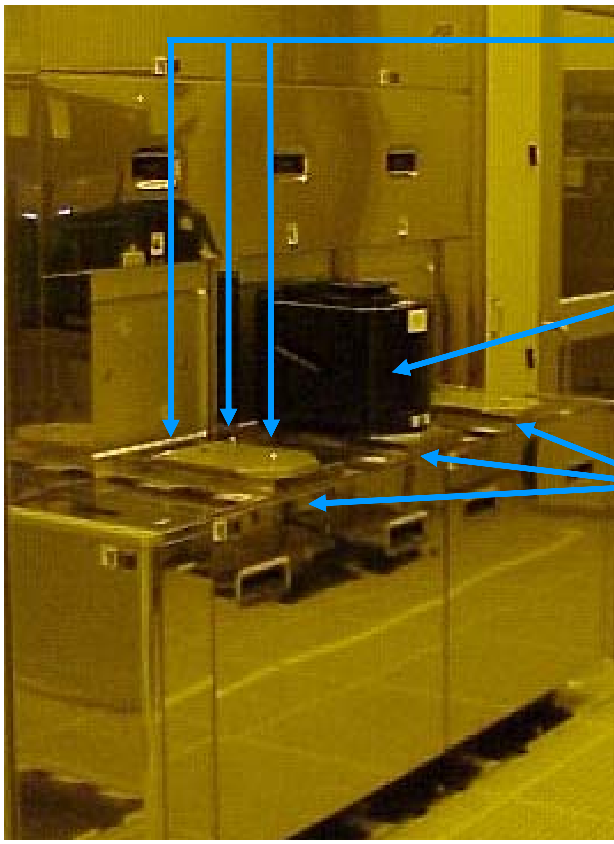
1. See www.sematech.org. SEMATECH was founded in 1982 in response to the declining competitiveness of US semiconductor manufacturers. When the manufacturers got together and compared their super secret confidential processes, they found that they were all actually doing basically the same thing, and that by sharing their notes with one-another, they could eliminate problems and once again become competitive.

2. One can lead a dehydrated mammal to an aqueous beverage, but one cannot always succeed in making them imbibe without sufficient coaxing.
3. SEMI E57-1296 kinematic coupling standard for wafer transport pods

Kinematic Couplings: *300mm Wafer Transport*

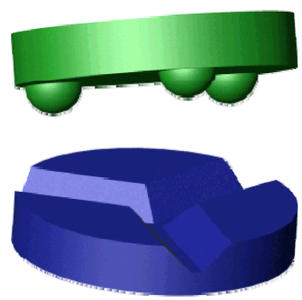
- How to precisely locate a plastic wafer carrying structure (FOUP) on a tool, so a robot can precisely load/unload wafers?
  - Exactly constrain it of course with an interface that contacts the FOUP at 6 unique points!
  - Success requires management of contact stresses, and standards upon which manufacturers agree
  - SEMI E57-1296 kinematic coupling standard for wafer transport pods



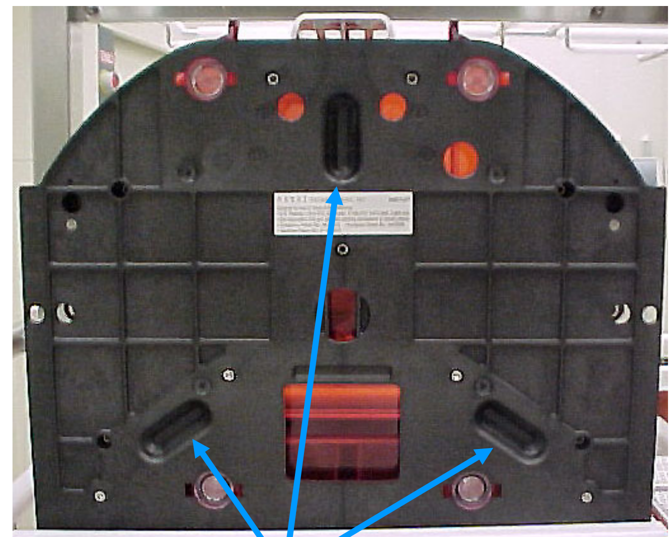
Kinematic coupling pins on loadport based on SEMI E57 standard

300mm Wafer carrier (FOUP) precisely positioned on kinematic coupling pins on loadport

Production equipment loadports based on SEMI E15.1 standard



Base of the FOUP



Mating kinematic coupling grooves on the FOUP, permitting precise alignment on load ports, so robots can precisely access 300 mm wafers

Quasi-Kinematic Couplings

Quasi-kinematic couplings are a type of low-cost alignment coupling that can deliver micron level repeatability. They are designed with elastic/plastic deformation to emulate the performance of kinematic couplings whose grooves are mounted on flexures. Kinematic couplings consist of three balls attached to a first component that mate with three V-grooves in a second component. The balls and V-grooves form small-area contacts.

QKCs consist of three axi-symmetric balls attached to a first component that mate with three axi-symmetric grooves (e.g. conical grooves called A-grooves) in a second component. The mating of these surfaces of revolution defines a circular line contact. In an effort to make the ball-groove joints emulate those in a kinematic coupling, material is removed from the conical, axi-symmetric groove surface to define two make an *A-groove* which can form two arcs of contact per ball-groove joint. Quasi-kinematic couplings use ball and groove geometries which are symmetric, making them easier to manufacture for less cost. The trade-off between KCs and QKCs is cost vs. constraint. The lower-cost geometries depart from the constraint characteristics of ideal kinematic couplings, yielding some degree of over constraint. With careful design, quasi-kinematic couplings can be made to emulate the performance of kinematic couplings.[1]

QKC's arc contacts differentiate the constraint characteristics of the quasi-kinematic coupling from the kinematic coupling. The diagrams in the table shows the constraint forces (arrows) between the balls and grooves of each coupling type. In an ideal, perfectly constrained coupling, the constraint forces are (1) perpendicular to the bisectors of the coupling triangle's angles and (2) permit unobstructed freedom of motion parallel to the bisectors. Any constraint parallel to the bisectors (e.g. along the grooves) would over constrain the coupling. Ideally the ratio between the parallel and perpendicular constraint, called the Constraint Metric (*CM*) should be zero.

Point and small area contacts, such as those found in kinematic couplings, primarily provide constraint in one direction normal to the surface. As such, they can easily be aligned so that their constraint is perpendicular to the bisectors of the coupling triangle's angles. For the ideal kinematic coupling, these constraints are normal to the bisectors of the coupling triangle and freedom of motion is permitted parallel to the bisectors. The arc contacts of the quasi-kinematic coupling provide constraint perpendicular to the bisector and some constraint along the bisectors. Without unobstructed freedom of motion parallel to the bisector, the QKC will have some degree of over constraint.

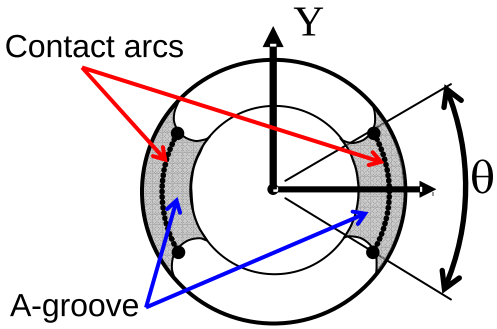
The key to designing a good quasi-kinematic coupling is to minimize over constraint by minimizing the unwanted constraint parallel to the bisectors. We can do this by changing the value of contact angle θ. The effect of the angle θ on the constraint metric (*CM*), which is ideally zero, is shown in the graph. By inspection, we can see that constraint contributions that are parallel to the angle bisectors (in the y direction) can be reduced by making the contact angle smaller. This in turn reduces the degree of over constraint the joint may impose on the coupling. Unfortunately, this reduces the amount of ball and groove in contact and therefore reduces the coupling's stiffness. The plot on shows how the relationship between *CM* and stiffness *K* vary with contact angle for a given design. Analytical methods exist to derive the relationships between θ, *CM*, and *K*.

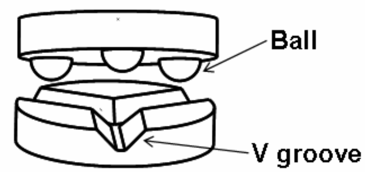
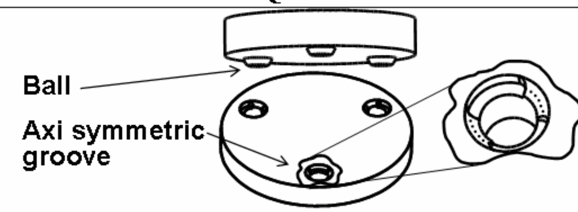
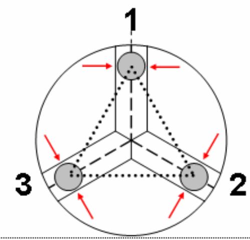
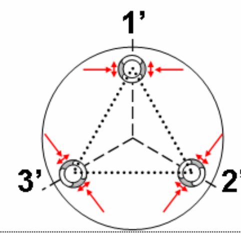
As designers, it is our job to achieve a relationship in which the stiffness and over constraint for a given θ are adequate for our design. Designs with contact angles less than 60 degrees (CM < 0.1) are typically safe choices. However larger contact angles may be used when one is willing to trade over constraint for higher stiffness.

How does the evolution of a regular kinematic coupling into a quasi-kinematic coupling stimulate your brain with respect to easy design and manufacture of repeatable mounting modules? Read on....

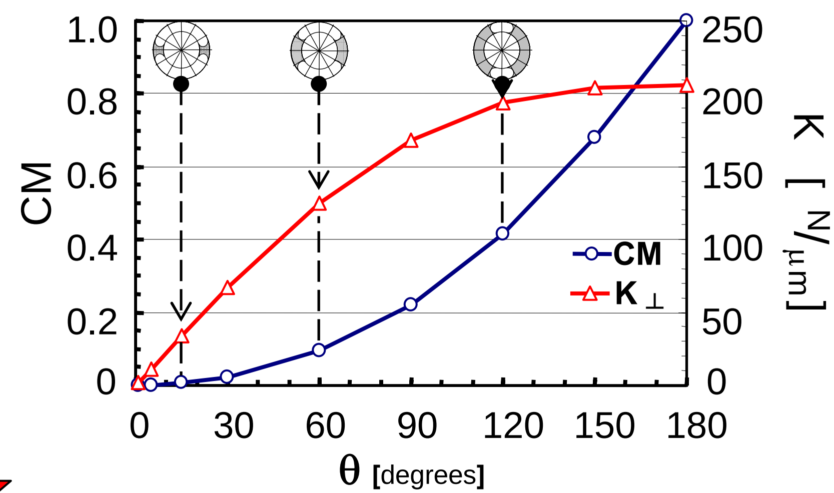
1. QKCs were the subject of Prof. Culpepper's Doctoral thesis in Prof. Slocum's research group. See Culpepper, M.L., "Design of Quasi-Kinematic Couplings," *Precision Engineering*, Vol. 28, Issue 3, July 2004, Pages 338-57; 31)Culpepper M., Slocum A., Shaikh F., "Quasi-kinematic Couplings for Low-cost Precision Alignment of High-volume Assemblies", Journal of Mechanical Design, Vol. 126 (4), pp. 456-63, and US Patent 6,193,430.

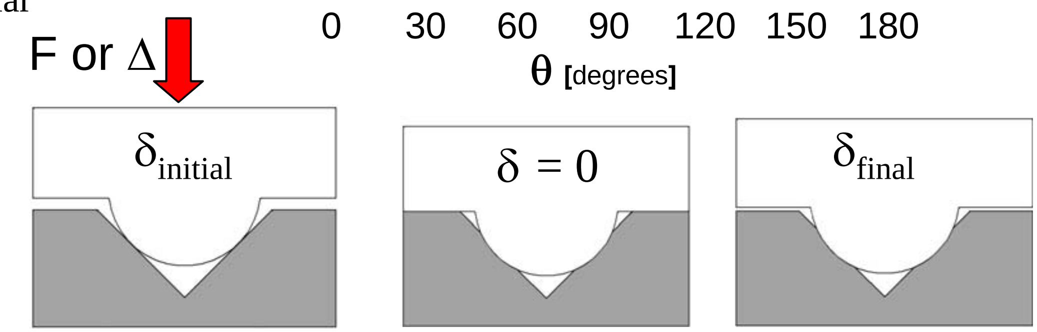
Quasi-Kinematic Couplings



	Kinematic coupling	QKCs
Coupling geometry		
Coupling constraints		

- **KC geometry**
  - 3 balls (hemispheres)
  - 3 *v-grooves*
  - 6 “point” contacts
- **QKC geometry**
  - 3 balls (hemispheres)
  - 3 *A-grooves* (surface of revolution)
  - Circular arc contact
  - Remove groove material to form arc contacts
- **Ideal constraint:**
  - Desire all constraints to be perpendicular to bisectors of triangle angles
  - Desire no constraint parallel to bisectors of triangle vertices
  - Constraint metric (CM) = $\frac{\text{constraint parallel}}{\text{constraint perpendicular}}$
  - Ideal CM = 0
- **QKCs are weakly over constrained**
  - Contact angle q defines arc geometry
  - Larger θ = stiffer joint but more over constraint
  - Choose design which delivers adequate stiffness, K, and minimizes CM
  - θ < 60 degrees typically emulate a true kinematic coupling





Quasi-Kinematic Couplings: *Details*

Quasi-kinematic coupling A-grooves are axi-symmetric, thus they can be made in a "plunge/drill" operation using a form tool. Groove reliefs can be cast, formed, milled, or drilled in place. The tools and processes required to form the groove seats are comparable to those required to make the holes for pin-hole joints. Quasi-kinematic coupling balls can be made from low-cost, polished spheres (i.e. bearings) or by turning the appropriate shape on a lathe.

A cross section of a QKC joint in various steps of mating is shown on the opposing page. The steps of assembly/mating are:

Step 1: The "ball" is pressed into the top component.

Step 2: The balls are mated into the A-grooves. The QKC must be designed so that a small separating gap (a few thousandths of an inch) exists between the mated components when the balls are mated into the A-grooves. This gap exists to ensure that all ball-groove contacts engage.

Step 3: A preload is applied (by bolt or other means) to force the ball and grove materials to comply, thereby closing the gap. Application of sufficient preload causes contact between the aligned components to assume all motion constraint and provide a sealed interface.

Step 4: The preload has been removed and a portion of the gap restored by elastic recovery of the ball and groove materials. This is necessary to prepare the QKC for subsequent alignment cycles.

This three-piece joint design is used in the engine case study discussed on the following page. The plastic deformation of the bulk materials in the ball and groove allow the ball-groove patterns to reduce the mismatch between them. In essence they are deformed into a better-matching pattern.[1] This enables one to avoid tight ball-groove placement tolerances, but limits (less than 10) the number of subsequent mates due to the increased likelihood of fatigue. It is impossible to achieve this "self-forming" behavior with dowel pins that fit into holes. This is one reason why a QKC is not as tolerance sensitive as a pin-hole joint.

During Step 3, the balls are pressed into the surfaces of the A-grooves, thereby plastically deforming the ball-groove surfaces and smoothing them as shown by the profilometer output. This process makes it possible for the ball to settle in the most stable position within a groove without having the ball and groove randomly snag on microscopically jagged asperities. This is important, as the repeatability of a coupling scales with the surface finish (~ 1/3 RA) of the mating surfaces.

As a designer it is important to specify the QKC design and fabrication processes correctly so as to achieve a successful "flattening" or burnishing of the ball-groove surfaces. A successful burnishing operation has two important requirements. First use a ball with polished (or ground if sufficient) surface finish and Young's modulus greater than that of the groove[2] and second, enable tangential sliding between the ball and groove surfaces[3] during mating. Tangential sliding happens naturally as the ball is forced down into the groove, therefore it is up to the designer to specify the correct ratio of hardness for the ball and groove.

Is there a “poor designer” version of the QKC that you can imagine for use with modular elements that you need to mount and dismount? or are you better off with a regular three-groove kinematic coupling? Or is their a hybrid: a regular kinematic coupling where you do not worry about the balls plastically deforming the V-grooves?

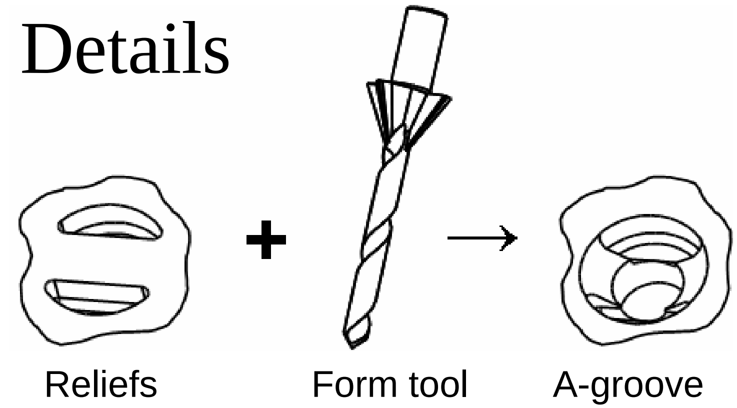
1. Culpepper M.L., Design and Application of Compliant Quasi-kinematic Couplings. Ph.D. Thesis. Cambridge, MA. MIT. 2000.

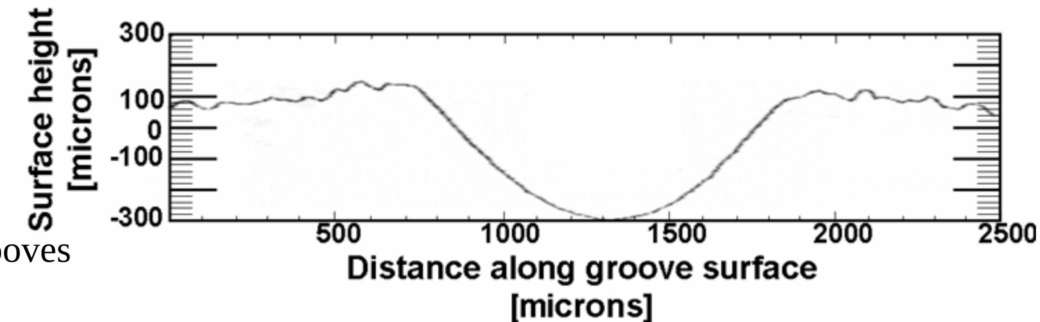
2. Johnson, K.L., "Deformation of a Plastic Wedge by a Rigid Flat Die Under the Action of a Tangential Force", Journal of Mechanics and Physics of Solids, 1968, pp. 395 - 402.

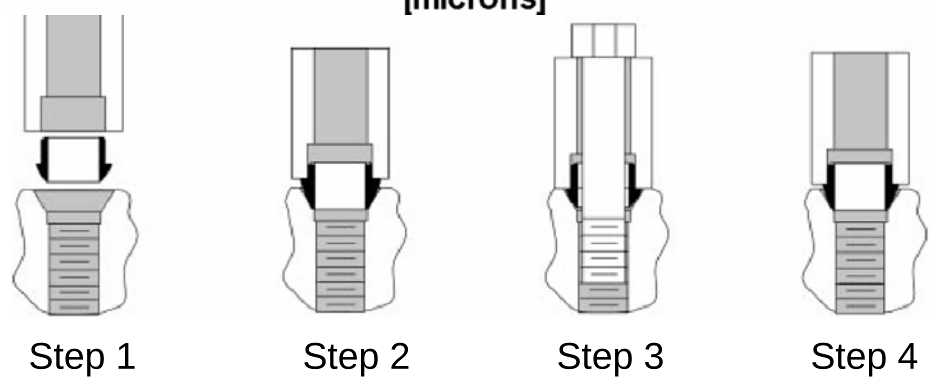
3. Childs, T.H., "The Persistence of Asperities in Indentation Experiments", Wear, 1973, pp. 3 - 17.

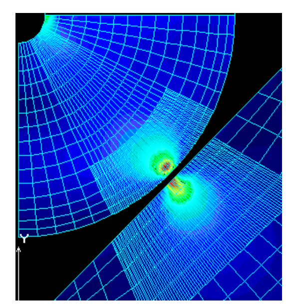
Quasi-Kinematic Couplings: Details

- **Fabricating QKC geometry**
  - Pre-cast or machine reliefs
  - Form tool machines axisymmetric *A-grooves*
  - Balls can be ball bearings or may be ground
- **QKC mating cycle**
  - Step 1: Balls are assembled into top part
  - Step 2: Balls mate A-grooves; finite gap between components
  - Step 3: Balls are preloaded into A-grooves
    - Gap is closed allowing interface to seal
  - Step 4: When preload is released, balls and grooves elastically recover
- **Ball and groove deformation**
  - During Step 3, grooves plastically deform
  - Plastic deformation reduces mismatch between ball and groove patterns
  - Balls and grooves elastically recover in Step 4
  - Recovery restores gap between parts
- **Surface finish**
  - Repeatability of coupling scales as 1/3 RA
  - Rough finish = poor repeatability
  - Grinding and/or polishing are expensive
  - Press hard, fine-surfaced ball surface into rough groove surface









Quasi-Kinematic Couplings: *Automotive Example*

Ford's *Duratec*^{TM} is a high-performance, six cylinder engine made in quantities of ~ 300, 000 units per year. The Duratec's crank shaft and main journal bearings are housed between the block and bedplate. A block and bedplate pair is shown partially assembled on the opposing page. A monolithic bedplate design, with cast-in main bearing caps, is used to decrease the number of parts which must be assembled. These main bearing caps hold the journal bearings which in turn constrain the engine's crank shaft. The block and bedplate contain four corresponding half-bores which form a full bore when the block and bedplate are assembled.

During fabrication, the block and bedplate are aligned, bolted together, and then simultaneously machined to produce four bores between the two components. The components are then disassembled, the crank shaft and bearings are installed and the components are realigned with the aim to match the original alignment. Re-alignment errors, δ, between the half-bore centerlines must not exceed 5 microns. Failure to do so reduces the journal bearing fluid film between the crank shaft and bearing surfaces. This in turn adversely affects the bearing friction which increases fuel consumption.

In the original design, alignment was achieved using eight pin-hole joints. The parts were then clamped in place via the assembly bolts shown on the opposing page. The rough geometry of the half bores were then simultaneously finished machined. Although two dowel pin joints may seem sufficient to align this design, the elastic averaging of eight pin-hole joints was required to achieve reasonable repeatability. The location and configuration of the pin-hole joints are shown on the opposing page. Although the eight pin-hole design meets the five micron alignment specifications, 4.85 microns, the design is grossly over constrained and thus tight tolerances on the size (± 0.008 mm) and placement (± 0.04 mm) of 16 holes are required. The cost and time required to machine kinematic coupling V-grooves into the engine were unacceptable. A QKC was designed for this application using the methods described in the preceding sections. The size, shape, materials and location of the balls and A-grooves were optimized using the parametric design tools.[1]

1. Culpepper, M. L., A. H. Slocum and F. Z. Shaikh, "Quasi-kinematic Couplings for Low-cost Precision Alignment of High-volume Assemblies," Journal of Mechanical Design, Vol. 126 (4), pp. 456-63.

The QKC joints are positioned such that they form the largest coupling triangle possible, thereby maximizing the rotational stiffness of the coupling. In doing so, forces that act upon the block and bedplate produce small misalignment errors. Using these positions allows the QKC balls to be press fit into holes which previously held the old dowel pins. Given the form-tooling previously shown, the grooves can be simultaneously produced when holes are drilled for the assembly bolts. The contact angle chosen for this application was 120 degrees. With reference to the chart in the preceding pages, this yields a CM = 0.4, which is not ideal. However it was found through experimentation that a high contact angle was required to achieve the desired coupling stiffness. This high coupling stiffness was required to counteract the torques from the assembly bolts which acted to misalign the bedplate and block.

The performance and design metrics for the pin-hole and QKC joints are shown in the table. The results are very favorable; however, despite the fact that the design was proven in the lab on test engines, several issues remained which prevented it from being adopted for production:

1) There was a perception that dowel pins add stiffness to the joint in addition to the clamping force of the bolts: This perception, however, effectively assumes the bolts are not providing preload to the joint, which provides more stiffness than the dowels ever could.

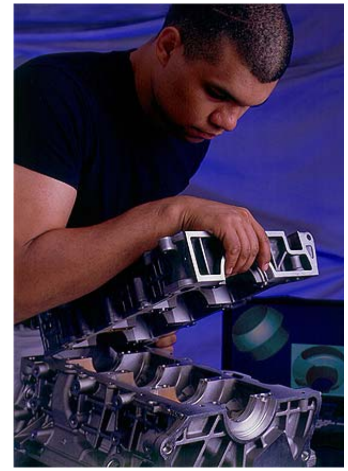
2) When the bearings are placed into the bore, they need to be "crushed", and the dowel pins allow for more vertical travel in the joint than the QKC elements.

3) Dowel pins have been used for many years, and if a new system is used and some unforeseeable thing happens, having to replace all the engines would bankrupt the company.

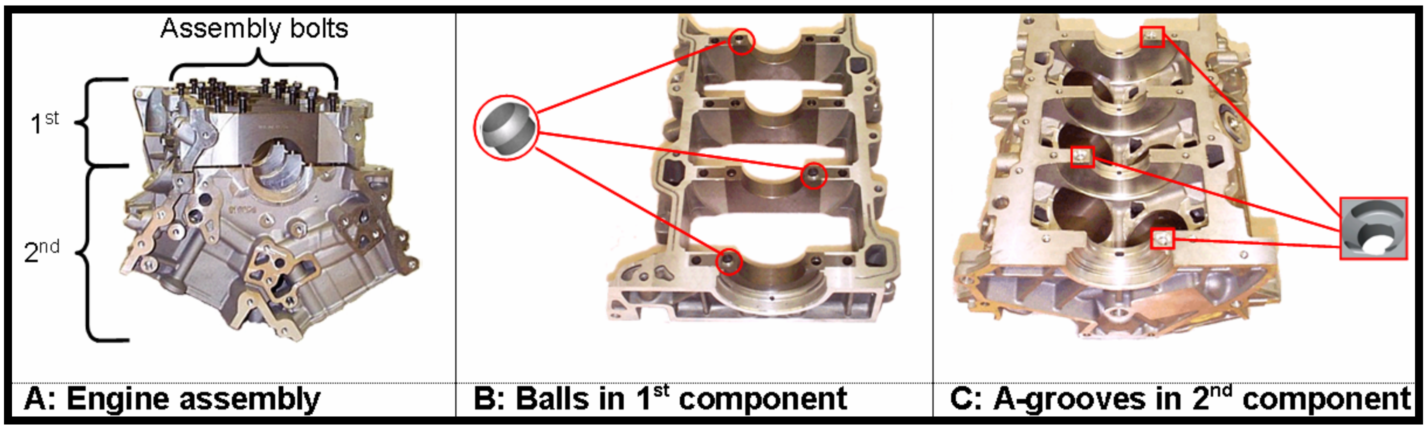
The best ideas can be thwarted by perceptions of reality even when the data indicates success. Do you have any great ideas that are not even yet proven that are critical to the functioning of your design? And you have how many weeks left until the contest? And how many other commitments do you have?

Quasi-Kinematic Couplings: *Automotive Example*

- Original alignment design
  - Components were aligned with 8 pin-hole joints
  - This design is very over constrained
  - Pin-hole patterns requires tight tolerances
  - 8 precision ground dowels required
  - 16 precision holes are bored
- QKC design
  - 8 pins => 3 balls
  - 16 holes =>
    - 3 holes
    - 3 A-grooves

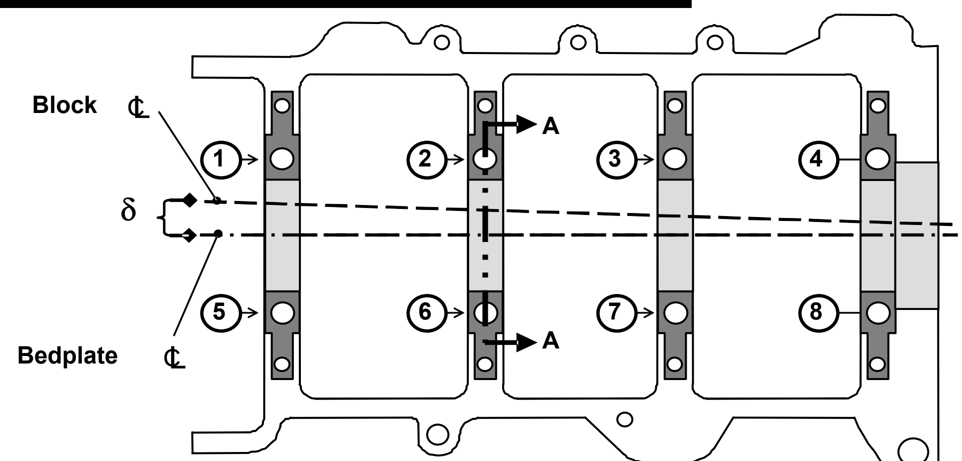


Prof. Martin Culpepper with his Ph.D. thesis, the QKC



A: Engine assembly | B: Balls in 1st component | C: A-grooves in 2nd component

Engine QKC	8 dowels	QKC
Precision pieces	**8**	**3**
Precision features	**16**	**6**
Tolerance [microns]	**40**	**80**
Repeatability [microns]	**5**	**1.5**
Cost reduction/engine	**N/A**	**$1**



Kinematic Couplings: *Servo-Controlled*[1]

During the manufacture of semiconductor components, the components are tested at various stages. Manufacturers have significant economic incentive to detect and discard faulty components as early in the manufacturing process as possible. For example, numerous semiconductor integrated circuits are fabricated on a large silicon wafer. The wafer is diced and the integrated circuits are separated into dies. The dies are then mounted into frames. Bond wires are attached to connect the die to leads which extend from the frame. The frame is then encapsulated in plastic or other packaging material, producing a finished product.

This manufacturing process is relatively expensive. Accordingly, most semiconductor manufacturers test each integrated circuit on the wafer before the wafer is diced. The defective integrated circuits are marked and discarded after the wafer is diced. In this way, the cost of packaging the defective dies is saved. As a final check, most manufacturers test each finished product before it is shipped. Manufacturers who guarantee that a very high percentage of the semiconductor components delivered to their customers will function properly can charge higher prices for their products.

To rapidly test large quantities of semiconductor components, Automatic Test Equipment (generally *testers* or *ATE*) are used. A tester rapidly generates input signals for application to the integrated circuit and can determine whether the appropriate response signals are generated. Because testers are highly automated, they can run through a series of millions of test cases in a few seconds.

To efficiently test integrated circuits, a device is needed to move and quickly connect the device being tested to the tester. To move wafers, a machine called a *prober* is used. To move packaged parts, a machine called a *handler* is used. These machines precisely position the component being tested so that it makes electrical contact with outputs of the tester. Probers, handlers and other devices for positioning a device under test relative to the test head are called generically "handling devices."

1. See US Patents 5,821,764 and 6,104,202; and Chiu, M.A., Slocum, A.H., "Improving Testhead Interfaces with Kinematic Docking", Presented at IEEE Southwest Test Workshop, San Diego, CA 1995, and Chiu, M.A., Slocum, A.H., "Improvements in the Prober/Test Head Mechanical Interface", Presented at IEEE Southwest Test Workshop, San Diego, CA, 1996.

Connecting the handling device to the tester poses several challenges. First, semiconductor circuits have many inputs and outputs. Typical circuits might have between 20 and 100 inputs and outputs. However, some larger circuits have as many as 500 inputs and outputs and circuits with over 1,000 inputs and outputs are being contemplated. Thus, the tester must generate and receive hundreds of signals. The electronic circuitry needed for driving and receiving these signals must be as close to the device being tested as possible to allow high speed operation.

Consequently, most testers are designed with a test head that includes all of the driving and receiving circuitry. The test head is connected via a cable bundle to an electronics cabinet which contains data processing circuitry which determines which signals should be driven and compares the received signals to expected values. The test head can be up to a few feet in diameter in order to hold all of the driving and receiving circuitry. The device being tested is on the order of a square inch in the case of a finished product and is even smaller in the case of an integrated circuit on a wafer. To make electrical contact, the hundreds of signals leaving the test head must be squeezed into a very small area.

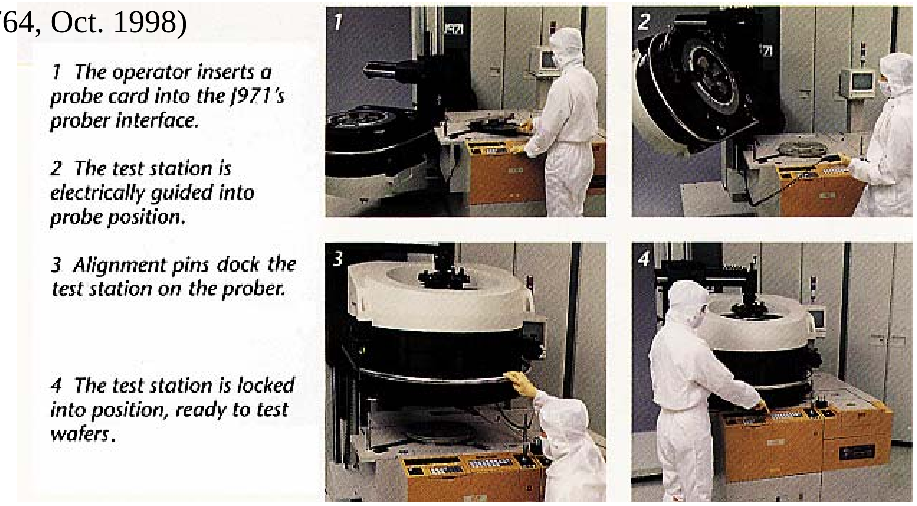
The figure shows how a probe card's small probe needles must contact the small bond pads on the device under test (DUT). In the past, the test head was leveled to the handler using a cumbersome manually adjusted interface, which could take an hour to tweak each time the tester was redocked to the handler. Given that the system cost could be upwards of a million dollars, this was prohibitive.

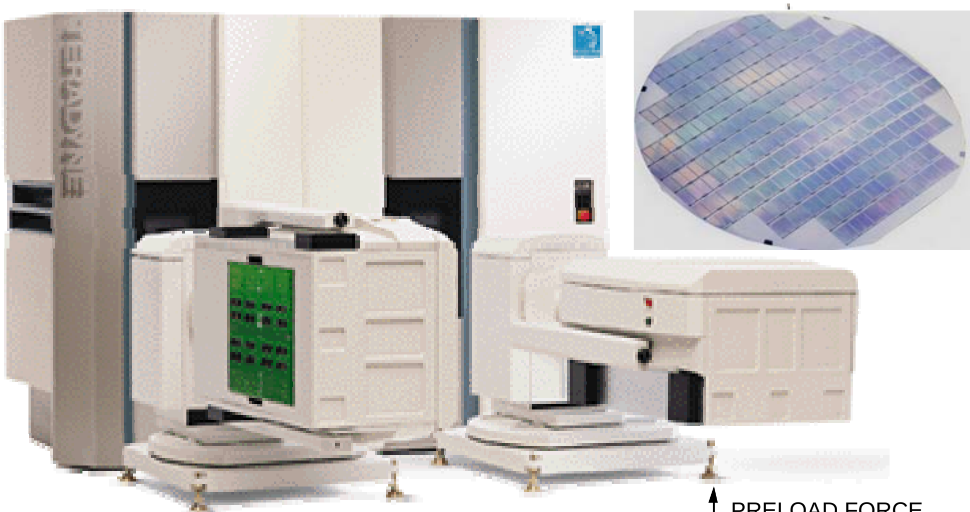
Kinematic couplings to the rescue! KCs can repeatedly locate the testhead, but the planarity between the handler and a newly installed probe card might not be acceptable. If the height of the kinematic coupling contact points could be controlled based on feedback between the probe needles and a test wafer, then planarity could be set at the beginning of the cycle using servo control. Hence was born the idea for a servo-controlled kinematic coupling.

You are not likely to need such a sophisticated system for a robot design contest, but one never knows. Still, this example shows you how a simple machine element can form a critical part of a precision mechatronic system that you can look forward to designing as a practising engineer.

Kinematic Couplings: *Servo-Controlled*

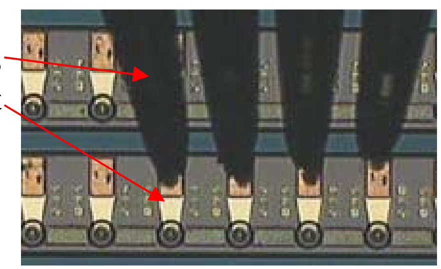
- Automatic Test Equipment (ATE) is used to test computer chips during their manufacture
  - Testing wafers requires a very high precision interface between the tester and wafer
- Sevro-controlled kinematic couplings automatically level ATE test heads to wafer plane
  - Michael Chiu's Doctoral Thesis (US Patent #5,821,764, Oct. 1998)
- Teradyne has shipped over 500 systems

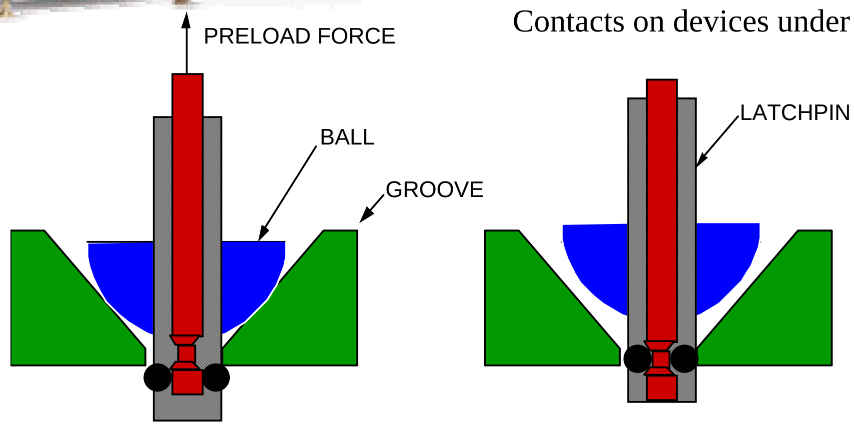




Tester probes

Contacts on devices under test





DOCK UNDOCK

Servo-Controlled Kinematic Couplings: *Details*

The first step in the design of the servo controlled kinematic coupling system was to define the functional requirements and then develop a ***strategy*** and ***concepts*** (e.g., by the process as described in Topic 1[1]). Once the details are worked out, driven by the appropriate engineering calculations such as determining the Hertz contact stresses, the optimal transmission ratio, and the stiffness of the structural loop, there remains the daunting task of making all the parts fit within not enough space. Next, the design must be life tested, and invariably long term lubrication and wear issues must be addressed. This often requires finding a lubrication or materials expert to help select the correct alloys and lubricants.

One of the important functional requirements was for the coupling to coarsely locate itself, and then with what essentially amounts to an anchor, to grab hold of the mating Vee-groove and pull the ball into contact and preload the testhead to the handler. One might think that the weight of the testhead should be sufficient; however, the cable that connects the testhead to the tester's mainframe computer may be 300 mm in diameter and 3 meters long and weigh nearly as much as the testhead: the tail can sometimes wag the dog. Hence the kinematic coupling must be preloaded to withstand forces in all directions.

This preloading requirement was obtained by decoupling the design: The leadscrew controlled the position of a slide supported by crossed-roller bearings, to which was attached the ball. The preloading post passed through the center of the ball, and at its end was a ball-lock device much like that which is found on quick-connect pneumatic couplings. A separate actuator controlled the action of the preloading mechanism.

Once designed, built, and tested at the manufacturer, Teradyne Corp., one must still rely on careful implementation in the field. In fact, some customers initially wanted to add a fourth Vee Groove and ball unit "just in case three was not enough" which showed they just were not grasping the power of kinematic design. In another instance, an installer did not bother to read the instructions, and used a round bar to make sure that two of the vee-grooves were exactly aligned to each other: the customer called to say "those darn couplings are not working!" Once properly fitted, however, they worked 10x better than any previous system.[2]

1. It is a very long way from the initial "AHA!" flash of inspiration to delivery of a reliable product.

The plots compare the accuracy and repeatability of the servo controlled kinematic coupling, the *K-Dock™*, and a standard interface known as a *J-Ring*. The results are exceptionally good. and are the reason that teradyne was able to sell hundreds and hundreds of K-Docks making it virtually a standard interface despite its costing tens thousand dollars more. On a million dollar tester that costs hundreds of dollars per hour to operate, the payback time is a matter of months. This highlights another important aspect of design: the design engineer is often best suited to define the benefit that can be achieved from using the product, and thus must work with marketing to create and present a *cost-of-ownership* model to the customer.

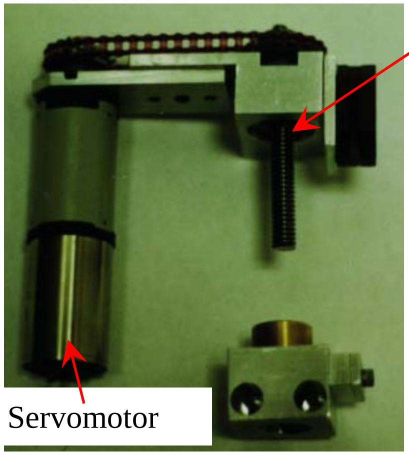
A good product must not only offer superior performance, it must do so for a lower cost of ownership. This means that the product can itself cost more than an alternative or currently used device, if the product can lower the over cost of ownership (or operation) of the total system. This can sometimes be an uphill battle because sometimes customers just see the initial fixed cost of a system and do not always factor in the long-term operating costs.

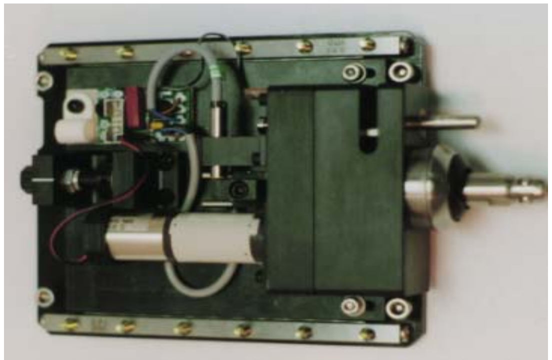
There is a similar lesson to be learned in the context of designing and building a machine for a robot design competition: haste-makes waste. All too often, a novice designer will overlook creating a careful model or performing a bench-level experiment and instead rush into creating their exciting idea. Once the idea is made, however, it often has major problems, but it is difficult to change details. But the student is reluctant to start over because they have already spent so much time on the design...the reluctance to let go and resort to a simpler contingency plan is often the cause of major failure. Although this is still a learning experience, if the student is open-minded enough to realize it is their fault, and not their initial perception that the course is too rushed. Soooo review your designs and early tests and calculations and objectively decide if its time to resort to your countermeasures!

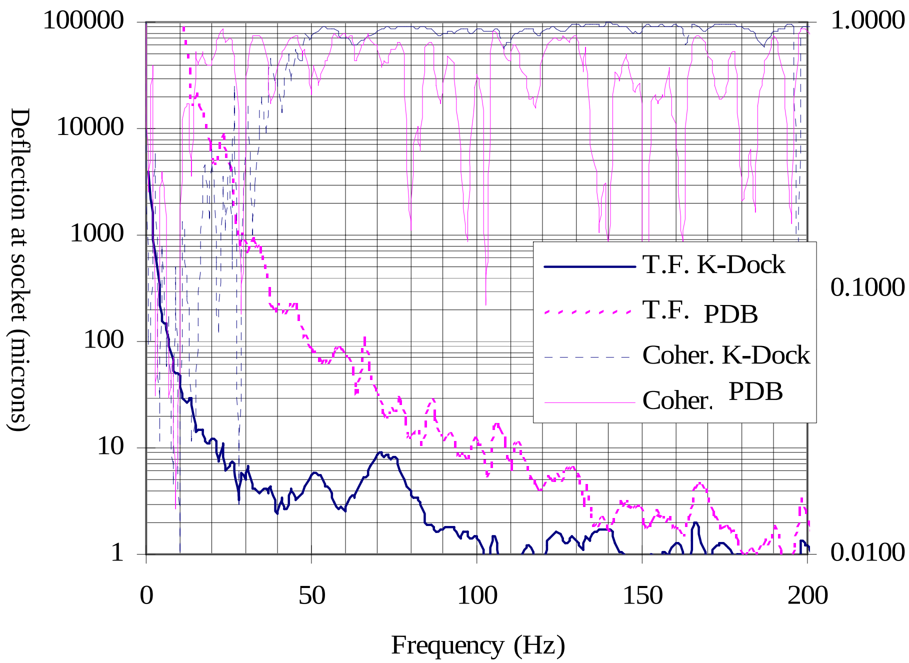
2. You can lead a dehydrated mammal to an aqueous solution, but you cannot always make them rehydrate.

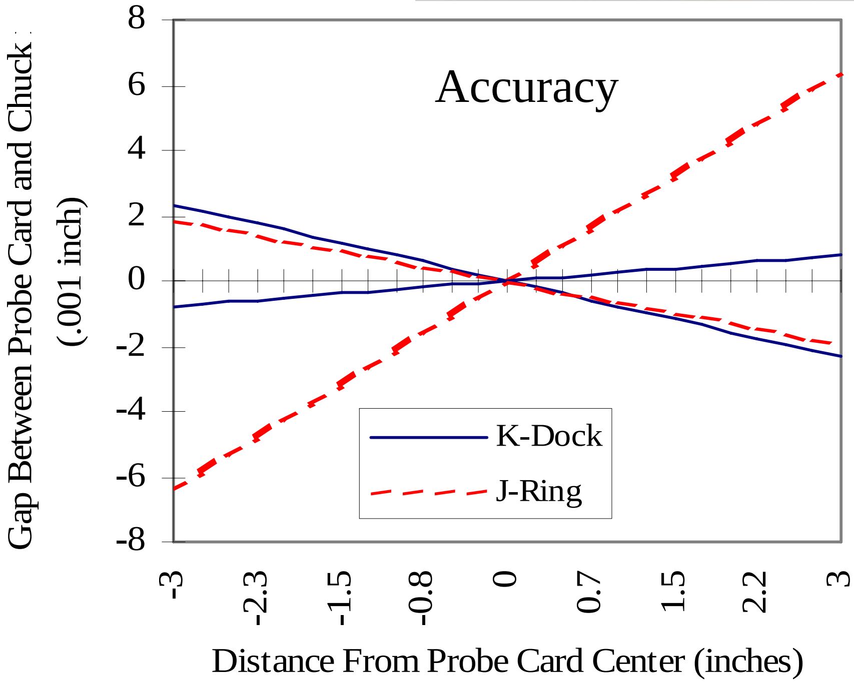
SCKC: *Details*

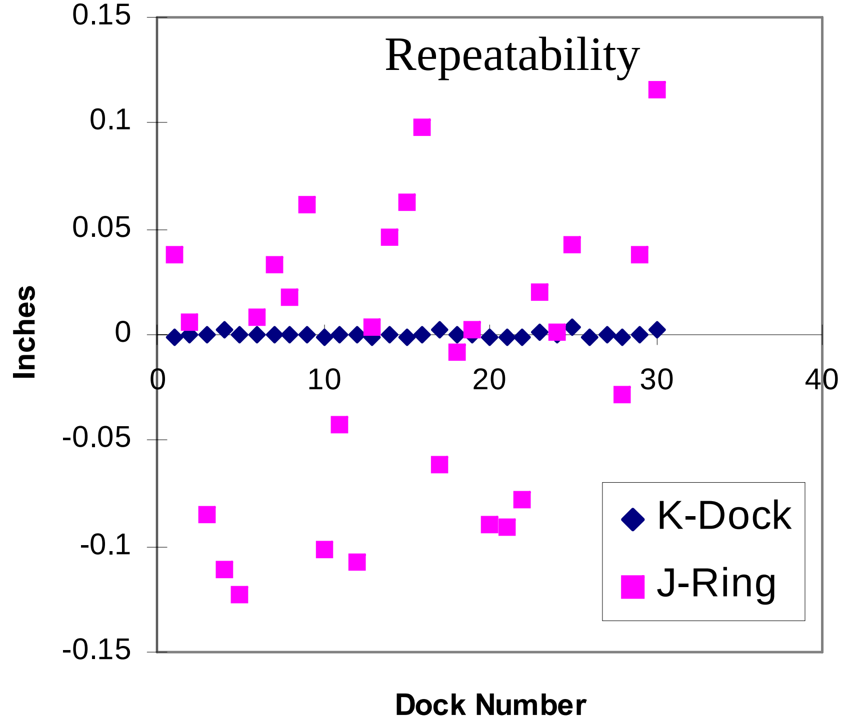
- Improved repeatability, accuracy, dynamic stiffness











Elastic Averaging

It is essentially impossible to create a perfect joint between two surfaces, even if one wanted to assume that they were flat. Overall bow, local waviness, and surface finish effects all combine to thwart the formation of the perfect joint. Only if the features that seek to prevent a perfect joint can be made elastic enough, and high enough forces applied, can they be elastically forced together to yield a near-perfect joint or structural interface.

In fact, the achievement of perfectly flat planes has been a prime goal of the manufacturing industry for many centuries, because from a flat plane one can derive a straight edge... If you give a manufacturer a flat plane...[1] It is not known exactly who figured out how to use three plates to be rubbed against each other in a round-robin manner, such that the high points would show up after rubbing and then could be scraped off; however, this method enabled precision machine tools to be built. Having flat surfaces allowed precision machine tools' structural joints to be precise and very rigid which increased the machines' accuracy. With precision machine tools, accurate interchangeable parts could be made, and this was a critical catalyst for the industrial revolution.

Even when plates are scraped flat, they never really touch everywhere. As noted on page 3-27, if *Exact Constraint Design*, has good and bad points, then Maxwell's reciprocity would indicate that *Inexact Constraint Design*, which would require systems to be statically indeterminate, might also have bad and good points. *Elastic averaging* or *controlled compliance*: Elastic deformation compensates for geometric errors; however, the forces are managed so yielding does not occur. When there are many compliant elements, each of which locally deforms to accommodate an error, in total they can form a very rigid and accurate system, and the design is called an *Elastically Averaged Design*.

In fact, many machine components are made far more accurate than any of their components by *elastic averaging*. *Curvic couplings* use two specially ground face gears that are forced together to allow one surface to be indexed with respect to another and achieve an accuracy (square root of the number of gear teeth) better than either gear itself, because of the high forces used to preload the gears together. spline-type flexible couplings can eliminate backlash if their elements radially flex to create a preload effect[2].

The footprints of the 3, 4, and 5 legged chairs illustrate this point most comfortably. The red arrows show the minimum radius from the center-of-stiffness, in this case the center of the chair, to the edge of the supports. This radius indicates the stability of the chair. If the center-of-mass shifts outside this radius' point, the chair will tip. Thus despite the fact that all three chairs have the same radius circle that contains all the legs, the more legs, the greater the stability. However, in order to prevent the chair from rocking back and forth on three legs, because not all the legs' feet can ever lie in the exact same plane even if the floor was perfectly flat, the legs have to be compliant enough so that a modest load causes them to deflect and make them all contact the floor. On the other hand, the legs cannot be so soft that the sitter feels unstable when planting their mass onto the chair. Thus in order to design a five legged chair, the engineer has to have an idea of the potential variance in the floor and chair-leg planarity and the weight of the person.

Forcing joints together with very high forces in an attempt to make them "perfectly" mate, can cause high stresses and deformations in other parts of the system. Thus it is important that when elastically averaging two components together, that the forces be applied in a uniform manner, with their net resultant acting through the center of stiffness of the structural interface.

What surfaces do you have that will be forced into planar contact? Will this cause warping of critical components that could decrease performance? Will this cause an averaging effect that can increase performance? What might appear to be elastically averaged, may in fact be grossly over constrained...

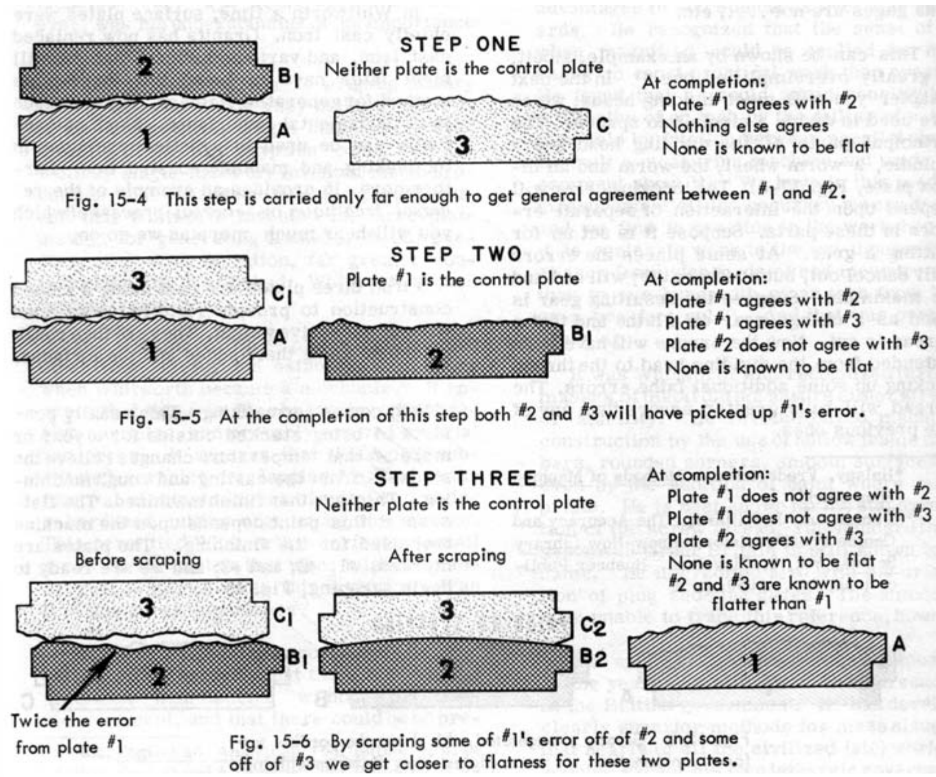
1. See J. Roe, English and American Tool Builders, © 1916 Yale University Press. Also see Chris Evans, Precision Engineering: An Evolutionary View, Cranfield Press, Cranfield, Bedford, England.

2. M. Balasubramaniam, H. Dunn, E. Golaski, S. Son, K. Sriram, A. Slocum, "An anti backlash two-part shaft with interlocking elastically averaged teeth", Precis. Eng., Volume 26, No. 3, pp. 314-330, 2002.

Elastic Averaging

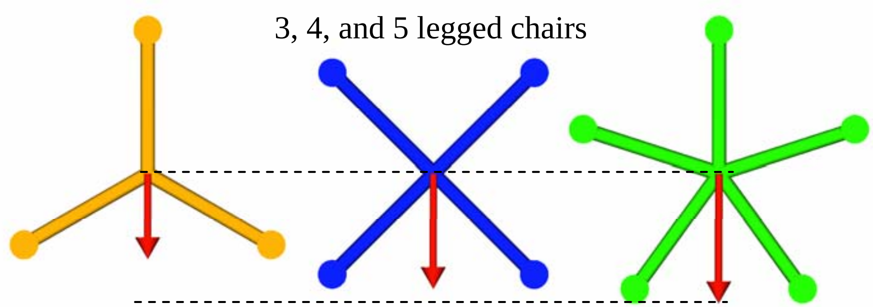
- Any one error can be averaged out by having many similar features
  - As in gathering data with random errors, the accuracy of the reading is proportional to the square root of the number of samples taken
- Local errors are accommodated by elastically deforming the members
  - Overall high stiffness is obtained by the sum of many compliant members

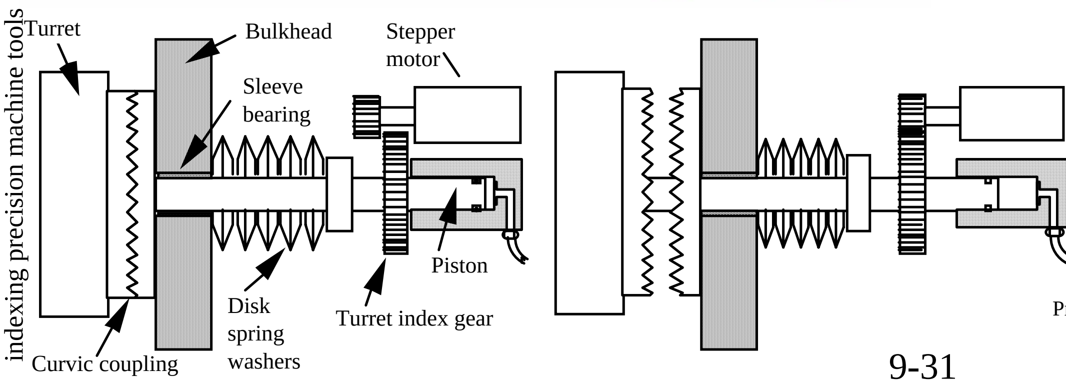
Scraping plates flat, the genesis of all precision machines



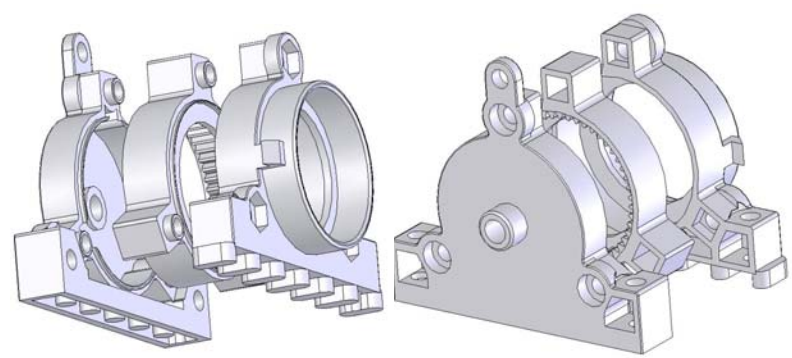
From T. Busch, Fundamentals of Dimensional Metrology, Delmar Publishers, Albany, NY, 1964

3, 4, and 5 legged chairs





Curvic coupling mechanism for indexing precision machine tools



Prof. Slocum, Nevan Hanumara, and Radu Gogoana redesigned the Tamiya planetary gearbox (see Topic 7) to be deterministic with the use of Lego™-like bumps and sockets between stages so they snap together and are precisely aligned via the principle of Elastic Averaging

Elastic Averaging: *Over-Constraint*

The issue of proper constraint underlies any discussion of a mechanism or structure. Any moving mechanism that is over constrained may be subject to undesirable loads as the mechanism moves. The forced displacements acting on finite stiffnesses creates forces which can cause overloading and/or fatigue. This is not to be confused with the use of many small effective stiffnesses combined to yield an overall large stiffness while locally accommodating variations. Over-constraint increases loads on a structure or mechanism without providing any real benefit to accuracy or stiffness, especially when the compliance is too high.

Return to the multi legged chair example. A three legged chair will always have three legs in contact with the ground. A four legged chair will always have at least three legs in contact with the ground. If the chair is too stiff, then even when a heavy person sits on it, there is a good chance that only three legs will ever touch the ground. The chair will totter and customer dissatisfaction will occur even if the chair was perfect and it was the floor that was not flat! This is an over constrained system: The chair seat lacks the compliance necessary to allow the four legs to all contact the ground.

Extremes are not to be taken in either direction. Too stiff and the system is over constrained and performance suffers. Too compliant, and the system will have poor accuracy, or it may buckle and fail.

Consider the interface between an actuator, such as a ballscrew and its nut, and a carriage supported by linear bearings. If the screw shaft is mounted in support bearings whose axis of rotation is not perfectly aligned with the axis of motion of the carriage, then as the carriage moves, the distance between the carriage and the screw shaft will vary. This can only be accommodated by bending of the screw, and deflections of the bearings. This places a large radial load on the nut which does nothing to increase accuracy of motion. It is a *parasitic load.*

This parasitic load can be kept manageable by making the leadscrew sightly longer than the desired stroke, so the screw shaft radial compliance provides the coupling action. How much? It is a question of springs. The engineer has to create a model for the system that has the spring stiffness of the carriage bearings, carriage, leadscrew nut, leadscrew, leadscrew support bearings, and the machine structure. In other words, the engineer needs to model the structural loop. Then, manufacturing alignment errors, linear and angular displacements, can be imposed at nodes between the springs, and a force equilibrium can be determined. Just because there is elastic deformation in a system does not mean that a beneficial averaging effect is taking place!

A tolerance study therefore not only helps to determine if parts will fit together geometrically, it helps to determine the forces between the objects when the bolts are tightened. These forces then go into the assessment of the life of the bearings. In fact, as discussed in the next chapter, one of the principle loads on bearings is not just the intended loads of operation, but also the loads caused by unintended over constraint incurred during assembly. As discussed on page 6-7, it also makes a big difference where the over constraint is applied: The center of stiffness can allow parts to be coupled together such that over-constraint will at least cause minimal angular errors.

The simplest way to deal with over constraint is to model the compliances in the system and make sure that when displacements are imposed, the resulting loads are not too high. If they are, then additional mechanism may be required. In this case, couplings must be used.[1]

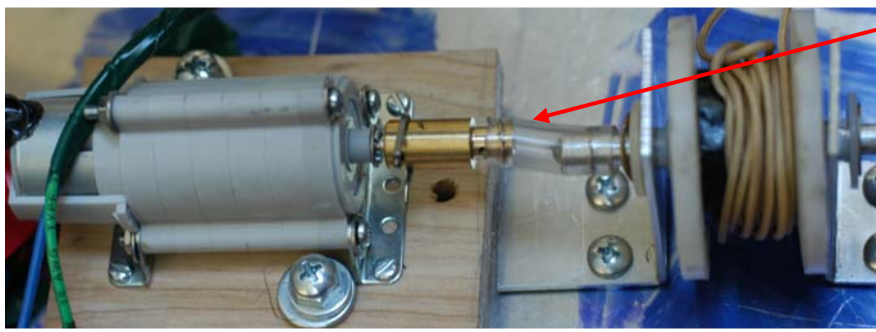
With respect to structural joints, nominally planar surfaces are often bolted together. Here it is important that enough bolts are used with sufficient preload, i.e., force generated by tightening the bolts, to cause the structure to deform such that the intended contact surfaces truly are in contact, and clamped with the desired force. Remember, as discussed on page 9-8, lubricated threads enable higher preload forces to be obtained, and it is the preload on the joint acting with the coefficient of friction between the joint surfaces, that enables a joint to withstand shear loads.

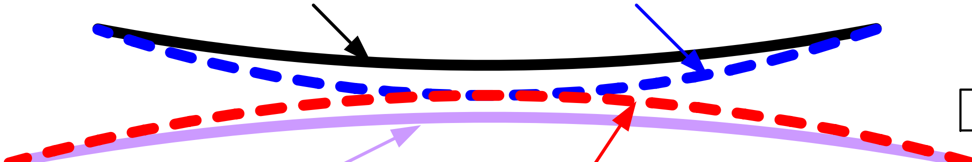
Check your design for potentially over-constrained joints, and make sure there is sufficient compliance and preload in the system so they will act as elastically averaged joints. A systematic review in the design phase, while the machine still is just a collection of parts in the solid modeler, can save you countless hours in the shop, and even more hours fixing things when they later do not work.

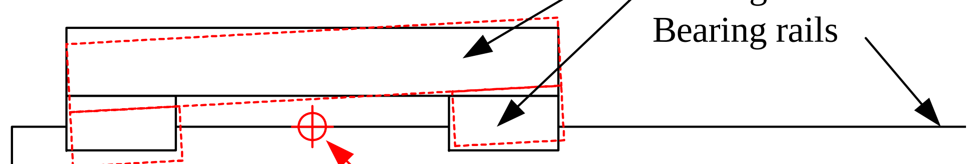
1. E.g., see pages 5-29, 5-30, and 10-19 through 10-28

Elastic Averaging: *Overconstraint?*

- Over-constraint is NOT Elastic Averaging
  - Example: One component (a carriage) wants to move along one path and another (ballscrew nut) along another, but they are attached to each other
    - They will resist each other, and high forces can result which accelerates wear
    - Either more accurate components and assembly are required, or compliance, or clearance (pin in oversized hole) must be provided between the parts
  - Designers should always be thinking of not just an instant along motion path, but along the entire motion path







Topic 9 Study Questions

Which suggested answers are correct (there may be more than one, or none)? Can you suggest additional and/or better answers?

1. Interfaces must enable parts to fit together with the desired accuracy, but you cannot create two sets of exactly matching holes in two components:
   <u>True</u>
   False
2. A countermeasure to the problem of holes not lining up is you can oversize the holes:
   <u>True</u>
   False
3. Clearance between bolts and holes means that the components will not have a unique assembly position:
   <u>True</u>
   False
4. "Error budgets" keep track of interferences & misalignments and help predict the overall accuracy of an assembly or a machine:
   <u>True</u>
   False
5. Structural joints (non moving) transfer loads between members, and are a necessary part of almost all structures:
   <u>True</u>
   False
6. A good weld is as strong as the base metal, but heat treated alloys require re-heat treatment:
   <u>True</u>
   False
7. Surface preparation is VERY IMPORTANT including cleanliness and on thicker parts, bevel edges to be welded:
   <u>True</u>
   False
8. Adhesives are often used to bond thin edges:
   True
   <u>False</u>
9. Epoxy is often used for making laminates:
   <u>True</u>
   False
10. Adhesive joints are usually not meant to be moment connections:
    <u>True</u>
    False
11. Thread locking agents are used to keep screw threads from coming undone:
    <u>True</u>
    False
12. CLEANLINESS IS OF UTMOST IMPORTANCE FOR ADHESIVE JOINTS:
    <u>True</u>
    False
13. To reduce the need for precision tolerances or hand scraping to fit precision machines together, components can often be positioned with respect to each other and then rigidly fixed using a potting epoxy:
    <u>True</u>
    False
14. When using epoxy to pot structures together, it is important to consider that the epoxy shrinks, so the components must still be well-supported:
    <u>True</u>
    False
15. Wooden shims make good adjustment elements when potting machine components together:
    True
    <u>False</u>
16. Epoxy can be used to replicate a precision surface onto a rough surface, thereby achieving a nearly perfect match between the two surfaces:
    <u>True</u>
    False
17. Bolts and screws ONLY clamp one element to another!:
    <u>True</u>
    False
18. Friction and the clamping force are what hold the joint together:

True
False

19. Bolts and screws DO NOT themselves take shear loads (unless you use a shoulder bolt):
    True
    False
20. Clean lubricated threads can make a factor of 2 difference in the force created by a bolt:
    True
    False
21. Preloading a bolted joint is critical to keep the ratio of pre-stress/ alternating stress high to reduce fatigue:
    True
    False
22. Bolts act in parallel with the stiffness of the joint:
    True
    False
23. By tightening bolts to create a preload higher than the applied load, the effects of alternating stresses created by a load are reduced:
    True
    False
24. As bolts are tightened (preloaded), their stiffness acts in series with the flange stiffness:
    True
    False
25. As external loads are applied to the joint, bolts' stiffness acts in parallel with flange stiffness:
    True
    False
26. Preloading bolts allows large loads to be applied to a joint while minimally affecting the bolt stress:
    True
    False
27. A bolted joint can be designed so it "leaks" before a bolt breaks:
    True
    False
28. The stress cones under bolts' heads must never overlap:
    True
    False
29. Bolted joints are a good source of damping in a machine:
    True
    False
30. Bolted joints can cause local deformations in the surrounding material which can sometimes degrade bearing accuracy and in extreme cases, degrade bearing life:
    True
    False
31. Bolt torques can induce residual stresses in clamped-flat-spring flexural bearing elements and cause parasitic error motions:
    True
    False
32. Pinned joints use pins pressed into holes to transmit forces (or torque):
    True
    False
33. Pinning parts together can help during alignment during manufacturing or assembly:
    True
    False
34. Line-bore holes for shafts and bearings by pinning or clamping plates together and drilling all the holes at once:
    True
    False
35. A riveted joint uses expanded members to transmit shear forces and resist peeling forces:
    True
    False
36. The expanding nature of a rivet allows many holes to be drilled in parts to be fastened together:
    True
    False

37. Deterministic designs are created using financial, time, and error budgets:
    <u>True</u>
    False
38. Hertz contact pressure is proportional to:
    <u>Force to the 1/3rd power</u>
    <u>Radius to the –2/3rd power</u>
    <u>Elastic Modulus to the 2/3rd power</u>
39. Hertz contact deflection is proportional to:
    <u>Force to the 2/3rd power</u>
    <u>Radius to the –1/3rd power</u>
    <u>Elastic Modulus to the –2/3rd power</u>
40. Hertz contact ellipse diameter is proportional to:
    <u>Force to the 1/3rd power</u>
    <u>Radius to the 1/3rd power</u>
    <u>Elastic Modulus to the –1/3rd power</u>
41. Kinematic couplings use linkages to compensate for errors between coupled components:
    True
    <u>False</u>
42. Three-groove kinematic couplings nominally have their grooves aligned along the angle bisectors of the imaginary triangle that connects the mating ball's centers:
    <u>True</u>
    False
43. A properly designed press or shrink fit generally is one of the best connections means between a shaft and component:
    <u>True</u>
    False
44. Extreme care should be taken when press or shrink-fitting a bearing onto a shaft because the strains could cause too much preload in the bearing:
    <u>True</u>
    False